ANIL

Super Digest MATHEMATICS

For Class 10

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Preface

We feel great pleasure in bringing out this thoroughly revised and updated edition of **'Anil Super Digest Mathematics'** for the students of **Class X.** Salient features of this book are :

- It has been updated according to latest syllabus prescribed by the Board of High School and Intermediate Education, Allahabad U. P. for Class X.
- It can be used by the elite group of scholars as well as the students and each user can go as far with it as he cares to.
- The principles and the subject-matter have been explained in simple and easy to understand language, without the sacrifice of any depth and precision.
- A number of solved examples have been provided alongwith each chapter so that the student may understand each and every type of question related with that chapter.

In spite of our best efforts, the possibilities of certain errors cannot be ruled out for which suggestions to improve upon will be gratefully acknowledged.

—Authors

CONTENTS

Unit 1 : Number System

1 Real Number

EXERCISE 1.1

Multiple Choice Type Questions

1. H.C.F of 650 and 1170 is :

(a) 130 (b) 140

(c) 80 (d) 160.

Sol. We start with the larger no. 1170. By euclid's division algorithm, we have

Dividend = (Divisor × Quotient + Remainder)

$$1170 = 650 \times 1 + 520$$

650)1170(1
650
520

We apply euclid's division algorithm on divisor 650 and the remainder 520.

Dividend = (Divisor × Quotient + Remainder)

$$650 = 520 \times 1 + 130$$

520)650(1
520
130

Again, we apply Euclid's division algorithm on divisor 520 and remainder 130.

Dividend = (Divisor × Quotient + Remainder)

$$520 = 130 \times 4 + 0$$

130)520(4
520
0

HCF (650, 1170) = 130. **Ans.**

2. HCF of 120, 224 and 256 is :

(a) 6 (b) 8

(c) 14 (d) 16.

Sol. We start with larger no. 256. By euclid's division algorithm, we have,

Dividend = (Divisor × Quotient + Remainder)

$$256 = 224 \times 1 + 32$$

224)256(1
224
32

We apply euclid's division algorithm on divisor 224 and the remainder 32.

Dividend = (Divisor × Quotient + Remainder)

$$120 = 32 \times 7 + 0$$

32)224(7
224
0

HCF (224, 256) = 32.

We apply euclid's division algorithm on divisor 24 and the remainder 8.

Dividend = (Divisor × Quotient + Remainder)

$$32 = 24 \times 1 + 8$$

24)32(1
24
8

We apply euclid's division algorithm on divisor 32 and the remainder 24.

Dividend = (Divisor × Quotient + Remainder)

$$24 = 8 \times 3 + 0$$

8)24(3
24
0

HCF (120, 224, 256) = 8. **Ans.**

3. HCF of 455 and 42 is :

(a) 5 (b) 11

(c) 7 (d) 9.

Sol. We start with larger no. 256. By euclid's division algorithm, we have,

Dividend = (Divisor × Quotient + Remainder)

$$455 = 42 \times 10 + 35$$

42)455(10
420
35

We apply euclid's division algorithm on divisor 35 and the remainder 7

Dividend = (Divisor × Quotient + Remainder)

$$42 = 35 \times 1 + 7$$

35)42(1
35
7

We apply euclid's division algorithm on divisor 42 and the remainder 35

Dividend = (Divisor × Quotient + Remainder)

$$35 = 7 \times 5 + 0$$

$$\begin{array}{r} 7\overline{)35}(5 \\ 35 \\ \hline 0 \end{array}$$

HCF (455, 42) = 7. **Ans.**

4. Find the greatest number which divides 260, 1314, and 1331 and leaves remainder 5 in each case.

Sol. Required number

= HCF of (260 – 5), (1314 – 5) and (1331 – 5) *i.e.,* required number

= HCF of 255, 1309, 1326

To find HCF of 255, 1309, 1326 first we will find HCF of 255 and 1309 and then we will find HCF of (255, 1309) and 1326.

Step 1. HCF of 255 and 1309 by using euclid's division algorithm is :

$$1309 = 255 \times 5 + 34$$
$$255 = 34 \times 7 + 17$$
$$34 = 17 \times 2 + 0$$

HCF of 1309, 255 is 17.

Step 2. Now, HCF of 17 and 1326 will be :

$$1326 = 17 \times 78 + 0$$

Hence, HCF (255, 1309, 1326) = 17

5. Find the greatest number which divides 121, 226 and 259 and leaves remainder 1,2 and 3 respectively.

Sol. Required number

= HCF of (121 – 1), (226 – 2) and (259 – 3) *i.e.* required number

= HCF of 120, 224, 256.

Step 1. To find HCF of 120, 224, 256 first we will find HCF of 120 and 224 and then we will find HCF of (120, 224) and 256.

by using euclid's division algorithm is :

$$224 = 120 \times 1 + 104$$
$$120 = 104 \times 1 + 16$$
$$104 = 16 \times 6 + 8$$
$$16 = 8 \times 2 + 0$$

HCF of 120, 224 is 8.

Step 2. Now HCF of 256 and 8 will be :

$$256 = 8 \times 32 + 0$$

Hence, HCF of (120, 244, 256) = 8.

Greatest number which divides 121, 226 and 259 and leaves remainder 1, 2 & 3 respectively is = 8.

6. Find the greatest possible number which can divide 76, 132 and 160 and leaves remainder same in each case.

Sol. Let the greatest number which can divide 76, 132 and 160 is = p

let the remainder = r

The quotient is equal to q_1, q_2 and q_3.

Dividend = Divisor × Quotient + Remainder

$$76 = p \times q_1 + r \quad ...(1)$$
$$132 = p \times q_2 + r \quad ...(2)$$
$$160 = p \times q_3 + r \quad ...(3)$$

From 1, 2 and 3 we get :

$$p(q_2 - q_1) = 132 - 76 = 56$$
$$p(q_3 - q_2) = 160 - 132 = 28$$
$$p(q_3 - q_1) = 160 - 76 = 84$$

Therefore the HCF of 56, 28 & 84 are :

$$56 = 2 \times 2 \times 2 \times 7$$
$$28 = 2 \times 2 \times 7$$
$$84 = 2 \times 2 \times 3 \times 7$$
$$\text{HCF} = 2 \times 2 \times 7 = 28$$

Therefore the greatest number which can divide 76, 132 & 160 leave the same remainder = 28.

7. Using Euclid's division algorithm find HCF of 274170 and 17017.

Sol. We have,

Dividend = 274170

Divisor = 17017

We start with larger number 274170.

By euclid's division algorithm, we have

Dividend = Divisor × Quotient + Remainder

$$274170 = 17017 \times 16 + 1898$$

$$\begin{array}{r} 17017\overline{)274170}(16 \\ 17017 \\ \hline 104000 \\ 102102 \\ \hline 1898 \end{array}$$

We apply euclid's division algorithm on divisor 17017 and the remainder 1898.

Dividend = Divisor × Quotient + Remainder

$17017 = 1898 \times 8 + 1833$

$$\begin{array}{r|l} 1898) & 17017 \,(8 \\ & \underline{15184} \\ & 1833 \end{array}$$

Again,we apply euclid's division algorithm on divisor 1898 and remainder 1833.

Dividend = Divisor × Quotient + Remainder

$1898 = 1833 \times 1 + 65$

$$\begin{array}{r|l} 1833) & 1898 \,(1 \\ & \underline{1833} \\ & 65 \end{array}$$

Again,we apply euclids division algorithm on divisor 1833 and remainder 65.

Dividend = Divisor × Quotient + Remainder

$1833 = 65 \times 28 + 13$

$$\begin{array}{r|l} 65) & 1833 \,(28 \\ & \underline{130} \\ & 533 \\ & \underline{520} \\ & 13 \end{array}$$

Now, we apply euclids division algorithm on divisor 65 and remainder 13.

Dividend = Divisor × Quotient + Remainder

$65 = 13 \times 5 + 0$

$$\begin{array}{r|l} 13) & 65 \,(5 \\ & \underline{65} \\ & 0 \end{array}$$

Thus, HCF of 274170 and 17017 is = 13.

8. Using euclid's division algorithm find HCF of 255, 1309 and 1326.

Sol. We have to find HCF of 255, 1309 and 1326 first we find HCF of 255 and 1309 then HCF of (225, 1309) and 1326.

We apply euclid's division on 255 and 1309.

Dividend = Divisor × Quotient + Remainder

$1309 = 255 \times 5 + 34$

$255 = 34 \times 7 + 17$

$17 = 17 \times 1 + 0$

Now, we find HCF of 17 and 1326 we apply euclid's division algorithm on 1326 and 17.

Dividend = Divisor × Quotient + Remainder

$1326 = 17 \times 78 + 0$

$\therefore$ Thus, HCF of 255, 1309, 1326 is = 17.

9. Show that the square of any positive integer can not be of the form $6q + 2$ or $6q + 5$ for any integer q.

Sol. Let a be an arbitary positive integer, then by Euclid's division algorithm, corresponding to the positive integer a and 6, there exist non-negative integers q and r such that,

$a = 6q + r$, where $0 < r < 6$

$a = 6q + r$, where $0 \le r < 6$

$\Rightarrow a^2 = (6q + r)^2$

$= 36q^2 + 12qr + r^2 [\because (a + b)^2 = a^2 + b^2 + 2ab]$

$\Rightarrow a^2 = 6(6q^2 + 2qr) + r^2$ ...(i)

where, $0 \le r < 6$

Case I : When $r = 0$, then putting $r = 0$ in eq. (i), we get $a^2 = 6(6q^2) = 6m$

Where $m = 6q^2$ is an integer.

Case II : When $r = 1$, then putting $r = 1$ in eq. (i), we get

$a^2 + 6(6q^2 + 2q) + 1 = 6m + 1$

Where, $m = (6q^2 + 2q)$ is an integer.

Case III : When $r = 2$, then putting

$r = 2$ in eq. (i), we get

$a^2 = 6(6q^2 + 4q) + 4 = 6m + 4$

Where, $m = (6q^2 + 4q)$ is an integer.

Case IV : When $r = 3$, then putting

$r = 3$ in eq. (i), we get

$a^2 = 6(6q^2 + 6q) + 9$

$= 6(6q^2 + 6q) + 6 + 3$

$\Rightarrow a^2 = 6(6q^2 + 6q + 1) + 3 = 6m + 3$

Where, $m = (6q^2 + 6q + 1)$ is an integer.

Case V : When $r = 4$, then putting

$r = 4$ in eq. (i), we get

$a^2 = 6(6q^2 + 8q) + 16$

$= 6(6q^2 + 8q) + 12 + 4$

$\Rightarrow a^2 = 6(6q^2 + 8q + 2) + 4 = 6m + 4$

Where, $m = (6q^2 + 8q + 2)$ is an integer.

Case VI : When $r = 5$, then putting $r = 5$ in eq. (i), we get

$$a^2 = 6(6q^2 + 10q) + 25$$
$$= 6(6q^2 + 10q) + 24 + 1$$
$$\Rightarrow a^2 = 6(6q^2 + 10q + 4) + 1 = 6m + 1$$

Where, $m = (6q^2 + 10q + 4)$ is an integer.

Hence,the square of any positive integer cannot be of the form of $6q + 2$ or $6q + 5$ for any integer q.

10. Show that the square of any positive integer can be in the form of $6q$, $6q + 1$, $6q + 3$, $6q + 4$.

Sol. Let a be any positive integer & $b = 6$ then, by euclid's division algorithm

$$a = 6m + r \; ; \; 0 \le r < 6$$

odd positive integer will be of the form

$$a = 6m + 1, 6m + 3, 6m + 5$$

Case I $\Rightarrow$Where $r = 1$ & $bq = 6m$

$$a^2 = (6m + 1)^2$$
$$= 36m^2 + 12m + 1$$
$$= 6(6m^2 + 2m) + 1$$
$$= 6q + 1;$$

where $q = 6m^2 + 2m$

Case II $\Rightarrow$Where $r = 3$ & $bq = 6m$

$$a^2 = (6m + 3)^2$$
$$= 36m^2 + 36m + 9$$
$$= 6(6m^2 + 6m + 1) + 3$$
$$= 6q + 3;$$

where $a = 6m^2 + 6m + 1$

Case III $\Rightarrow$Where $r = 5$ & $bq = 6m$

$$a^2 = (6m + 5)^2$$
$$= 36m^2 + 60m + 25$$
$$= 6(6m^2 + 10m + 4) + 1$$
$$= 6q + 1;$$

where $q = 6m^2 + 10m + 4$

Hence, passed that $6q$, $6q + 1$, $6q + 3$, $6q + 4$ can be is the form of squares of positive integer.

11. Show that cube of any positive integer is either of the form $4q$, $4q + 1$ or $4q + 3$.

Sol. Let a be the positive integer and $b = 4$.

Then, by Euclid's algorithm, $a = 4q + r$, for some integer $q \ge 0$ and $r = 0, 1, 2, 3$ because $0 \le r < 4$.

So, $a = 4q$ or $4q + 1$ or $4q + 2$ or $4q + 3$.

$$(4q)^3 = 64q^3 = 4(16q)^3$$
$$= 4q, \text{ where } q \text{ is some integer.}$$
$$(4q + 1)^3 = 64q^3 + 48q^2 + 12q + 1$$
$$= 4(16q^3 + 12q^2 + 3q) + 1$$
$$= 4q + 1, \text{ where } q \text{ is some integer.}$$
$$(4q + 3)^3 = 64q^3 + 144q^2 + 108q + 27$$
$$= 4(16q^3 + 36q^2 + 27q + 6) + 3$$
$$= 4q + 3, \text{ where } q \text{ is some integer.}$$

Hence, the cube of any positive integer is of the form $4q$, $4q + 1$, or $4q + 3$ for some integer q.

12. Show that one and only one out of n, $n + 4$, $n + 8$, $n + 12$ and $n + 16$ is divisible by 5 where n is any positive integer.

Sol. Let n, $n + 4$, $n + 8$, $n + 12$, $n + 16$ be integers. where n can take the form $5q$, $5q + 1$, $5q + 2$, $5q + 3$, $5q + 4$.

Case I : Where $n = 5q$, Then n is divisible by 5, but neither of $5q + 1$, $5q + 2$, $5q + 3$ & $5q + 4$, is divisible by 5.

Case II : Where $n = 5q + 1$, Then n is not divisible by 5

$$n + 4 = 5q + 1 + 4$$
$$= 5q + 5$$
$$= 5(q + 1)$$

which is divisible by 5.

Case III : Where $n = 5q + 2$, Then n is not divisible by 5

$$n + 8 = 5q + 2 + 8$$
$$= 5q + 10$$
$$= 5(q + 2)$$

which is divisible by 5.

Case IV : Where $n = 5q + 3$, Then n is not divisible by 5

$$n + 12 = 5q + 3 + 12$$
$$= 5q + 15$$
$$= 5(q + 3)$$

which is divisible by 5.

Case V : Where $n = 5q + 4$, Then n is not divisible by 5

$$n + 16 = 5q + 4 + 16$$
$$= 5q + 20$$
$$= 5(q + 4)$$

which is divisible by 5.

Hence, n, $n + 4$, $n + 8$, $n + 12$ and $n + 16$ is divisible by 5.

13. Prove that one and only one out of n, $n + 2$, $n + 4$ is divisible by 3 where n is an positive integer.

Sol. We applied euclid's division algorithm on n and 3.

$a = bq + r$ on putting $a = n$ and $b = 3$

$$n = 3q + r, 0 < r < 3 \text{ i.e.,}$$
$$n = 3q \quad ...(1)$$
$$n = 3q + 1 \quad ...(2)$$
$$n = 3q + 2 \quad ...(3)$$

$n = 3q$ is divisible by 3 or $n + 2 = 3q + 1 + 2 = 3q + 3$ also divisible by 3 or $n + 4 = 3q + 2 + 4 = 3q + 6$ is also divisible by 3.

Hence, n, $n + 2$, $n + 4$ are divisible by 3.

14. Show that square of any positive integer is of the form $4q$ or $4q + 1$ for some integer q.

Sol. Let positive integer $q = 4m + r$, by division algorithm we know here $0 \leq r$ 4, So, when $r = 0$

$$a = 4m$$

Squaring both side, we get

$$a^2 = (4m)^2$$
$$a^2 = 4(4m)^2$$
$$a^2 = 4q, \text{ where } q = 4m^2.$$

when, $r = 1$

$$a = 4m + 1$$

Squaring both side, we get

$$a^2 = (4m + 1)^2$$
$$a^2 = 16m^2 + 1 + 8m$$
$$a^2 = 4(4m^2 + 2m) + 1$$
$$a^2 = 4q + 1,$$

where $q = 4m^2 + 2m$

when $r = 2$

$$a = 4m + 2$$

Squaring both side, we get

$$a^2 = (4m + 2)^2$$
$$a^2 = 16m^2 + 4 + 16m$$
$$a^2 = 4(4m^2 + 4m + 1)$$
$$a^2 = 4a$$

when $q = 4m^2 + 4m + 1$

when $r = 3$

$$a = 4m + 3$$

Squaring both side, we get

$$a^2 = (4m + 3)^2$$
$$a^2 = 16m^2 + 9 + 24m$$
$$a^2 = 16m^2 + 24m + 8 + 1$$
$$a^2 = 4(4m^2 + 6m + 2) + 1$$
$$a^2 = 4q + 1$$

Where, $q = 4m^2 + 6m + 2$,

Hence, square of any positive integer is in form of $4q$ or $4q + 1$, where q is any integer.

15. There are 120 boys and 114 girls in class X of a school. Principal of the school decided as a policy matter to have maximum number of mixed sections, each section has to accommodate equal number of boys and equal number of girls. What is the maximum number of such sections ?

Sol. we have 120 boys and 114 girls.

To find maximum number of each section. We have to find H.C.F of 120 & 114.

First we take H.C.F. of largest number 120.

We apply euclid's division algorithm on 120 & 114.

Dividend = (Divisor × Quotient + Remainder)

$$120 = 114 \times 1 + 6$$

```
114)120(1
    114
    ---
      6
    ---
```

Now, we apply euclid's division algorithm on 114 & 6.

Dividend = (Divisor × Quotient + Remainder)

$$114 = 6 \times 19 + 0$$

```
6)114(19
   6
  ---
   54
   54
  ---
    0
  ---
```

Hence, maximum number of each section is = 6.

16. Show that square of any odd integer is of the form $4q + 1$ for some integer q.

Sol. We know that any positive odd integer of the form

$2m + 1$, $2m + 3$..........

Let a be any odd integer, then

$$a = 2m + 1$$

Squaring both sides, we get

$$a^2 = (2m + 1)^2$$
$$a^2 = 4m^2 + 4m + 1$$
$$a^2 = 4(m^2 + m) + 1$$
$$a^2 = 4q + 1$$

[where $(m^2 + m) = q$]

Hence, proved

17. HCF of 126 and 35 is H. If H is expressed as

$$\mathbf{H = 126 \times A + 35 \times B}$$

then prove that $\frac{A \times B}{H} = -2$.

Sol. HCF of 126 and 35 :

Prime factor of 35 = 5×7

Prime factor of 126 = $2 \times 3 \times 3 \times 7$

$\therefore$ Common factor = 7

$\therefore$ HCF = common factor = 7

Now, according to question,

$$\text{HCF} = 126A + 35B = 7$$
$$18 \times 7A + 5 \times 7B = 7$$

$18A + 5B = 1$, here many solutions possible because given one equation and two variables

Let $A = 2$ and $B = -7$

then, $18 \times 2 - 5 \times 7 = 1$

So, $A = 2$ and $B = -7$ is a solution of equation

Now, LHS = $\frac{A \times B}{H}$

Put $A = 2, B = -7$ and $H = 7$

Then, $\frac{A \times B}{H} = \frac{2 \times -7}{7} = -2$ = RHS

Hence, proved.

18. If the HCF of 210 and 55 is expressible in the form of 210 × 5 + 55 × A, then find the value of [2 – A].

Sol. Let us first find the HCF of 210 and 55. Applying euclid division algorithm on 210 and 55 we get,

$$210 = 55 \times 3 + 45$$
$$55 = 45 \times 1 + 10$$
$$45 = 10 \times 4 + 5$$
$$10 = 5 \times 2 + 0$$

We observe that the remainder at this stage is zero. So, the last division *i.e.*, 5 is the H.C.F of 210 and 55.

$\therefore$ $5 = 210 \times 5 + 55 \times A$

$$55 \times A = 5 - 1050$$
$$55 \times A = -1045$$
$$A = \frac{-1045}{55}$$
$$A = -19$$

$\therefore$ $2 - A = 2 - (-19)$

$= 2 + 19$

$= 21.$ **Ans.**

19. Find HCF of 81 and 237. Express the HCF in the form of 237 × *p* + 81 × *q*. Find the value of (3*p* + *q*).

Sol. H.C.F. of 81 and 237, by using Euclid's division Legorithm, As

$237 = 81 \times 2 + 75$ ...(1)

$81 = 75 \times 1 + 6$ ...(2)

$75 = 6 \times 12 + 3$ ...(3)

$6 = 3 \times 2 + 0$ ...(4)

Hence, H.C.F. of 81 and 237 is 3.

From equation (3), we get

$$3 = 75 - 6 \times 12$$

And, from eq. (2), we get

$$6 = 81 - 75 \times 1$$

So, $3 = 75 - (81 - 75 \times 1) \times 12$

$3 = 75 - (81 \times 12 - 75 \times 12)$

$3 = 75 \times 13 - 81 \times 12$

From eq. (1), we get

$$75 = 237 - 81 \times 2$$

So, $3 = (237 - 81 \times 2) \times 13 - 81 \times 12$

$3 = (237 \times 13 - 81 \times 26) - 81 \times 12$

$3 = (237 \times 13) (+ 81) \times -38$

[We need an expression of $237 \times p + 81 + q$]

Therefore, $p = 13, q = -38$

$3p + q = 3 \times 13 + (-38)$

$= 39 - 38$

$3p + q = 1$ **Ans.**

20. A mason has to fit a bathroom with square marble tiles of the largest possible size. The size of the bathroom is 14·5 ft by 12·5 ft. What would be the size of tiles in inches ? How many such tiles are required ?

Sol. Size of bathroom = 14·5 ft × 12·5 ft

1 feet = 12 inches

Hence, 14·5 feet = 14·5 × 12

= 174 inches

12·5 feet = 12·5 × 12

= 150 inches

To find the side of a square we need HCF (174 & 150) we apply euclid's division algorithm on 174 & 150

Dividend = Divisor × Quotient + Remainder

$174 = 150 \times 1 + 24$

$150 = 24 \times 6 + 6$

$24 = 6 \times 4 + 0$

Hence, HCF (174, 150) = 6

The size of tiles in inches = 6 inches.

Number of square tiles

$= \dfrac{\text{area of bathroom}}{\text{area of square tiles}}$

$= \dfrac{174 \times 150}{6 \times 6}$

= 725 tiles. **Ans.**

21. A bookseller purchased 117 books out of which 45 books are of mathematics and the remaining 72 books are of physics. Each book has same size. Mathematics and physics books are to be packed in saparate bundles and each bundle must contain same number of books. Find the least number of bundles which can be made for these 117 books.

Sol. We have given

maths books = 45

physics books = 72

To find least number of bundles we have to find HCF of 45 and 72.

To find HCF we apply euclid's division algorithm on 72 & 45

Dividend = Divisor × Quotient + Remainder

$72 = 45 \times 1 + 27$

$45 = 27 \times 1 + 18$

$27 = 18 \times 1 + 9$

$18 = 9 \times 2 + 0$

$\therefore$ HCF (72, 45) = 9

Therefore each bundle contain 9 books.

Bundles can be made

$= \dfrac{117}{9}$

= 13 Bundles. **Ans.**

EXERCISE 1.2

Multiple Choice Type Questions

1. Prime factors of 256 is :

(a) 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2

(b) 2 × 2 × 2 × 2 × 2

(c) 2 × 2 × 2 × 2 × 2 × 2

(d) 2 × 2 × 2

Sol. We use division method to find prime factor

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Prime factor of 256 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2. **Ans.**

2. Prime factor of 176 is :

(a) 2 × 2 × 2 × 11

(b) 2 × 2 × 2 × 2 × 11

(c) 3 × 3 × 9

(d) 2 × 2 × 2 × 13

Sol. We use division method to find prime factor

2	176
2	88
2	44
2	22
11	11
	1

Prime factor of 176 = 2 × 2 × 2 × 2 × 11. **Ans.**

3. Find the prime factor of 12673.

Sol. We use division method to find prime factor

19	12673
23	667
29	29
	1

Prime factor of 12673 = 19 × 23 × 29. **Ans.**

4. Given that HCF (1261, 1067) = 97, find L.C.M (1261, 1067).

Sol. We have,

HCF (1267, 1067) = 97

We know that,

Product of LCM & HCF = Product of two number

$97 \times \text{LCM} = 1261 \times 1067$

$\text{LCM} = \frac{1261 \times 1067}{97}$

$= 13871$

Hence, LCM (1261, 1067) = 13871. **Ans.**

5. Find the LCM and HCF of the following pairs of integers and verify : LCM × HCF = Product of two numbers.

(i) 26 and 91 (ii) 510 and 92

(iii) 336 and 54

Sol. To find LCM and HCF we find prime factors. To find prime factors we use division method :

(i)

2	26
13	13
	1

7	91
13	13
	1

Prime factors of 26 = 2 × 13.

Prime factors of 91 = 7 × 13.

LCM (26, 91) = 2 × 7 × 13 = 182.

HCF (26, 91) = 13.

LCM × HCF = Product of two numbers

182 × 13 = 26 × 91

2366 = 2366.

(ii)

2	510
3	255
5	85
17	17
	1

2	92
2	46
23	23
	1

Prime factors of 510 = 2 × 3 × 5 × 17.

Prime factors of 92 = 2 × 2 × 23.

LCM (510, 92) = 2 × 2 × 3 × 5 × 17 × 23 = 23460.

HCF (26, 9) = 2.

LCM × HCF = Product of two numbers

23460 × 2 = 510 × 92

46920 = 46920.

(iii)

2	336
2	168
2	84
2	42
3	21
7	7
	1

2	54
3	27
3	9
3	3
	1

Prime factors of 336 = 2 × 2 × 2 × 2 × 3 × 7.

Prime factors of 54 = 2 × 3 × 3 × 3

LCM (336, 54) = 2 × 2 × 2 × 2 × 3 × 3 × 3 × 7 = 3024.

HCF (336, 54) = 2 × 3. = 6.

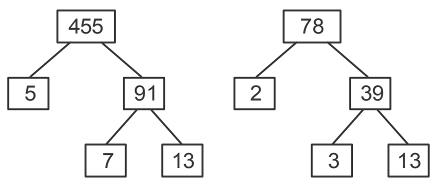
LCM × HCF = Product of two numbers

3024 × 6 = 336 × 54

18144 = 18144.

6. Find the LCM and HCF of the following pairs of integers by applying the fundamental theorem of arithmetic method : (i) 455, 78, (ii) 408, 170 (iii) 13, 11.

Sol. (i)



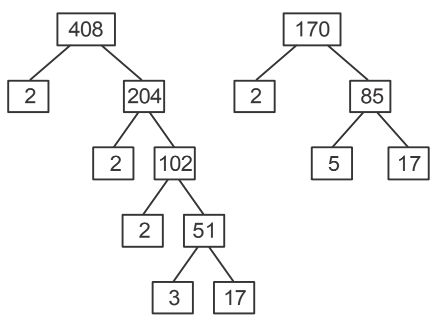
Prime factors of 455 = 5 × 7 × 13

Prime factors of 78 = 2 × 3 × 13

LCM (455, 78) = 2 × 3 × 5 × 7 × 13 = 2730

HCF (455, 78) = 13

(ii)



Prime factors of 408 = 2 × 2 × 2 × 3 × 17

170 = 2 × 5 × 17

LCM (408, 170)

= 2 × 2 × 2 × 3 × 5 × 17

= 2040.

HCF (408, 170) = 2 × 17 = 34.

(iii) 13 and 11 are prime numbers

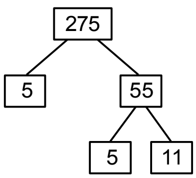
∴ Therefore, there are no prime factors of 13 and 11. Hence,

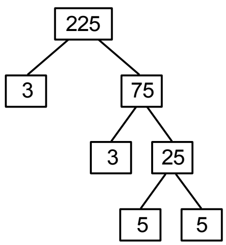
HCF (13, 11) = 1

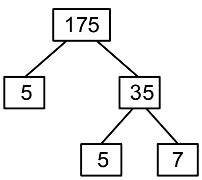
LCM (13, 11) = 13 × 11 = 143.

7. Find the LCM and HCF of the following integers by applying the prime factorisation method :
(i) 275, 225, 175
(ii) 765, 510, 408
(iii) 19, 13, 7

Sol. (i)







Prime factors of 275= 5 × 5 × 11

225 = 3 × 3 × 5 × 5

175 = 5 × 5 × 7

Hence, LCM (275, 225, 175)

= 5 × 5 × 3 × 3 × 7 × 11

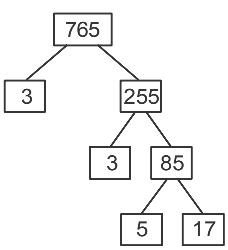
= 17325.

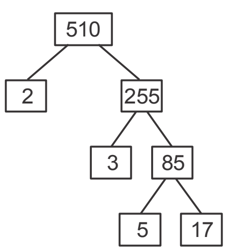
HCF (275, 225, 175)

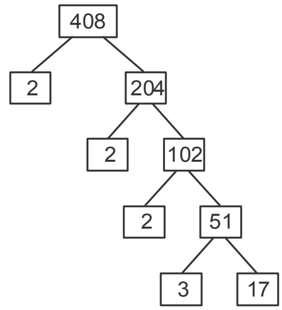
= 5 × 5

= 25.

(ii)







Prime factors of

765 = 3 × 3 × 5 × 17

510 = 2 × 3 × 5 × 17

408 = 2 × 2 × 2 × 3 × 17

LCM (765, 510, 408) = 2 × 2 × 2 × 3 × 3 × 5 × 17 = 6120.

HCF (765, 510, 408) = 3 × 17

= 51.

(iii) 19, 13, 7 are prime numbers. Therefore there are no prime factors of 19, 13, 7

∴ HCF (19, 13, 7) = 1

LCM (19, 13, 7) = 19 × 13 × 7

= 1729. **Ans.**

8. Explain why : 3 × 5 × 7 + 7 × 11 is a composite number.

Sol. A composite number is a number that is divisible by another number other then by itself and one. The number 2 is the only even prime number. All other even number are composite.

We have,

⇒ 3 × 5 × 7 + 7 × 11

⇒ 105 + 77

⇒ 182

182 is an even number and therefore it is a composite number. other than this 182 is the product of 2 × 7 × 13

Hence, it is a prime number.

9. Explain why : 5 × 7 × 11 + 13 × 17 is a composite number. Also find smallest divisor.

Sol. We have,

5 × 7 × 11 + 13 × 17

385 + 221

606.

Prime factors of 606 = 2 × 3 × 101

So, it is the product of more then two prime numbers 2, 3 and 101
Hence, it is a composite number and 2 is a smallest divisor of 606.

10. Check whether following numbers are prime or composite :
(i) 2 × 3 + 5
(ii) 7 × 11 × 13 + 13
(iii) 3 × 7 + 2
(iv) 13 × 17 × 19 + 23 × 38

Sol. (i) We have, 2 × 3 + 5
6 + 5 = 11
11 has no prime factors.
Hence, 11 is a prime numbers.
(ii) We have, 7 × 11 × 13 + 13
1001 + 13 = 1014
So, it is the product of more than two prime numbers 2, 3, and 13.
Hence, it is a composite number.
(iii) We have, 3 × 7 + 2
21 + 2 = 23
23 has no prime factors
Hence, 23 is a prime number.
(iv) We have, 13 × 17 × 19 + 23 × 38
4199 + 874 = 5073.
So, it is a product of more than two prime factors 3,19,89.
Hence, it is a composite number.

11. Check whether $(15)^n$ can end with the digit 0 for any $n \in$ N.

Sol. If $(15)^n$ ends with digit zero. Then number should be divisible by 2 and 5.
As, 2 × 5 = 10
⇒ This means the prime factorisation of $(15)^n$ should contain prime factors 2 and 5.
⇒ $(15)^n = (3 \times 5)^n$
It does not have prime factor 2 but have 3 and 5.
Since, 2 is not present in the prime factorisation there is no natural numbers nor which $(15)^n$ ends with digit zero.
So, $(15)^n$ can not end with digit zero.

12. Check whether $(28)^n$ can end with the digit 0 for any $n \in$ N.

Sol. If any number ends with the digit 0, It should be divisible by 10 or in other words, It will also be divisible by 2 and 5 as 2 × 5 = 10
It can be observed that 5 is not in the prime factorization of
28 = 2 × 2 × 7 or $(28)^n$
Hence, for any value of n, 28^n will not be divisible by 5.
Therefore, 28^n cannot end with the digit 0 for any natural number n.

13. Check whether $(26)^n$ can end with the digit 5 for any $n \in$ N.

Sol. A number to end with digit 5, the prime factorisation of a number must have 5 as its prime factor.
So, calculating prime factorisation of 26 we get,
26 = 2 × 13
Since, 26 has no prime factor 5 in its prime factorisation there is no natural number 'n' for which 26^n will end with 5.

Short Answer Type Questions

14. A rectangular field is 150 m × 60 m. Two cyclists Karan and Vijay start together and can cycle at speed of 21 m/min and 28 m/min, respectively. They cycle along the rectangular track, around the field from the same point and at the same moment. After how many minutes will they meet again at the starting point ?

Sol. $$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$
Speed ⇒ Karan = 21 m/min
Vijay = 28 m/min
Distance = Perimeter of rectangle
= (150 × 2) + (60 × 2)
= 300 + 120
Time taken by Karan = 420 m.
$$21 = \frac{420}{\text{Time}}$$
$$\text{Time} = \frac{420}{21}$$
= 20 min.
Time taken by Vijay

$$28 = \frac{420}{\text{Time}}$$

$$\text{Time} = \frac{420}{28}$$

$$= 15 \text{ min.}$$

So, they will meet : At the LCM of (20, 15)

Prime factors of

$$20 = 2 \times 2 \times 5$$

$$15 = 3 \times 5$$

$$\therefore \quad \text{LCM } (20, 15) = 2 \times 2 \times 3 \times 5$$

$$= 60.$$

∴ Therefore, they will meet again at the starting point after 60 minutes.

15. Radius of a circular track is 63 m. Two cyclists Amit and Ajit start together from the same position, at the same time and in the same direction with speeds 33 m/min and 44 m/min. After how many minutes they meet again at the starting point ?

Sol. Given speed : Amit = 33 m/min

Ajit = 44 m/min

Distance : Perimeter of circle

$$= 2\pi r$$

$$= 2 \times \frac{22}{7} \times 63$$

$$= 396 \text{ m.}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Time taken by Amit :

$$33 = \frac{396}{\text{Time}}$$

$$\text{Time} = \frac{396}{33}$$

$$= 12 \text{ min.}$$

Time taken by Ajit :

$$44 = \frac{396}{\text{Time}}$$

$$\text{Time} = \frac{396}{44}$$

$$= 9 \text{ min.}$$

So, they will meet = At the L.C.M. of (12, 9)

Prime factors of

$$12 = 2 \times 2 \times 3$$

$$9 = 3 \times 3$$

$$\text{LCM } (12, 9) = 2 \times 2 \times 3 \times 3$$

$$= 36.$$

Therefore, they will meet after 36 minutes.

16. Three sets of english, hindi and mathematics books have to be stacked in such a way that all the books are stored topic wise and the height of each stack is the same. The number of english books is 96, the number of hindi books is 240 and the number of mathematics books is 336. Assuming that the books are of the same thickness, determine the number of stacks of english, hindi and mathematics books. Depict values.

Sol. There are 3 sets of english, hindi and maths books which have to be stacked in such a way that the height of each stack is same.

There are 96 english books

240 hindi books

336 maths books

First we will find HCF of (96, 240, 336)

Prime factorisation

$$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$$

$$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$$

$$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$$

HCF (96, 240, 336)

$$= 2 \times 2 \times 2 \times 2 \times 3 = 48$$

Now we are going to divide the total number of books of each subject by the HCF which is 48.

English books (number of stacks)

$$= 96 \div 48$$

$$= 2.$$

Hindi books (number of stacks)

$$= 240 \div 48$$

$$= 5.$$

Maths books (number of stacks)

$$= 336 \div 48$$

$$= 7.$$ **Ans.**

EXERCISE 1.3

Short Answer Type Questions

1. Prove that 2 + $\sqrt{6}$ is an irrational number.

Sol. Let us assume, to the contrary $2+\sqrt{6}$ as rational number

Now, let $2+\sqrt{6} = \frac{p}{q}$

(where p & q are co-primes and $q \neq 0$)

$$\sqrt{6} = \frac{p}{q}-2$$

$$\sqrt{6} = \frac{p-2q}{q}$$

Since, p and q are integers, therefore, $\frac{p-2q}{q}$ is a rational number.

By fact $\sqrt{6}$ is irrational number. our assumption is wrong.

Hence, it is proved that $2+\sqrt{6}$ is irrational no.

2. Prove that $3\sqrt{11}$ is an irrational number.

Sol. We have to prove $3\sqrt{11}$ is irrational number

Let us assume the opposite.

i.e., $3\sqrt{11}$ is a rational number

Hence, $3\sqrt{11}$ can be written in form of $\frac{p}{q}$

Where p and q ($q \neq 0$) are co-prime (no common factor other then 1)

Hence, $3\sqrt{11} = \frac{p}{q}$

$$\sqrt{11} = \frac{1}{3}\times\frac{p}{q}$$

$$\sqrt{11} = \frac{p}{3q}$$

Here, $\frac{p}{3q}$ is a rational no.

But $\sqrt{11}$ is a irrational no.

Since irrational ≠ rational

∴ Our assumption is wrong

Hence, $3\sqrt{11}$ is irrational.

Proved

3. Prove that $(\sqrt{3}-\sqrt{8})$ is an irrational no.

Sol. Let $(\sqrt{3}-\sqrt{8})$ be any rational number x.

$$x = \sqrt{3}-\sqrt{8}$$

Squaring both the sides

$$x^2 = (\sqrt{3}-\sqrt{8})^2$$

$$x^2 = 3 + 8 - 2\sqrt{24}$$

$$x^2 = 11 - 2\sqrt{24}$$

$$x^2 - 11 = -2\sqrt{24}$$

$$\frac{x^2-11}{2} = -\sqrt{24}$$

as x is a rational no. So x^2 is also a rational number, 11 & 2 are rational numbers. So $-\sqrt{24}$ must also be a rational number as a quotient of two rational numbers is also a rational number. But $-\sqrt{24}$ is a irrational number.

So, we arrive to a contradiction

This shows our assumption is wrong.

So, $\sqrt{3}-\sqrt{8}$ is an irrational number.

4. Prove that $3\sqrt{10}$ is an irrational number.

Sol. Let us assume $3\sqrt{10}$ is an rational no.

So, it can be expressed in the form of $\frac{a}{b}$ form where a and b are integers.

So; $3\sqrt{10} = \frac{a}{b}$

$$\sqrt{10} = \frac{a}{3b}$$

Since, a and b are integer, so $\frac{a}{3b}$ is a rational number. Which implies that $3\sqrt{10}$ is also a rational number.

But it contridicts the facts that $\sqrt{10}$ is an irrational number.
Therefore, out assumption is wrong.

Hence, it is proved that $3\sqrt{10}$ is an irrational number.

5. Prove that $\sqrt{5}$ is an irrational number.

Sol. We have to prove $\sqrt{5}$ is an irrational numbers.

Let us assume the opposite *i.e.* $\sqrt{5}$ is a rational number.

Hence, $\sqrt{5}$ can be written in the form of $\frac{a}{b}$ where a & b $(b \neq 0)$ be are co-prime.
(non common factor other than 1)

Hence, $\sqrt{5} = \frac{a}{b}$

$$\sqrt{5}b = a$$

Squaring both sides

$$(\sqrt{5}b)^2 = a^2$$
$$5b^2 = a^2$$
$$b^2 = \frac{a^2}{5}$$

Hence, 5 divides a^2
By theorem : If p is a prime *no* and p divides a where a is positive numbers
So, 5 shall divide 'a' also ...(1)
Hence, we can say that

$\frac{a}{5} = c$, where c is same integer

So, $a = 5c$
Now, we know that
$$5b^2 = a^2$$
Putting $a = 5c$
$$5b^2 = (5c)^2$$
$$5b^2 = 25c^2$$
$$b^2 = 5c^2$$
$$\frac{b^2}{5} = c^2$$

Hence, 5 divides b^2
By theorem : If p is prime number and p divides a^2, then p divides a, where a is positive number
So, 5 divides b also ...(2)
By (1) and (2)
5 divides both a & b
Hence, 5 is a factor of a & b
So, a & b have a factor of 5.
Therefore, a & b are not co-prime.
Hence, our assumption is wrong.
$\therefore$ By contradiction it is proved that $\sqrt{5}$ is an irrational number.

6. Prove that $\sqrt{7}-8$ is an irrational number.

Sol. Let us assume that $\sqrt{7}-8$ is irrational no.

$$\Rightarrow \therefore \sqrt{7}-8 = 8 = \frac{a}{b}$$
$$\sqrt{7} = \frac{a}{b}+8$$

$\frac{a}{b}$ and 8 are rational numbers

But $\sqrt{7}$ is an irrational number.
It is a contradiction
Therefore, our assumption $\sqrt{7}-8$ is an irrational number.

7. Prove that $\frac{5}{\sqrt{2}}$ is an irrational number.

Sol. To prove $\frac{5}{\sqrt{2}}$ is irrational

Let us assume that $\sqrt{2}$ as rational

$$\frac{1}{\sqrt{2}} = \frac{p}{q} \text{ (where } p \text{ and } q \text{ are co-prime)}$$
$$q = \sqrt{2}p$$

Squaring both the sides

$$q^2 = 2p^2 \quad ...(1)$$

By theorem

a is divisible by 2

$\therefore \quad a = 2c$

(where c is an integer)

Putting the value of 2 in equation 1

$$2p^2 = q^2 = 2c^2 = 4c^2$$

$$p^2 = \frac{4c^2}{2}$$

$$= 2c^2$$

$$\frac{p^2}{2} = c^2$$

By theorem p is also divisible by 2

But p are co-prime.

This is contradiction which has arisen due the our assumption wrong.

$\therefore \frac{5}{\sqrt{2}}$ is an irrational number.

8. Prove that $\sqrt{13}+\sqrt{17}$ is an irrational number.

Sol. We can prove this by method of contradiction.

Let us assume that $\sqrt{13}$ is a rational number.

$$\sqrt{13} = \frac{q}{p}$$

Squaring both the sides

$$13 = \frac{q^2}{p^2}$$

$$13p^2 = q^2 \quad ...(1)$$

q^2 is a multiple of 13

$\therefore q$ is also a multiple of 13

Let $q^2 = 13\,x$, where x is an integer

Put in (1)

$$13p^2 = (13\,x)^2$$

$$13p^2 = 169\,x^2$$

$$p^2 = \frac{169}{13}\,x^2$$

$$p^2 = 13x^2$$

p^2 is a multiple of 13

$\therefore p$ is also a multiple of 13

But this contradicts our assumption

Hence, our assumption is wrong.

So, $\sqrt{13}$ is an irrational number.

Hence, it is proved that $\sqrt{13}+\sqrt{17}$ is an irrational number.

EXERCISE 1.4

Multiple Choice Type Questions

1. The decimal expansion of $\frac{141}{120}$ will terminate after how many places of decimals ?

(a) 3 (b) 4

(c) 1 (d) 2.

Sol. $\frac{141}{120}$ first it can be expressed in simplest form $\frac{47}{40}$.

Now, $\frac{47}{40} = 1{\cdot}175.$

So, it will terminate after 3 digit of decimals. **Ans.**

2. The decimal expansion of $\frac{131}{120}$ will terminate after how many places of decimals.

(a) 3 (b) 4

(c) 1 (d) 2.

Sol. $\frac{131}{120} = 1{\cdot}091.$

So, it will terminate after 3 digits. **Ans.**

3. The decimal expansion of the rational number $\frac{11}{2^3{\cdot}5^2}$ will terminate after :

(a) One decimal place

(b) Two decimal places

(c) Three decimal places

(d) More than three decimal places.

Sol.
$$\frac{11}{2^3 \times 5^2} = \frac{11}{2\times2\times2\times5\times5}$$
$$= \frac{11}{200}$$
$$= 0{\cdot}055.$$

So, $\frac{11}{2^3{\cdot}5^2}$ will terminate after 3 decimal places.

4. The decimal expansion of the rational number $\frac{43}{2^4 \times 5^3}$ will terminate after :

(a) 3 places (b) 4 places

(c) 5 places (d) 1 place.

Sol. $\frac{43}{2^4 \times 5^3} = \frac{43}{2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5}$

$= \frac{43}{2000}$

$= 0{\cdot}0215$

So, it will terminate after 4 places. **Ans.**

5. The decimal expansion of the rational numbers $\frac{23}{2^2 \cdot 5}$ will terminate after :

(a) one decimal place

(b) two decimal places

(c) three decimal places

(d) more than three decimal places.

Sol. $\frac{23}{2^2 \times 5} = \frac{23}{2 \times 2 \times 5}$

$= \frac{23}{20}$

$= 1{\cdot}15.$

So, it will terminate after 2 decimal places.

Short Answer Type Questions

6. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion.

(i) $\frac{54}{343}$ (ii) $\frac{35}{800}$

(iii) $\frac{127}{910}$ (iv) $\frac{23}{2048}$

(v) $\frac{19}{250}$

(vi) $\frac{922}{2^1 \times 5^2 \times 7^3 \times 11^4}$

(vii) $\frac{1001}{2^0 \times 5^3 \times 11^5}$

(viii) $\frac{328}{2^2 \times 5^3 \times 7^4}$

(ix) $\frac{1111}{7^4 \times 13^2}$

(x) $\frac{52}{2^3 \times 5^4}$

Sol. (i) $\frac{54}{343}$ = Since, the factor of denominator 343 is not in the form $2^n \times 5^m$. Therefore, $\frac{54}{343}$ is a non terminating repeating decimal.

(ii) $\frac{35}{800}$ = Since, the factor of denominator 800 are $2^5 \times 5^2$. Therefore, $\frac{35}{800}$ is a terminating.

(iii) $\frac{127}{910}$ = Since, the factor of denominator 910 are $5^1 \times 2^1 \times 7^1 \times 13^1$. Therefore, $\frac{127}{910}$ is a non terminating repeating decimal.

(iv) $\frac{23}{2048}$ = Since, the factor of denominator 2048 are $2^{11} \times 5^0$. Therefore, $\frac{23}{2048}$ is terminating decimal.

(v) $\frac{19}{250}$ = Since, the factor of denominator 250 are $2^1 \times 5^3$. Therefore, $\frac{19}{250}$ is a terminating decimal.

(vi) $\frac{922}{2^1 \times 5^2 \times 7^3 \times 11^4}$ = Since, the factor of denominator are $2^1 \times 5^2 \times 7^3 \times 11^4$. Therefore, $\frac{922}{2^1 \times 5^2 \times 7^3 \times 11^4}$ are non terminating repeating decimal.

(vii) $\frac{1001}{2^0 \times 5^3 \times 11^5}$ = Since, the factor of denominator are $2^0 \times 5^3 \times 11^5$. Therefore, $\frac{1001}{2^0 \times 5^3 \times 11^5}$ are non terminating repeating decimal.

(viii) $\frac{328}{2^2 \times 5^3 \times 7^4}$ = Since, the factor of denominator are $2^2 \times 5^3 \times 7^4$.

Therefore, $\frac{328}{2^2 \times 5^3 \times 7^4}$ are non terminating repeating decimal.

(ix) $\frac{1111}{7^4 \times 13^2}$ = Since, the factor of denominator are $7^4 \times 13^2$

Therefore, $\frac{1111}{7^4 \times 13^2}$ are non terminating repeating decimal.

(x) $\frac{52}{2^3 \times 5^4}$ = Since, the factor of denominator are $2^3 \times 5^4$. Therefore $\frac{52}{2^3 \times 5^4}$ is terminating factor.

7. Which of the following decimal expansions are rational. If they are rational $\frac{p}{q}$, then write down the prime factors of q.

(i) 0·240240024000240000

(ii) $11 \cdot 214\overline{78}$

(iii) $12.\overline{123456789}$

(iv) 125·085

(v) 23·243456789

Sol. (i) 0·240240024000240000 is non terminating non repeating decimal. Therefore, it is irrational number.

(ii) $11 \cdot 214\overline{78}$ is repeating decimal. So it represents as rational number.

$$11 \cdot 214\overline{78} = \frac{1121478}{99000}$$

$q = 99000$
$= 2^3 \times 3^2 \times 5^3 \times 11.$

(iii) $12.\overline{123456789}$ → It is repeating decimal. Therefore, it is a rational no.

$$12.\overline{123456789} = \frac{12123456789}{999999999}$$

$q = 999999999$
$= 3^2 \times 111111111.$

(iv) 125·085 is terminating decimal. So, it represents a rational number.

$$125 \cdot 085 = \frac{125085}{1000}$$

$q = 1000$
$= 2^3 \times 5^3.$

(v) 23·243456789 is terminating decimal. So, it represents a rational number.

$$23 \cdot 243456789 = \frac{23243456789}{1000000000}$$

$q = 1000000000$
$= 2^9 \times 5^9.$

8. A rational number in its decimal expansion is 327·7081. What can you say about the prime factor of q. When this number is expressed in the form p/q ? Give reasons.

Sol. We know that terminating decimal must be the one which has a finite decimal digits which ends exactly at a point and we can not find any repetitions of the decimal values thereafter. The terminating decimal can be represented in the form of a fraction. Since, the value of the fraction terminates at some point. Since, the rational number 327·7081 is a terminating decimal, it has to be in the form of a/b where, b is the denominator must be of structure $2^x \times 5^y$.

9. Without actually performing the long division, find if $\frac{987}{10500}$ will have terminating or non terminating (repeating) decimal expansion. Give reasons for your answers.

Sol.
$$\frac{987}{10500} = \frac{47}{500}$$
$$= \frac{47}{5^3 \times 2^2} \times \frac{2}{20}$$
$$= \frac{94}{5^3 \times 2^3}$$
$$= \frac{94}{10^3}$$
$$= \frac{94}{1000}$$
$$= 0 \cdot 094$$

A rational no. in the form of p/q with p & q co-primes $q \neq 0$ can be expressed in the form of terminating decimal expansion if q has either only 2^5 or only 5^5 or both.

Hence, this is a terminating decimal factor.

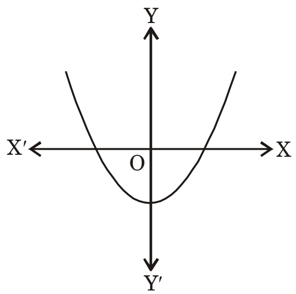
❑

Unit 2 : Algebra

2 Polynomials

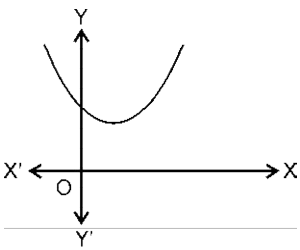
EXERCISE 2.1

1. The graph of $y = p(x)$ are given below. For which of these $p(x)$ is linear or quadratic ? Also find the number of zeroes of $p(x)$ in each case.



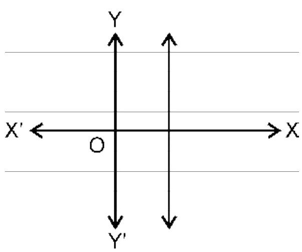
Sol. It is quadratic polynomial.
The number of zeroes is 2 as the graph intersects the x-axis at 2 points.

2.



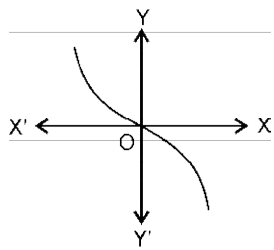
Sol. It is a quadratic polynomial.
The number of zeroes is 0, as the graph does not intersects at x-axis.

3.



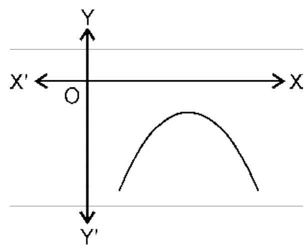
Sol. It is a linear polynomial.
The number of zeroes is 1 as the graph intersect the x-axis at one point only.

4.



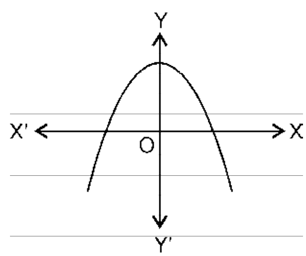
Sol. This polynomial is neither quadratic non linear.
The number of zeroes is 1 as the graph intersect the x-axis at one point only.

5.



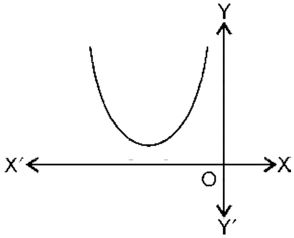
Sol. This polynomial is a quadratic polynomial.
The number of zeroes is 0 as the graph does not intersect the x-axis.

6.



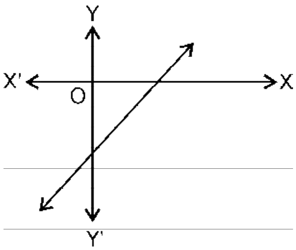
Sol. This polynomial is a quadratic polynomial.
The number of zeroes is 2, as the graph intersect the x-axis at two points.

7.



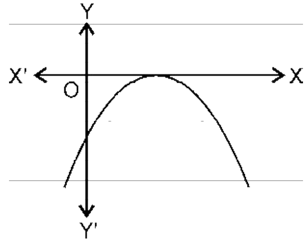
Sol. This polynomial is quadratic polynomial.
The number of zeroes is 0 as it does not touches the x-axis.

8.



Sol. This polynomial is linear polynomial.
The number of zeroes in this polynomial is 1 as it touches x-axis at one point only.

9.



Sol. This polynomial is quadratic polynomial.
The number of zeroes in this polynomial is 1 as it touches x-axis at one point only.

EXERCISE 2.2

Multiple Choice Type Questions

1. The number of polynomials having zeroes -2 and 5 is :

(a) 1 (b) 2
(c) 3 (d) More than 3.

Sol. Let $p(x) = ax^2 + bx + c$ be the required polynomial whose zeroes are –2 and 3.

$\therefore$ Sum of zeroes $= \frac{-b}{a}$

$$\Rightarrow \quad \frac{-b}{a} = -\left(\frac{-2+5}{1}\right)$$

$$= \frac{2-5}{1}$$

$$= -\frac{3}{1} \quad \text{...(1)}$$

and product of zeroes $= \frac{c}{a}$

$$\frac{c}{a} = -2 \times 5$$

$$= \frac{-10}{1} \quad \text{...(2)}$$

From equations (i) and (ii)

$$a = 1, b = -3$$

and $$c = -10$$

$$p(x) = ax^2 + bx + c$$
$$= 1 \cdot x^2 - 3x - 10$$
$$= x^2 - 3x - 10$$

But we know that, if we multiply or divide any polynomial by any arbitary constant. Then, the zeroes of polynomial never change.

$\therefore \quad p(x) = kx^2 - 3kx - 10k$

(where, k is real no.)

$$p(x) = \frac{x^2}{k} - \frac{3}{k}x - \frac{10}{k}$$

(where k is non zero real no.)

Hence, the required number of polynomial are infinite *i.e.,* more than 3.

2. If 1 is zero of the polynomial $p(x) = ax^2 - 3(a-1)x - 1$, then the value of '$a$' is :

(a) 1 (b) – 1
(c) 2 (d) – 2.

Sol. $p(x) = ax^2 - 3(a-1)x - 1$

$$a(1)^2 - 3(a-1)1 - 1 = 0$$
$$a - 3a + 3 - 1 = 0$$
$$-2a + 2 = 0$$
$$2 = 2a$$
$$a = 1.$$

3. If α, β are zeroes of $x^2 - 6x + k$. What is the value of k if $3\alpha + 2\beta = 20$.

(a) – 16 (b) 8

(c) – 2 (d) – 8.

Sol. Polynomial $p(x) = x^2 - 6x + k$

Given α & β are the roots.

To find k, if $3\alpha + 2\beta = 20$ ...(1)

From the quadratic expression

$$\alpha + \beta = 6 \quad ...(2)$$

and $$\alpha\beta = k \quad ...(3)$$

Multiply equation (2) by 2 and subtracting by equ. (1) :

$$\alpha = 20 - 12 = 8.$$

Substitute the value of α in (2) to get :

$$\beta = 6 - 8 = -2.$$

Substitute these value of α and β in equ. (3) we get :

$$k = \alpha\beta = -16.$$ **Ans.**

4. If one zero of $2x^2 - 3x + k$ is reciprocal to the other, then the value of k is :

(a) 2 (b) $\frac{-2}{3}$

(c) $\frac{-3}{2}$ (d) – 3.

Sol. Given—$2x^2 - 3x + k$ ...(1)

Let the 2 zeroes are a & $\frac{1}{a}$

Quadratic form :

$ax^2 + bx + c$ ...(2)

On comparing equation (1) & (2) we get :

$$a = 2,\ b = -3,\ c = k$$

$$\text{Product of zeroes} = a \times \frac{1}{a} = 1$$

$$\text{Product of zeroes} = \frac{c}{a}$$

$$1 = \frac{k}{2}$$

$$k = 2$$

$\therefore$ The value of $k = 2$. **Ans.**

5. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficient.

(a) $3z^2 - 2z - 1$ (b) $8y^2 - 3y$

(c) $9x^2 - 5$ (d) $x^2 - 7x - 8$.

Sol. (a) $3z^2 - 2z - 1$

we have, $3z^2 - 2z - 1$,

where $a = 3$, $b = -2$, $c = -1$

$$3z^2 - 2z - 1 = 0$$

$$3z(z - 1) + 1(z - 1) = 0$$

$$(3z + 1)(z - 1) = 0$$

$$3z + 1 = 0$$

$$z - 1 = 0$$

$$z = \frac{-1}{3}$$

$$z = 1.$$

Verification :

(1) Sum of zeroes

$$= -\frac{\text{coefficient of } z}{\text{coefficient of } z^2}$$

$$\frac{-1}{3} + 1 = -\frac{(-2)}{3}$$

$$\frac{2}{3} = \frac{2}{3}.$$

(2) Product of zeroes.

$$= \frac{\text{constant term}}{\text{coefficient of } z^2}$$

$$\frac{-1}{3} \times 1 = \frac{-1}{3}$$

$$\frac{-1}{3} = \frac{-1}{3}.$$

Sol. (b) $8y^2 - 3y$

we have, $8y^2 - 3y$; where $a = 8$, $b = -3$, $c = 0$

$$8y^2 - 3y = 0$$

$$y(8y - 3) = 0$$

$$8y - 3 = \frac{0}{y}$$

$$8y - 3 = 0$$

$$8y = 3$$

$$y = \frac{3}{8}$$

$$y = 0$$

Verification :

(1) Sum of zeroes

$$= -\frac{\text{coefficient of } y}{\text{coefficient of } y^2}$$

$$\frac{3}{8} + 0 = -\frac{(-3)}{8}$$

$$\frac{3}{8} = \frac{3}{8}.$$

(2) Product of zeroes

$$= \frac{\text{constant term}}{\text{coefficient of } y^2}$$

$$\frac{3}{8} \times 0 = \frac{0}{8}$$

$$0 = 0.$$

Sol. (c) $9x^2 - 5$

we have, $9x^2 - 5$, where $a = 9$, $b = -5$, $c = 0$

$$9x^2 - 5 = 0$$

$$9x^2 = 5$$

$$x^2 = \frac{5}{9}$$

$$x = \pm\sqrt{\frac{5}{9}}$$

$$x = \frac{\sqrt{5}}{3}$$

$$x = -\frac{\sqrt{5}}{3}.$$

Verification :

(1) Sum of zeroes

$$= -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\frac{\sqrt{5}}{3} - \frac{\sqrt{5}}{3} = -\frac{0}{9}$$

$$0 = 0.$$

(2) Product of zeroes

$$= \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$$\frac{\sqrt{5}}{3} \times \frac{-\sqrt{5}}{3} = \frac{-5}{9}$$

$$\frac{-5}{9} = \frac{-5}{9}.$$

Sol. (d) $x^2 - 7x - 8$

we have, $x^2 - 7x - 8$, where $a = 1$, $b = -7$, $c = -8$

$$x^2 - 7x - 8 = 0$$

$$x^2 - 8x + x - 8 = 0$$

$$x(x - 8) + 1(x - 8) = 0$$

$$(x + 1)(x - 8) = 0$$

$$x + 1 = 0$$

$$x - 8 = 0$$

$$x = -1$$

$$x = 8$$

Verification :

(1) Sum of zeroes

$$= -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$-1 + 8 = -\frac{(-7)}{1}$$

$$7 = 7.$$

(2) Product of zeroes

$$= \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$$-1 \times 8 = \frac{-8}{1}$$

$$-8 = -8.$$

6. Find the polynomials whose zeroes are given as under :

$\alpha = 2$, $\beta = 3$

Sol. Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β

Here $\alpha + \beta = 2 + 3 = 5$

& $\alpha\beta = 2 \times 3 = 6$

Thus, the polynomial formed = x^2 – (sum of zeroes) x + product of zeroes

$x^2 - 5x + 6.$

7. **$\alpha = -4$, $\beta = 5$**

Sol. Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β

Here $\alpha + \beta = -4 + 5 = 1$

$\alpha\beta = -4 \times 5 = -20$

Thus, the polynomial formed = x^2 – (sum of zeroes) x + product of zeroes

$x^2 - x - 20.$

8. **$\alpha = -1$, $\beta = -2$**

Sol. Let the polynomial be $ax^2 + bx + c$ and the zeroes be α and β

Here $\alpha + \beta = -1 + (-2) = -3$

$\alpha\beta = -1 \times -2 = 2$

Thus, the polynomial formed = x^2 – (sum of zeroes) x + product of zeroes

$x^2 - (-3x) + 2.$

$x^2 + 3x + 2.$

9. **$\alpha = 0$, $\beta = 5$**

Sol. Let the polynomial be $ax^2 + bx + c$ and the zeroes be α and β

Here $\alpha + \beta = 0 + 5 = 5$

$\alpha\beta = 0 \times 5 = 0$

Thus, the polynomial formed =

$= x^2 -$ (sum of zeroes) x + product of zeroes

$= x^2 - 5x + 0$

$= x^2 - 5x.$ **Ans.**

10. $\alpha = 4, \beta = -4$

Sol. Let the polynomial be $ax^2 + bx + c$ and the zeroes be α and β

Here, $\alpha + \beta = 4 + (-4) = 0$

$\alpha\beta = 4 \times -4 = -16$

Thus, the polynomial formed

$= x^2 -$ (sum of zeroes) x + product of zeroes

$x^2 - 0x + (-16)$

$x^2 - 16.$ **Ans.**

11. $\alpha = \frac{1}{2}\ \beta = \frac{1}{2}$

Sol. Let the polynomial be $ax^2 + bx + c$ and the zeroes be α and β

Here, $\alpha + \beta = \frac{1}{2} + \frac{1}{2} = 1$

$\alpha\beta = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Thus, the polynomial formed

$x^2 -$ (sum of zeroes) x + product of zeroes

$x^2 - 1x + \frac{1}{4}$

$x^2 - x + \frac{1}{4}.$ **Ans.**

12. $\alpha = \frac{-5}{3}, \beta = \frac{-5}{3}$

Sol. Let the polynomial be $ax^2 + bx + c$ and the zeroes be α and β

Here, $\alpha + \beta = \frac{-5}{3} + \left(\frac{-5}{3}\right) = -\frac{10}{3}$

$\alpha\beta = \frac{-5}{3} \times \frac{-5}{3} = \frac{25}{9}$

Thus, the polynomial formed

$= x^2 -$ (sum of zeroes) x + product of zeroes

$= x^2 - \left(\frac{-10}{3}\right)x + \frac{25}{9}$

$= x^2 + \frac{10}{3}x + \frac{25}{9}.$ **Ans.**

EXERCISE 2.3

Multiple Choice Type Questions

1. If $p(x) = x^2 + 6x + 9$ and $q(x) = x + 3$ then remainder will be when $p(x)$ is divided by $q(x)$:

(a) – 1 (b) 0

(c) 11 (d) 2.

Sol. Dividend $= x^2 + 6x + 9$, Divisor $= x + 3$

Here, dividend and divisor both in standard form so, we have

$$\begin{array}{r|l} x+3 & x^2+6x+9 \;(\; x+3 \\ & x^2+3x \\ & -\quad - \\ \hline & 3x+9 \\ & 3x+9 \\ & -\quad - \\ \hline & 0 \end{array}$$

So, the remainder will be 0.

2. Dividing $(x^3 + 1)$ by $(x + 1)$ the remainder will be :

(a) – 1 (b) 11

(c) 0 (d) – 2.

Sol.

$$\begin{array}{r|l} x+1 & x^3+1 \;(\; x^2-x+1 \\ & x^3+x^2 \\ & -\quad - \\ \hline & -x^2+1 \\ & -x^2-x \\ & +\quad + \\ \hline & x+1 \\ & x+1 \\ & -\quad - \\ \hline & 0 \end{array}$$

So, the remainder is 0.

3. Dividing $x^3 + 3x + 3$ by $(x + 2)$, the remainder will be :

(a) – 2

(b) – 11

(c) 0

(d) 1.

Sol.

$$\begin{array}{r|l} & x^2 - 2x + 7 \\ \hline x+2 & x^3 + \quad 3x + 3 \\ & x^3 + 2x^2 \\ & - \quad - \\ \hline & -2x^2 + 3x + 3 \\ & -2x^2 - 4x \\ & + \quad + \\ \hline & 7x + 3 \\ & 7x + 14 \\ & - \quad - \\ \hline & -11 \end{array}$$

So, the remainder is –11.

4. **Dividing $x^3 - 7x + 6$ by $(x - 1)$, the remainder will be :**

(a) 6 (b) – 6

(c) 1 (d) 0.

Sol.

$$\begin{array}{r|l} & x^2 + x - 6 \\ \hline x-1 & x^3 - 7x + 6 \\ & x^3 - x^2 \\ & - \quad + \\ \hline & x^2 - 7x + 6 \\ & x^2 - x \\ & - \quad + \\ \hline & -6x + 6 \\ & -6x + 6 \\ & + \quad - \\ \hline & 0 \end{array}$$

So, the remainder will be 0.

5. **Dividing $x^3 - 3x^2 - x + 3$ by $x^2 - 4x + 3$ the remainder will be :**

(a) – 3 (b) 3

(c) 1 (d) 0.

Sol.

$$\begin{array}{r|l} & x + 1 \\ \hline x^2 - 4x + 3 & x^3 - 3x^2 - x + 3 \\ & x^3 - 4x^2 + 3x \\ & - \quad + \quad - \\ \hline & x^2 - 4x + 3 \\ & x^2 - 4x + 3 \\ & - \quad + \quad - \\ \hline & 0 \end{array}$$

So, the remainder will be 0.

Short Answer Type Questions

6. **In each of the following, divide the polynomial *p* by *g* and find the quotient and the remainder. Find in which case *g* is the factor of *p*.**

(i) $p(x) = (x^4 + 1)$ and $g(x) = (x + 1)$

Sol.

$$\begin{array}{r|l} & x^3 - x^2 + x - 1 \\ \hline x+1 & x^4 + 1 \\ & x^4 + x^3 \\ & - \quad - \\ \hline & -x^3 + 1 \\ & -x^3 - x^2 \\ & + \quad + \\ \hline & x^2 + 1 \\ & x^2 + x \\ & - \quad - \\ \hline & -x + 1 \\ & -x - 1 \\ & + \quad + \\ \hline & 2 \end{array}$$

∴ The quotient is $x^3 - x^2 + x - 1$ and the remainder is 2.

(ii) $p(x) = 10x^2 - 5x + 2$ and $g(x) = 5x$.

Sol.

$$\begin{array}{r|l} & 2x - 1 \\ \hline 5x & 10x^2 - 5x + 2 \\ & 10x^2 \\ & - \\ \hline & -5x + 2 \\ & -5x \\ & + \\ \hline & 2 \end{array}$$

∴ The quotient is $2x - 1$ and the remainder is 2.

(iii) $p(x) = 7x^2 - 3x + 9$ and $g(x) = x - 2$.

Sol.

$$\begin{array}{r|l} & 7x + 11 \\ \hline x-2 & 7x^2 - 3x + 9 \\ & 7x^2 - 14x \\ & - \quad + \\ \hline & 11x + 9 \\ & 11x - 22 \\ & - \quad + \\ \hline & 31 \end{array}$$

∴ The quotient is $7x + 11$ and the remainder is 31.

Long Answer Type Questions

7. In each of the following divide the polynomial p by g and find the quotient and the remainder. Find in which cases g is a factor of p.

(i) $p(t) = t^3 - 3t^2 + 4t + 2$ and $g(t) = t - 1$.

Sol.

$$\begin{array}{r|l} & t^2-2t+2 \\ \hline t-1 & t^3-3t^2+4t+2 \\ & t^3-t^2 \\ & -\quad + \\ \hline & -2t^2+4t+2 \\ & -2t^2+2t \\ & +\quad - \\ \hline & 2t+2 \\ & 2t-2 \\ & -\quad + \\ \hline & 4 \end{array}$$

$\therefore$ The quotient is $t^2 - 2t + 2$ and remainder is 4.

(ii) $p(u) = u^3 - 14u^2 + 37u - 60$ and $g(u) = u - 2$

Sol.

$$\begin{array}{r|l} & u^2-12u+13 \\ \hline u-2 & u^3-14u^2+37u-60 \\ & u^3-2u^2 \\ & -\quad + \\ \hline & -12u^2+37u-60 \\ & -12u^2+24u \\ & +\quad - \\ \hline & 13u-60 \\ & 13u-26 \\ & -\quad + \\ \hline & -34 \end{array}$$

$\therefore$ The quotient is $u^2 - 12u + 13$ and the remainder is -34.

(iii) $p(y) = y^3 + 3y^2 - 12y + 4$ and $g(y) = y - 2$

Sol.

$$\begin{array}{r|l} & y^2+5y-2 \\ \hline y-2 & y^3+3y^2-12y+4 \\ & y^3-2y^2 \\ & -\quad + \\ \hline & 5y^2-12y+4 \\ & 5y^2-10y \\ & -\quad + \\ \hline & -2y+4 \\ & -2y+4 \\ & +\quad - \\ \hline & 0 \end{array}$$

$\therefore$ The quotient is $y^2 + 5y - 2$ and the remainder is 0.

Hence, the remainder is 0 so, $g(y)$ is the factor of $p(y)$.

(iv) $p(t) = t^6 + 3t^2 + 10$ and $g(t) = t^3 + 1$.

Sol.

$$\begin{array}{r|l} & t^3-1 \\ \hline t^3+1 & t^6+3t^2+10 \\ & t^6+t^3 \\ & -\quad - \\ \hline & -t^3+3t^2+10 \\ & -t^3\qquad -1 \\ & +\qquad + \\ \hline & 3t^2+11 \end{array}$$

$\therefore$ The quotient is $t^3 - 1$ and the remainder is $3t^2 + 11$.

8. (i) $p(x) = x^5 + x^4 + x^3 + x^2 + 2x + 2$ and $g(x) = x^3 + 1$.

Sol.

$$\begin{array}{r|l} & x^2+x+1 \\ \hline x^3+1 & x^5+x^4+x^3+x^2+2x+2 \\ & x^5\qquad +x^2 \\ & -\qquad - \\ \hline & x^4+x^3+2x+2 \\ & x^4\qquad +x \\ & -\qquad - \\ \hline & x^3+x+2 \\ & x^3\qquad +1 \\ & -\qquad - \\ \hline & x+1 \end{array}$$

The quotient is $x^2 + x + 1$ and the remainder is $x + 1$.

(ii) $p(y) = y^3 - 6y^2 + 11y - 6$ and $g(y) = y^2 - 5y + 6$.

Sol.

$$\begin{array}{r|l} & y-1 \\ \hline y^2-5y+6 & y^3-6y^2+11y-6 \\ & y^3-5y^2+6y \\ & -\quad +\quad - \\ \hline & -y^2+5y-6 \\ & -y^2+5y-6 \\ & +\quad -\quad + \\ \hline & 0 \end{array}$$

$\therefore$ The quotient is $y - 1$ and the remainder is 0.

Hence, the remainder as is zero. So, $g(y)$ is the factor of $p(y)$.

(iii) $p(y) = y^5 + 5y^3 + 3y^2 + 5y + 3$ and $g(y) = y^2 + 4y + 2$.

Sol.

$$
\begin{array}{l}
\qquad\qquad\quad y^3 - 4y^2 + 19y - 65 \\
y^2 + 4y + 2 \,\big)\; y^5 + 5y^3 + 3y^2 + 5y + 3 \,\big(\\
\qquad\qquad\quad y^5 + 4y^4 + 2y^3 \\
\qquad\qquad\quad - \quad - \quad - \\
\hline
\qquad\qquad\quad -4y^4 + 3y^3 + 3y^2 + 5y + 3 \\
\qquad\qquad\quad -4y^4 - 16y^3 - 8y^2 \\
\qquad\qquad\quad + \quad + \quad + \\
\hline
\qquad\qquad\qquad 19y^3 + 11y^2 + 5y + 3 \\
\qquad\qquad\qquad 19y^3 + 76y^2 + 38y \\
\qquad\qquad\qquad - \quad - \quad - \\
\hline
\qquad\qquad\qquad\quad -65y^2 - 33y + 3 \\
\qquad\qquad\qquad\quad -65y^2 - 260y - 130 \\
\qquad\qquad\qquad\quad + \quad + \quad + \\
\hline
\qquad\qquad\qquad 227y + 133 \\
\hline
\end{array}
$$

$\therefore$ The quotient is $y^3 - 4y^2 + 19y - 65$ and the remainder is $227y + 133$.

9. Show that :

(i) $(x - 2)$ is a factor of $(x^3 - 8)$.

Sol.

$$
\begin{array}{l}
\qquad\quad x^2 + 2x + 4 \\
x - 2 \,\big)\; x^3 - 8 \,\big(\\
\qquad\quad x^3 - 2x^2 \\
\qquad\quad - \quad + \\
\hline
\qquad\quad 2x^2 - 8 \\
\qquad\quad 2x^2 - 4x \\
\qquad\quad - \quad + \\
\hline
\qquad\quad 4x - 8 \\
\qquad\quad 4x - 8 \\
\qquad\quad - \quad + \\
\hline
\qquad\qquad 0 \\
\hline
\end{array}
$$

Hence, remainder = 0

$\therefore$ $(x - 2)$ is a factor of $(x^3 - 8)$.

(ii) $(t^2 - t + 2)$ is a factor of $(t^3 - 3t^2 + 4t - 4)$.

Sol.

$$
\begin{array}{l}
\qquad\qquad\quad t - 2 \\
t^2 - t + 2 \,\big)\; t^3 - 3t^2 + 4t - 4 \,\big(\\
\qquad\qquad\quad t^3 - t^2 + 2t \\
\qquad\qquad\quad - \quad + \quad - \\
\hline
\qquad\qquad\quad -2t^2 + 2t - 4 \\
\qquad\qquad\quad -2t^2 + 2t - 4 \\
\qquad\qquad\quad + \quad - \quad + \\
\hline
\qquad\qquad\qquad 0 \\
\hline
\end{array}
$$

$\therefore$ Remainder is 0.

$\therefore$ $(t^2 - t + 2)$ is a factor of $(t^3 - 3t^2 + 4t - 4)$.

(iii) $(u - 3)$ is a factor of $(u^3 - 2u^2 + 3u - 18)$.

Sol.

$$
\begin{array}{l}
\qquad\quad u^2 + u + 6 \\
u - 3 \,\big)\; u^3 - 2u^2 + 3u - 18 \,\big(\\
\qquad\quad u^3 - 3u^2 \\
\qquad\quad - \quad + \\
\hline
\qquad\qquad u^2 + 3u - 18 \\
\qquad\qquad u^2 - 3u \\
\qquad\qquad - \quad + \\
\hline
\qquad\qquad 6u - 18 \\
\qquad\qquad 6u - 18 \\
\qquad\qquad - \quad + \\
\hline
\qquad\qquad\quad 0 \\
\hline
\end{array}
$$

The remainder is 0. So, $(u - 3)$ is a factor of $u^3 - 2u^2 + 3u - 18$.

(iv) $(x + 1)$ is a factor of $(2x^2 + 5x + 3)$.

Sol.

$$
\begin{array}{l}
\qquad\quad 2x + 3 \\
x + 1 \,\big)\; 2x^2 + 5x + 3 \,\big(\\
\qquad\quad 2x^2 + 2x \\
\qquad\quad - \quad - \\
\hline
\qquad\quad 3x + 3 \\
\qquad\quad 3x + 3 \\
\qquad\quad - \quad - \\
\hline
\qquad\qquad 0 \\
\hline
\end{array}
$$

$\therefore$ Remainder is 0

$\therefore$ $(x + 1)$ is a factor of $(2x^2 + 5x + 3)$.

(v) $(x - 1)$ is a factor of $(x^3 - 6x^2 + 11x - 6)$.

Sol.

$$
\begin{array}{l}
\qquad\quad x^2 - 5x + 6 \\
x - 1 \,\big)\; x^3 - 6x^2 + 11x - 6 \,\big(\\
\qquad\quad x^3 - x^2 \\
\qquad\quad - \quad + \\
\hline
\qquad\quad -5x^2 + 11x - 6 \\
\qquad\quad -5x^2 + 5x \\
\qquad\quad + \quad - \\
\hline
\qquad\qquad\quad 6x - 6 \\
\qquad\qquad\quad 6x - 6 \\
\qquad\qquad\quad - \quad + \\
\hline
\qquad\qquad\quad 0 \\
\hline
\end{array}
$$

$\therefore$ The remainder is 0. So, $(x - 1)$ is factor of $(x^3 - 6x^2 + 11x - 6)$.

10. Divide polynomial $f(x) = 6x^5 + 4x^4 - 3x^3 + x + 1$ by polynomial $g(x) = 3x^2 - x + 1$ and state quotient and remainder. If $g(x)$ a factor of $f(x)$?

Sol.

$$3x^2 - x + 1 \overline{)\, 6x^5 + 4x^4 - 3x^3 + x + 1 \,(} \; 2x^3 + 2x^2 - x - 1$$

$$\begin{array}{l} 6x^5 - 2x^4 + 2x^3 \\ - \quad + \quad - \\ \hline 6x^4 - 5x^3 + x + 1 \\ 6x^4 - 2x^3 + 2x^2 \\ - \quad + \quad - \\ \hline -3x^3 - 2x^2 + x + 1 \\ -3x^3 + x^2 - x \\ + \quad - \quad + \\ \hline -3x^2 + 2x + 1 \\ -3x^2 + x - 1 \\ + \quad - \quad + \\ \hline x + 2 \\ \hline \end{array}$$

$\therefore$ The quotient $= 2x^3 + 2x^2 - x - 1$

Remainder $= x + 2$

$\therefore (3x^2 - x + 1)$ is not a factor of $(6x^5 + 4x^4 - 3x^3 + x + 1)$.

11. Obtain all zeroes of polynomial $p(x) = x^4 - 3x^3 - x^2 + 9x - 6$. If two of its zero are $(\sqrt{-3})$ and $(\sqrt{3})$.

Sol. Since, $\sqrt{3}$ and $\sqrt{-3}$ are zeroes $x - \sqrt{3}$ and $x + \sqrt{3}$ are factors of polynomial $x^4 - 3x^3 - x^2 + 9x - 6$

$(x + \sqrt{3})(x - \sqrt{3}) = x^2 - (\sqrt{3})^2$

$[(a + b)(a - b) = a^2 - b^2]$

$= x^2 - 3$

Now, divide the polynomial $x^4 - 3x^3 - x^2 + 9x - 6$ by $(x^2 - 3)$

$$x^2 - 3 \overline{)\, x^4 - 3x^3 - x^2 + 9x - 6 \,(} \; x^2 - 3x + 2$$

$$\begin{array}{l} x^4 - 3x^2 \\ - \quad + \\ \hline -3x^3 + 2x^2 + 9x - 6 \\ -3x^3 + 9x \\ + \quad - \\ \hline 2x^2 - 6 \\ 2x^2 - 6 \\ - \quad + \\ \hline 0 \\ \hline \end{array}$$

$x^4 - 3x^3 - x^2 + 9x - 6$

$= (x^2 - 3x + 2)(x^2 - 3)$

$x^2 - 3x + 2 = x^2 - 2x - x + 2$

$= x(x - 2) - 1(x - 2)$

$= (x - 2)(x - 1)$

$= x = 2, 1$

Therefore, the zeroes of polynomial $x^4 - 3x^3 - x^2 + 9x - 6$ other then $\sqrt{3}$ & $\sqrt{-3}$ is 1 and 2.

12. If two zeroes of the polynomial $x^4 + x^3 - 15x^2 - 29x - 6$ are $2 \pm (\sqrt{5})$. Find other zeroes.

Sol. As $x = 2 \pm \sqrt{5}$ are the zeroes of

$p(x) = x^4 + x^3 - 15x^2 - 29x - 6$

So, $x - (2 + \sqrt{5})$ are the factors of $p(x)$

Now, $[x - (2 + \sqrt{5}] [x - (2 - \sqrt{5})]$

$= [(x - 2) - \sqrt{5}] [(x - 2) + \sqrt{5}]$

$= (x - 2)^2 - (\sqrt{5})^2$

$= x^2 - 4x - 1$

Dividing $p(x)$ by $(x^2 - 4x - 1)$

$$x^2 - 4x - 1 \overline{)\, x^4 + x^3 - 15x^2 - 29x - 6 \,(} \; x^2 + 5x + 6$$

$$\begin{array}{l} x^4 - 4x^3 - x^2 \\ - \quad + \quad + \\ \hline 5x^3 - 14x^2 - 29x - 6 \\ 5x^3 - 20x^2 - 5x \\ - \quad + \quad + \\ \hline 6x^2 - 24x - 6 \\ 6x^2 - 24x - 6 \\ - \quad + \quad + \\ \hline 0 \\ \hline \end{array}$$

$x^2 + 5x + 6 = 0$

$x^2 + 2x + 3x + 6 = 0$

$x(x + 2) + 3(x + 2) = 0$

$(x + 2)(x + 3) = 0$

Other's Zeroes are $-3, -2$.

13. Given that $\sqrt{2}$ is a zero of the cubic polynomial $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$. Find its other two zeroes.

Sol. Given that $x = \sqrt{2}$ is a zero of

$p(x) = 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$

$\therefore (x - \sqrt{2})$ is a factor of $(6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2})$

Dividing $p(x)$ by $(x-\sqrt{2})$

$$\begin{array}{r|l} & 6x^2+7\sqrt{2}x+4 \\ \hline x-\sqrt{2} & 6x^3+\sqrt{2}x^2-10x-4\sqrt{2} \\ & 6x^3-6\sqrt{2}x^2 \\ & -\quad + \\ \hline & 7\sqrt{2}x^2-10x-4\sqrt{2} \\ & 7\sqrt{2}x^2-14x \\ & -\quad + \\ \hline & 4x-4\sqrt{2} \\ & 4x-4\sqrt{2} \\ & -\quad + \\ \hline & 0 \end{array}$$

$$6x^2+7\sqrt{2}x+4=0$$
$$6x^2+4\sqrt{2}x+3\sqrt{2}x+4=0$$
$$2x(3x+2\sqrt{2})+\sqrt{2}(3x+2\sqrt{2})=0$$
$$(2x+\sqrt{2})(3x+2\sqrt{2})=0$$
$$2x+\sqrt{2}=0$$
$$x=\frac{-\sqrt{2}}{2}$$
$$x=\frac{-2\sqrt{2}}{3}$$

$\therefore$ Other zeroes of $p(x)$ is $\frac{-\sqrt{2}}{2}$ & $\frac{-2\sqrt{2}}{3}$.

14. Find k so that x^2+2x+k is a factor of $2x^4+x^3-14x^2+5x+6$. Also find all the zeroes of the two polynomials.

Sol. Using long division method for the given polynomials as shown below :

$$\begin{array}{r|l} & 2x^2-3x-8-2k \\ \hline x^2+2x+k & 2x^4+x^3-14x^2+5x+6 \\ & 2x^4+4x^3+2kx^2 \\ & -\quad -\quad - \\ \hline & -3x^3-14x^2-2kx^2+5x+6 \\ & -3x^3-6x^2-3kx \\ & +\quad +\quad + \\ \hline & -8x^2-2kx^2+3kx+5x+6 \\ & -8x^2-8k-16x \\ & +\quad +\quad + \\ \hline & -2kx^2+3kx+8k+21x+6 \\ & -2kx^2-4kx-2k^2 \\ & +\quad +\quad + \\ \hline & 7kx+21x+2k^2+8k+6 \\ \hline & (21+7k)x+2k^2+8k+6 \\ \hline \end{array}$$

We get the remainder as $(21+7k)x+2k^2+8k+6$.

Since, x^2+2x+k is a factor of the given polynomial, the remainder should be zero.

Hence, $21+7k=0$

and $2k^2+8k+6=0$ at the same time $k=-3$ satisfy both the equation.

Hence, $k=-3$.

So, we can write the polynomial as

$2x^4+x^3-14x^2+5x+6$

$=(x^2+2x-3)(2x^2-3x-2)$

Hence, the zeroes of x^2+2x-3 are

x^2+2x-3

$\Rightarrow x^2+3x-x-3=0$

$\Rightarrow x(x+3)-1(x+3)=0$

$\Rightarrow (x+3)(x-1)=0$

$\Rightarrow x=-3$ or $x=1$

The above two factor of the 4th degree polynomial as well.

The other two roots of the 4th degree polynomial are roots of the quadratic.

$2x^2-3x-2=0$

$\Rightarrow 2x^2-4x+x-2=0$

$\Rightarrow 2x(x-2)+1(x-2)=0$

$\Rightarrow (2x+1)(x-2)=0$

$x=2$

or $x=\frac{-1}{2}$.

15. Given that $x-\sqrt{5}$ is a factor of the cubic polynomial $x^3-3\sqrt{5}x^2+13x-3\sqrt{5}$. Find all the zeroes of the polynomial.

Sol. $p(x)=x^3-3\sqrt{5}x^2+13x-3\sqrt{5}$

$x-\sqrt{5}$ is a root of $p(x)$

we will divide $p(x)$ by $x-\sqrt{5}$

$$\begin{array}{r|l} & x^2-2\sqrt{5}x+3 \\ \hline x-\sqrt{5} & x^3-3\sqrt{5}x^2+13x-3\sqrt{5} \\ & x^3-\sqrt{5}x^2 \\ & -\quad + \\ \hline & -2\sqrt{5}x^2+13x-3\sqrt{5} \\ & -2\sqrt{5}x^2+10x \\ & +\quad - \\ \hline & 3x-3\sqrt{5} \\ & 3x-3\sqrt{5} \\ & -\quad + \\ \hline & 0 \\ \hline \end{array}$$

Now we get a quadratic equation as quotient we will find the roots of the quotient

$x^2 - 2\sqrt{5}x + 3$

$$\text{Roots} = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$= \frac{-(-2\sqrt{5}) \pm \sqrt{(-2\sqrt{5})^2 - 4 \times 1 \times 3}}{2 \times 1}$$

$$= \frac{2\sqrt{5} \pm \sqrt{20 - 12}}{2}$$

$$= \frac{2\sqrt{5} \pm \sqrt{8}}{2}$$

$$= \frac{2\sqrt{5} \pm 2\sqrt{2}}{2}$$

$= \sqrt{5} + \sqrt{2}$ and $\sqrt{5} - \sqrt{2}$

All the roots are

$\sqrt{5}, \sqrt{5} + \sqrt{2}$ & $\sqrt{5} - \sqrt{2}$.

16. For which values of a and b, are the zeroes of $q(x) = x^3 + 2x^2 + a$ also the zeroes of the polynomial $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$? which the zeroes of $p(x)$ are not the zeroes of $q(x)$?

Sol. Given that the zeroes of

$$q(x) = x^3 + 2x^2 + a$$

are also the zeroes of polynomial

$$p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$$

i.e. $q(x)$ is a factor of $p(x)$. Then we use a division algorithm.

$$\begin{array}{r|l} & x^2 - 3x + 2 \\ \hline x^3 + 2x^2 + a & x^5 - x^4 - 4x^3 + 3x^2 + 3x + b \\ & x^5 + 2x^4 + ax^2 \\ & - \quad - \quad - \\ \hline & -3x^4 + 4x^3 + (3-a)x^2 + 3x + b \\ & -3x^4 - 6x^3 - 3ax \\ & + \quad + \quad + \\ \hline & 2x^3 + (3-a)x^2 + (3-3a)x + b \\ & 2x^3 + 4x^2 + 2a \\ & - \quad - \quad - \\ \hline & -(1+a)x^2 + (3+3a)x + (b-2a) \end{array}$$

If $(x^3 + 2x^2 + a)$ is a factor of $(x^5 - x^4 - 4x^3 + 3x^2 + 3x + b)$, then remainder should be zero.

i.e. $-(1 + a)x^2 + (3 + 3a)x + (b - 2a) = 0$

$= 0.x^2 + 0.x + 0$

on comparing the coefficient of x we get

$$a + 1 = 0$$

$\Rightarrow \quad a = -1$

and $\quad b - 2a = 0$

$$b = 2(-1) = -2$$

for $\quad a = -1$ & $b = -2$ the zero of $q(x)$ are also the zeroes of polynomial $p(x)$

$$q(x) = x^3 + 2x^2 - 1$$

$$p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x - 2$$

Now, $\quad p(x) = (x^3 + 2x^2 - 1)(x^2 - 3x + 2) = 0$

$(x^3 + 2x^2 - 1)(x^2 - 2x - x + 2)$

$(x^3 + 2x^2 - 1)(x - 1)(x - 1)$

Hence, the zero of $p(x)$ are 2 which are not zero of $q(x)$.

❑

3 Pair of Linear Equations in Two Variables

EXERCISE 3.1

1. Ritu went to 'SALE' to purchase some pants and skirts. When her mother asked her how many of each she had bought. She answered, "The number of skirts is seven less than eight times the number of pants purchased. Also, the number of skirts is four less than five times the number of pants purchased." For the sake of her mother, find how many pants and skirts Ritu bought ?

Sol. Let the skirts purchased = y

Pants purchased = x

According to the question

$$y = 8x - 7 \quad ...(1)$$
$$y = 5x - 4 \quad ...(2)$$

According to the condition given in the question

$$8x - 7 = 5x - 4$$
$$8x - 5x = 7 - 4$$
$$3x = 3$$
$$x = 1$$
$$y = 1$$

To find equivalent geometric representations, we find some points on the line representing each equation these solution are given below in the table.

From equation (i)

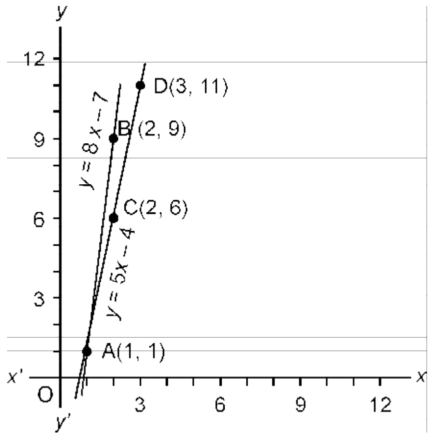
$$y = 8x - 7,\ 8x - y = 7$$

x	1	2
y	1	9
Points	A	B

From equation (ii)

$$y = 5x - 4,\ 5x - y = 4$$

x	2	3
y	6	11
Points	C	D



Solve the following equations :

2. $3x - y = 2$, $x + 2y = 10$.

Sol. The given equations are :

$$3x - y = 2$$
$$y = 3x - 2$$

When $x = 0$

$$y = 3 \times 0 - 2$$
$$y = -2$$

When $x = 2$

$$y = 3 \times 2 - 2$$
$$y = 4$$

$$x + 2y = 10$$
$$y = \frac{10 - x}{2}$$

When $x = 0$

$$y = \frac{10 - 0}{2}$$
$$y = 5$$

When $x = 10$

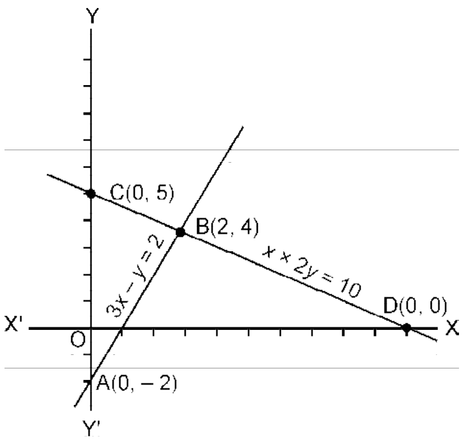
$$y = \frac{10 - 10}{2}$$
$$y = 0$$

In the tabular form

x	0	2
y	–2	4
Points	A	B

x	0	10
y	5	0
Points	C	D

Plotting the points A(0, –2), B(2, 4) and join them to form *a* line AB.



Similarly plot the points C(0, 5) and D(10, 0) and join them to get the line CD clearly the graphs of the given equations are parallel lines. As they have no common points, there is no common solution.

3. $\mathbf{2x + 3y = 15, 4x + 6y = 24}$

Sol. The given equations are :

$$2x + 3y = 15$$

$$x = \frac{15 - 3y}{2}$$

When $y = 0$

$$x = \frac{15 - 3 \times 0}{2}$$

$$x = 7{\cdot}5$$

When $x = 0$

$$0 = \frac{15 - 3y}{2}$$

$$y = 5$$

$$4x + 6y = 24$$

$$x = \frac{24 - 6y}{4}$$

when $y = 0$

$$x = \frac{24 - 6 \times 0}{4}$$

$$x = 6$$

when $x = 0$

$$0 = \frac{24 - 6y}{4}$$

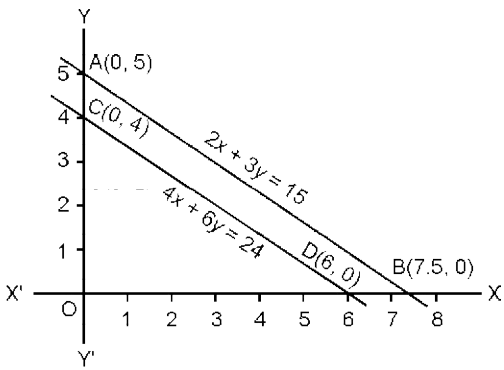
$$y = 4$$

Tabular form

x	0	7·5
y	5	0
Points	A	B

x	0	6
y	4	0
Points	C	D

Plotting the points on graph :



4. $\mathbf{4x + 5y = 9, 8x + 10y = 18.}$

Sol. The given equation are :

$$4x + 5y = 9$$

When $x = 0$

$$4 \times 0 + 5y = 9$$

$$y = 1{\cdot}8$$

When $y = 0$

$$4x + 5 \times 0 = 9$$

$$x = 2{\cdot}25$$

$$8x + 10y = 18$$

when $x = 0$

$$8 \times 0 + 10y = 18$$

$$y = 1{\cdot}8$$

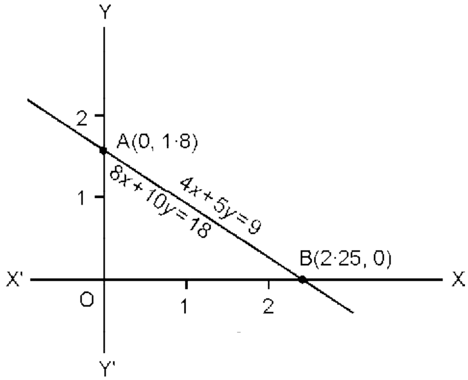
$8x + 10 \times 0 = 18$

$x = 2{\cdot}25$

$\therefore$ Hence, both the equations are equal.

Tabular form

x	0	2·25
y	1·8	0
Points	A	B



5. $2x + y - 5 = 0$, $x + y - 3 = 0$

Sol. The given equations are :

$2x + y = 5$

When $x = 0$

$2 \times 0 + y = 5$

$y = 5$

When $y = 0$

$2x + 0 = 5$

$x = 2{\cdot}5$

$x + y = 3$

when $x = 0$

$0 + y = 3$

$y = 3$

when $y = 0$

$x + 0 = 3$

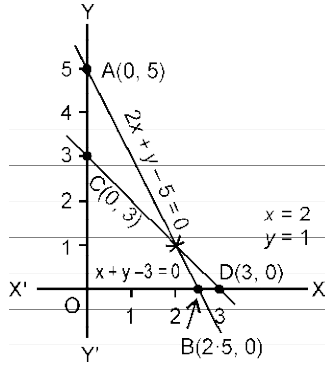
$x = 3$

Tabular form

x	0	2·5
y	5	0
Points	A	B

x	0	3
y	3	0
Points	C	D

Plotting the points on graph :



6. $2x + y - 11 = 0$, $x - y - 1 = 0$.

Sol. The given equations are :

$2x + y - 11 = 0$

When $x = 0$

$2 \times 0 + y - 11 = 0$

$y = 11$

When $y = 0$

$2x + 0 - 11 = 0$

$x = 5{\cdot}5$

$x - y - 1 = 0$

When $x = 0$

$0 - y - 1 = 0$

$y = -1$

When $y = 0$

$x - 0 - 1 = 0$

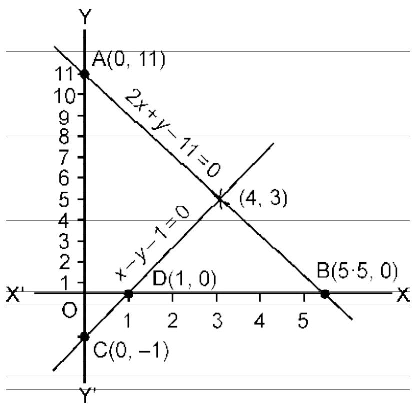
$x = 1$

Tabular form :

x	0	5·5
y	11	0
Points	A	B

x	0	1
y	–1	0
Points	C	D

Plotting the points on graph :



7. Solve the following system of linear equations graphically : $4x - 5y - 20 = 0$, $3x + 5y - 15 = 0$. Determine the vertices of the triangle formed by the lines, representing the above equations, and the y-axis.

Sol. The given equations are :

$$4x - 5y - 20 = 0$$

When $x = 0$

$$4 \times 0 - 5y - 20 = 0$$
$$y = -4$$

When $y = 0$

$$4x - 5 \times 0 - 20 = 0$$
$$x = 5$$

$$3x + 5y - 15 = 0$$

When $x = 0$

$$3 \times 0 + 5y - 15 = 0$$
$$y = 3$$

When $y = 0$

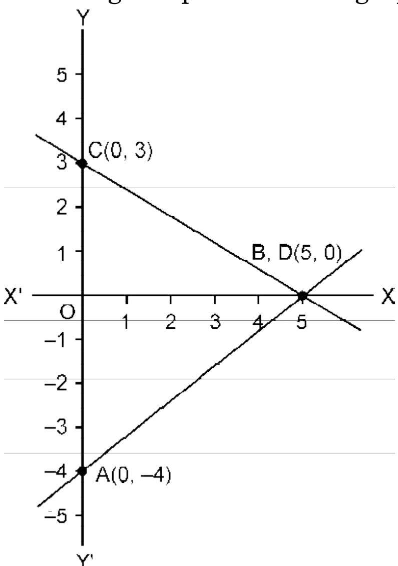
$$3x - 5 \times 0 - 15 = 0$$
$$x = 5$$

Tabular form

x	0	5
y	–4	0
Points	A	B

x	0	5
y	3	0
Points	C	D

Plotting the points on the graph :



The equation cut at point c (5,0).

8. $4x - 3y + 4 = 0$, $4x - 3y - 20 = 0$.

Sol. The given equations are :

$$4x - 3y + 4 = 0$$

When $x = 0$

$$4 \times 0 - 3y + 4 = 0$$
$$y = \frac{4}{3}$$
$$= 1{\cdot}33$$

When $y = 0$

$$4x - 3 \times 0 + 4 = 0$$
$$x = -1$$

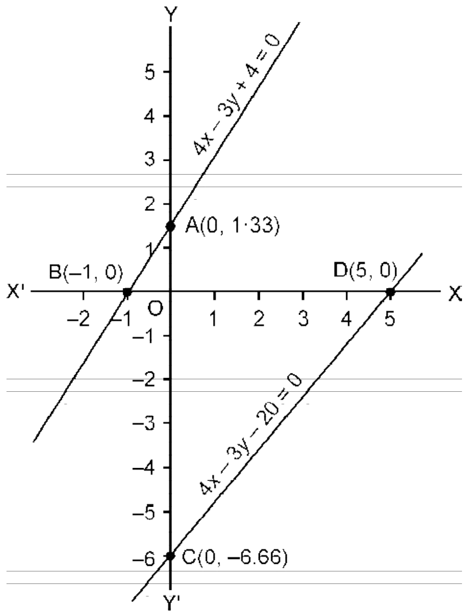
$$4x - 3y - 20 = 0$$
$$4 \times 0 - 3y - 20 = 0$$
$$y = \frac{-20}{3}$$
$$= 6{\cdot}66$$
$$4x - 3 \times 0 - 20 = 0$$
$$x = -5$$

Tabular form

x	0	–1
y	1·33	0
Points	A	B

x	0	5
y	–6·66	0
Points	C	D

Plotting the points on the graph :



9. $2x + y = 6$, $2x - y = -2$:

Sol. The given equation are :

$$2x + y = 6$$

when $x = 0$

$2 \times 0 + y = 6$

$y = 6$

when $y = 0$

$2x + 0 = 6$

$x = 3$

$2x - y = -2$

when $x = 0$

$2 \times 0 - y = -2$

$y = 2$

when $y = 0$

$2x - 0 = -2$

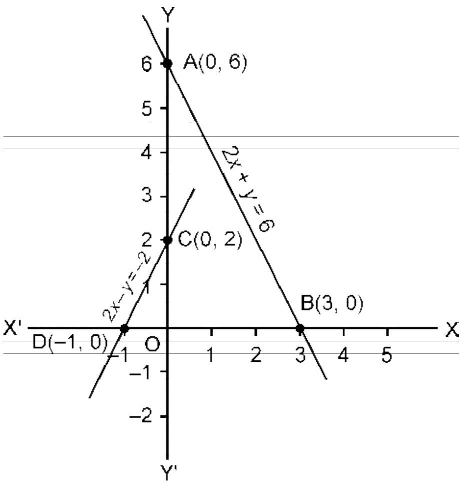
$x = -1$

Tabular form :

x	0	3
y	6	0
Points	A	B

x	0	–1
y	2	0
Points	C	D

Plotting the points on the graph :



10. Find the area of triangle formed by $x + y = 2$, x-axis and y-axis.

Sol. The given equation are :

$x + y = 2$

when $x = 0$

$0 + y = 2$

$y = 2$

when $y = 0$

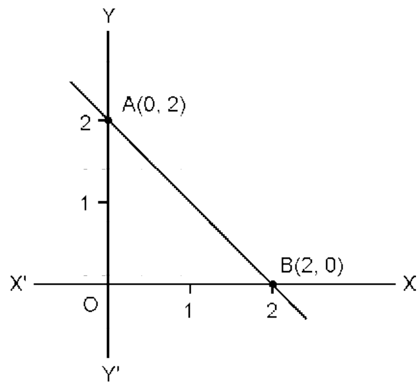
$x + 0 = 2$

$x = 2$

Tabular form :

x	0	2
y	2	0
Points	A	B

Plotting the points on the graph



Area of triangle formed

$= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 2 \times 2$

$= 2 \text{ unit}^2$. **Ans.**

EXERCISE 3.2

Solve for 'x' and 'y' by using method of substitution.

1. $3x - 4y = 10$, $4x + 3y = 5$

Sol. We have,

$3x - 4y = 10$ ...(1)

$4x + 3y = 5$ ...(2)

From equation (ii)

$y = \frac{5-4x}{3}$ ...(3)

On substituting the value of y from equation (iii) in equation (i), we get

$3x - 4\left[\frac{5-4x}{3}\right] = 10$

$3x - \left[\frac{20-16x}{3}\right] = 10$

$9x - 20 + 16x = 30$

$25x = 50$

$x = 2.$

On substituting $x = 2$ in equation (iii) We get

$$y = \frac{5-4\times 2}{3}$$

$$= \frac{5-8}{3}$$

$$= \frac{-3}{3}$$

$$= -1.$$ **Ans.**

2. $\frac{x}{2} + y = 0{\cdot}8, \quad \frac{7}{x+y/2} = 10$

Sol. We have,

$$\frac{x}{2} + y = 0{\cdot}8$$

$$x + 2y = 1{\cdot}6 \quad \text{...(i)}$$

$$\frac{7}{x+y/2} = 10$$

$$\frac{7\times 2}{2x+y} = 10$$

$$2x + y = 1{\cdot}4 \quad \text{...(ii)}$$

From equation (i)

$$x = 1{\cdot}6 - 2y \quad \text{...(iii)}$$

On substituting the value of x from equation (iii) in equation (ii), we get

$$2(1{\cdot}6 - 2y) + y = 1{\cdot}4$$

$$3{\cdot}2 - 4y + y = 1{\cdot}4$$

$$-3y = 1{\cdot}4 - 3.2$$

$$-3y = 1{\cdot}8$$

$$3y = -1{\cdot}8$$

$$y = 0{\cdot}6.$$

On substituting $y = 0{\cdot}6$ in equation (iii) $x = 1{\cdot}6 - 2 \times 0{\cdot}6$

$$= 0{\cdot}4.$$ **Ans.**

3. $x + y = a - b,\ ax - by = a^2 + b^2$

Sol. We have, $x + y = a - b$ ...(i)

$$ax - by = a^2 + b^2 \quad \text{...(ii)}$$

From equation (i)

$$y = a - b - x \quad \text{...(iii)}$$

On substituting the value of y from equation (iii) in equation (ii), we get

$$ax - b(a - b - x) = a^2 + b^2$$

$$ax - ab + b^2 + bx = a^2 + b^2$$

$$ax + bx = a^2 + b^2 - b^2 + ab$$

$$x(a + b) = a^2 + ab$$

$$x = \frac{a(a+b)}{(a+b)}$$

$$x = a$$

On substituting $x = a$ in equation (iii) we get,

$$y = a - b - a$$

$$y = -b.$$ **Ans.**

4. $0{\cdot}2x + 0{\cdot}3y = 1{\cdot}3,\ 0{\cdot}4x + 0{\cdot}5y = 2{\cdot}3$

Sol. We have,

$$0{\cdot}2x + 0{\cdot}3y = 1{\cdot}3 \quad \text{...(i)}$$

$$0{\cdot}4x + 0{\cdot}5y = 2{\cdot}3 \quad \text{...(ii)}$$

From equation (ii)

$$y = \frac{2{\cdot}3 - 0{\cdot}4x}{0{\cdot}5} \quad \text{...(iii)}$$

On substituting the value of y from equation (iii) in equation (i), we get

$$0{\cdot}2x + 0{\cdot}3\left[\frac{2{\cdot}3 - 0{\cdot}4x}{0{\cdot}5}\right] = 1{\cdot}3$$

$$0{\cdot}2x + \left[\frac{0{\cdot}69 - 0{\cdot}12x}{0{\cdot}5}\right] = 1{\cdot}3$$

$$0{\cdot}1x + 0{\cdot}69 - 0{\cdot}12x = 0{\cdot}65$$

$$x = \frac{0{\cdot}04}{0{\cdot}02}$$

$$= 2.$$

On substituting $x = 2$ in equation (iii)

$$y = \frac{2{\cdot}3 - (0{\cdot}4\times 2)}{0{\cdot}5}$$

$$y = \frac{2{\cdot}3 - 0{\cdot}8}{0{\cdot}5}$$

$$y = \frac{1{\cdot}5}{0{\cdot}5}$$

$$y = 3.$$ **Ans.**

5. $\sqrt{x} + \sqrt{3}y = 0,\ \sqrt{3}x - \sqrt{8}y = 0$

Sol. We have, $\sqrt{x} + \sqrt{3}y = 0$ ...(i)

$$\sqrt{3}x - \sqrt{8}y = 0 \quad \text{...(ii)}$$

From equation (i) we get

$$y = \frac{-\sqrt{x}}{\sqrt{3}} \quad \text{...(iii)}$$

On substituting the value of y from equation (iii) in equation (ii), we get

$$\sqrt{3}x - \sqrt{8}\left[\frac{-\sqrt{x}}{\sqrt{3}}\right] = 0$$

$$\sqrt{3}x + \frac{\sqrt{8}x}{\sqrt{3}} = 0$$

$$3x + \sqrt{8}x = 0$$

$$x = \frac{0}{3+\sqrt{8}}$$

$$= 0$$

On substituting value of x in equation (iii)

$$y = \frac{-\sqrt{0}}{\sqrt{3}}$$

$$= 0$$

6. $6x + 5y = 7x + 3y + 1 = 2(x + 6y - 1)$

Sol. We have,

$$6x + 5y = 7x + 3y + 1$$

$$7x - 6x + 3y - 5y + 1 = 0$$

$$x - 2y + 1 = 0 \quad ...(1)$$

$$6x + 5y = 2(x + 6y - 1)$$

$$6x + 5y = 2x + 12y - 2$$

$$2x - 6x + 12y - 5y - 2 = 0$$

$$-4x + 7y - 2 = 0 \quad ...(2)$$

From equation (1) we get

$$x = 2y - 1 \quad ...(3)$$

On substituting the value of x in equation (2)

$$-4(2y - 1) + 7y - 2 = 0$$

$$-8y + 4 + 7y - 2 = 0$$

$$-y + 2 = 0$$

$$-y = -2$$

$$y = 2.$$

On substituting the value of y in equation (3)

$$x = (2 \times 2) - 1$$

$$x = 3.$$ **Ans.**

7. $bx + ay = a + b,\ ax\left[\frac{1}{a-b} - \frac{1}{a+b}\right] + by\left[\frac{1}{b-a} - \frac{1}{b+a}\right] = 2$

Sol. We have,

$$bx + ay = a + b \quad ...(1)$$

$$ax\left[\frac{1}{a-b} - \frac{1}{a+b}\right] + by\left[\frac{1}{b-a} - \frac{1}{b+a}\right] = 2 \quad ...(2)$$

From equation (1) we get

$$x = \frac{a+b-ay}{b} \quad ...(3)$$

On substituting the value of x in equation (2)

$$a\left[\frac{a+b-ay}{b}\right]\left[\frac{a+b-a+b}{a^2-b^2}\right] + by\left[\frac{b+a-b+a}{b^2-a^2}\right] = 2$$

$$\frac{2a^2b+2ab^2-2a^2by}{b(a^2-b^2)} + \frac{2aby}{b^2-a^2} = 2$$

$$2a^2b + 2ab^2 - 2a^2by - 2ab^2y = 2b(a^2 - b^2)$$

$$2ab(a + b) - 2aby(a + b) = 2b(a^2 - b^2)$$

$$2ab(1 - y) = 2b(a + b)$$

$$1 - y = \frac{a+b}{a}$$

$$y = 1 - \frac{a+b}{a}$$

$$y = \frac{b}{a}.$$

Putting the value of y in equation (3)

$$x = \frac{a+b-a\left(\frac{b}{a}\right)}{b}$$

$$= \frac{a+b-b}{b}$$

$$x = \frac{a}{b}.$$ **Ans.**

EXERCISE 3.3

Solve the following for x and y by using method of elimination.

1. $8x - 3y = 13,\ 3x + 2y = 8$

Sol. We have, $8x - 3y = 13 \quad ...(1)$

$$3x + 2y = 8 \quad ...(2)$$

On multiply equation (1) by 2 and equation (2) by 3 and adding, we get,

$$16x - 6y = 26$$

$$9x + 6y = 24$$

$$25x = 50$$

$$x = 2$$

Putting the value of x in equation (1)

$$8 \times 2 - 3y = 13$$

$$16 - 3y = 13$$
$$-3y = 13 - 16$$
$$-3y = -3$$
$$y = 1$$

2. $\boldsymbol{x - y = 0{\cdot}9, \dfrac{11}{x+y} = 2\ (x \neq -y).}$

Sol. We have, $x - y = 0{\cdot}9$ ...(1)

$2x + 2y = 11$ ...(2)

On multiplying equation (1) by (2) and adding both the equation we get,

$$2x - 2y = 1{\cdot}8$$
$$2x + 2y = 11$$
$$4x = 12{\cdot}8$$
$$x = 3{\cdot}2.$$

Putting the value of x in equation (1)

$$3{\cdot}2 - y = 0{\cdot}9$$
$$-y = 0.9 - 3.2$$
$$-y = -2.3$$
$$y = 2{\cdot}3.$$

Ans.

3. $\boldsymbol{\dfrac{2x}{a} + \dfrac{y}{b} = 2, \dfrac{x}{a} - \dfrac{y}{b} = 4}$

Sol. $\dfrac{2x}{a} + \dfrac{y}{b} = 2$ ...(1)

$\dfrac{x}{a} - \dfrac{y}{b} = 4$ ...(2)

Put $\dfrac{x}{a} = g$ and $\dfrac{y}{b} = h$

$2g + h = 2$ ...(3)

$g - h = 4$ ...(4)

On adding the equation (3) and (4), we get

$\Rightarrow \quad 3g = 6$

$\Rightarrow \quad g = \dfrac{6}{3}$

$\Rightarrow \quad g = 2$

Put the value of g in equation (3)

$\Rightarrow 2 \times 2 + h = 2$

$\Rightarrow \quad 4 + h = 2$

$\Rightarrow \quad h = 2 - 4$

$\Rightarrow \quad h = -2$

$\because \quad g = \dfrac{x}{a} \Rightarrow \dfrac{x}{a} = 2$

$\Rightarrow \quad x = 2a$

$\because \quad h = \dfrac{y}{b} \Rightarrow \dfrac{y}{b} = -2$

$\Rightarrow \quad y = -2b.$ **Ans.**

4. $\boldsymbol{\dfrac{x}{3} + \dfrac{y}{4} = 4, \dfrac{5x}{6} + \dfrac{y}{8} = 4.}$

Sol. We have $\dfrac{x}{3} + \dfrac{y}{4} = 4$

$4x + 3y = 48$ ...(1)

$\dfrac{5x}{6} - \dfrac{y}{8} = 4$

$20x - 3y = 96$ ...(2)

By adding equation (1) and (2), we get

$$4x + 3y = 48$$
$$20x - 3y = 96$$
$$24x = 144$$
$$x = \frac{144}{24}$$
$$= 6.$$

On putting the value of x in equation (2)

$$20 \times 6 - 3y = 96$$
$$-3y = 96 - 120$$
$$-3y = -24$$
$$y = \frac{24}{3}$$
$$y = 8.$$

Ans.

5. $\boldsymbol{\dfrac{x}{a} + \dfrac{y}{b} = 2, ax - by = a^2 - b^2}$

Sol. We have $\dfrac{x}{a} + \dfrac{y}{b} = 2$ ...(1)

$ax - by = a^2 - b^2$ ...(2)

On dividing the equation (2) by a^2 and subtracting with equation (1) we get,

$$\frac{ax}{a^2} - \frac{by}{a^2} = \frac{a^2 - b^2}{a^2}$$
$$\frac{x}{a} - \frac{by}{b^2} = \frac{a^2 - b^2}{a^2}$$
$$\frac{x}{a} - \frac{by}{a^2} = \frac{a^2 - b^2}{a^2}$$
$$\frac{by}{a^2} + \frac{y}{b} = \frac{2}{1} - \frac{a^2 - b^2}{a^2}$$
$$\frac{b^2y + a^2y}{a^2b} = \frac{2a^2 - a^2 + b^2}{a^2}$$
$$\frac{y(a^2 + b^2)}{a^2b} = \frac{a^2 + b^2}{a^2}$$
$$y = b$$

On putting the value of y in equation (1) we get

$$\frac{x}{a} + \frac{b}{b} = 2$$

$$\frac{x}{a}+1 = 2$$

$$\frac{x}{a} = 2 - 1$$

$$\frac{x}{a} = 1$$

$$x = a.$$ **Ans.**

6. $(a + 2b)x + (2a - b)y = 2,$
$(a - 2b)x + (2a + b)y = 3$

Sol. We have,

$(a + 2b)x + (2a - b)y = 2$ ...(1)

$(a - 2b)x + (2a + b)y = 3$ ...(2)

Adding both the equation we get

$2ax + 4ay = 5$ ...(3)

Subtracting both the equation we get

$4bx - 2by = -1$ ...(4)

On multiplying the equation (3) by $2b$ and equation (4) by a, we get

$4abx + 8aby = 10b$ ...(5)

$4abx - 2aby = -a$ ...(6)

Subtracting equation (6) from equation (5) get

$$10aby = 10b + a$$

$$y = \frac{10b+a}{10ab}$$

On multiplying equation (6) by 5 and equation (5) we get

$$20abx = 10b - 4a$$

$$x = \frac{10b-4a}{20ab} = \frac{2(5b-2a)}{20ab}$$

$$= \frac{5b-2a}{10ab}$$

7. $2^x + 3^y = 17,\ 2^{x+2} - 3^{y+1} = 5$

Sol. We have, $2^x + 3^y = 17$ ...(1)

$2^{x+2} - 3^{y+1} = 5$ ...(2)

On multiplying the equation (1) by 2^2 and adding both the equations we get

$$2^x \times 2^2 + 3^y \times 2^2 = 17 \times 2^2$$

$$2^{x+2} + 3^y \times 2^2 = 68$$

$$2^{x+2} - 3^{y+1} = 5$$

$$3^y \times 2^2 + 3^{y+1} = 63$$

$$3^y \times 2^2 + 3^y \times 3 = 63$$

$$3^y(2^2 + 3^1) = 63$$

$$3^y(4 + 3) = 63$$

$$3^y = \frac{63}{7}$$

$$= 9$$

$$3^y = 9$$

$$\therefore \quad y = 2.$$

On putting the value of y in equation (1)

$$2^x + 3^2 = 17$$

$$2^x = 17 - 9$$

$$2^x = 8$$

$$\therefore \quad x = 3.$$ **Ans.**

EXERCISE 3.4

Solve for *x* and *y* by using method of cross-multiplication.

1. $x + y = 3,\ 2x + y = -2$

Sol. The given pair of equation is :

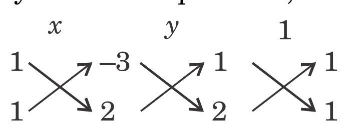
$$x + y = 3$$

$$x + y - 3 = 0$$

$$2x + y = -2$$

$$2x + y + 2 = 0$$

By cross multiplication, we get



$$\frac{x}{2+3} = \frac{y}{-6-2} = \frac{1}{1-2}$$

$$\frac{x}{5} = \frac{y}{-8} = \frac{1}{-1}$$

$$x = -5,\ y = 8.$$ **Ans.**

2. $\frac{x}{6}+\frac{y}{15} = 4,\ \frac{x}{3}-\frac{y}{12} = 4\frac{3}{4}$

Sol. We have $\frac{x}{6}+\frac{y}{15} = 4$

$$\frac{5x+2y}{30} = 4$$

$$5x + 2y = 120 \quad ...(1)$$

$$\frac{x}{3}-\frac{y}{12} = 4\frac{3}{4}$$

$$\frac{4x-y}{12} = \frac{19}{4}$$

$$16x - 4y = 228$$

$$4(4x - y) = 228$$

$$4x - y = \frac{228}{4}$$

$$4x - y = 57 \qquad ...(2)$$

So, $5x + 2y - 120 = 0$

$4x - y - 57 = 0$

By cross multiplication, we get

$$\begin{array}{cccccc} & x & & y & & 1 \\ 2 & & -120 & & 5 & & 2 \\ -1 & & -57 & & 4 & & -1 \end{array}$$

$$\frac{x}{-114-120} = \frac{y}{-480+285} = \frac{1}{-5-8}$$

$$\frac{x}{-234} = \frac{y}{-195} = \frac{1}{-13}$$

$x = 18, \quad y = 15.$ **Ans.**

3. $4x - 0{\cdot}5y = 12{\cdot}5$, $3x - 0{\cdot}8y = 8{\cdot}2$

Sol. We have,

$4x - 0{\cdot}5y = 12{\cdot}5; 4x - 0{\cdot}5y - 12{\cdot}5 = 0$

$3x - 0{\cdot}8y = 8{\cdot}2; 3x - 0{\cdot}8y - 8{\cdot}2 = 0$

By cross multiplication, we get

$$\begin{array}{cccccc} & x & & y & & 1 \\ -0{\cdot}5 & & -12{\cdot}5 & & 4 & & -0{\cdot}5 \\ -0{\cdot}8 & & -8{\cdot}2 & & 3 & & -0{\cdot}8 \end{array}$$

$$\frac{x}{4{\cdot}1-10} = \frac{y}{-37{\cdot}5+32{\cdot}8}$$

$$= \frac{1}{-3{\cdot}2+1{\cdot}5}$$

$$\frac{x}{-5{\cdot}9} = \frac{y}{-4{\cdot}7} = \frac{1}{-1{\cdot}7}$$

$x = 3.47, y = 2.76.$ **Ans.**

4. $\frac{2}{x} + \frac{3}{y} = 2$, $\frac{1}{x} - \frac{1}{2y} = \frac{1}{3}$

Sol. We have, $\frac{2}{x} + \frac{3}{y} = 2$

$$\frac{2y+3x}{xy} = 2$$

$$3x + 2y - 2xy = 0 \qquad ...(1)$$

$$\frac{1}{x} - \frac{1}{2y} = \frac{1}{3}$$

$$\frac{2y-x}{2xy} = \frac{1}{3}$$

$$-3x + 6y - 2xy = 0 \qquad ...(2)$$

On cross multiplication

$$\begin{array}{cccccc} & x & & y & & 1 \\ 2 & & -2 & & 3 & & 2 \\ 6 & & -2 & & -3 & & 6 \end{array}$$

$$\frac{x}{-4+12} = \frac{y}{6+6} = \frac{1}{18+6}$$

$$\frac{x}{8} = \frac{y}{12} = \frac{1}{24}$$

$$x = \frac{1}{3} \qquad \frac{1}{y} = 2$$

$$y = \frac{1}{2} \qquad x = \frac{1}{3}.$$

5. $\frac{2x}{3} + \frac{3y}{5} = 17$, $\frac{3x}{4} + \frac{2y}{3} = 19$

Sol. We have, $\frac{2x}{3} + \frac{3y}{5} - 17 = 0$

$$\frac{3x}{4} + \frac{2y}{3} - 19 = 0$$

On cross multiplication, we get

$$\begin{array}{cccccc} & x & & y & & 1 \\ \frac{3}{5} & & -17 & & \frac{2}{3} & & \frac{3}{5} \\ \frac{2}{3} & & -19 & & \frac{3}{4} & & \frac{2}{3} \end{array}$$

$$\frac{x}{\frac{57}{5}+\frac{34}{3}} = \frac{y}{-\frac{51}{4}+\frac{38}{3}} = \frac{1}{\frac{4}{9}-\frac{9}{20}}$$

$$\frac{x}{\frac{-171+170}{15}} = \frac{y}{\frac{-153+152}{12}} = \frac{1}{\frac{80-81}{180}}$$

$$\frac{x}{-1/15} = \frac{y}{-1/12} = \frac{1}{-1/180}$$

$x = 12, y = 15.$ **Ans.**

6. $\frac{2}{x-1} + \frac{3}{y+1} = 2$, $\frac{3}{x-1} + \frac{2}{y+1} = \frac{13}{6}$, $x \neq 1$; $y \neq -1$

Sol. We have, $\frac{2}{x-1} + \frac{3}{y+1} = 2 \qquad ...(i)$

$$\frac{3}{x-1} + \frac{2}{y+1} = \frac{13}{6} \qquad ...(ii)$$

On cross multiplication, we get

$$\begin{array}{cccccc} & x & & y & & 1 \\ 3 & & -2 & & 2 & & 3 \\ 2 & & \frac{13}{6} & & 3 & & 2 \end{array}$$

$$\frac{x}{-\frac{13}{2}+4} = \frac{y}{-6+\frac{13}{3}} = \frac{1}{4-9}$$

$$\frac{x\times 2}{-5} = \frac{y\times 3}{-5} = \frac{1}{-5}$$

$$x = \frac{1}{2} \qquad y = \frac{1}{3}$$

$$\frac{1}{x} = 2 \qquad \frac{1}{y} = 3$$

$$x - 1 = 2 \qquad y + 1 = 3$$

$$x = 2 + 1 = 3, \ y = 3 - 1 = 2.$$

7. $\boldsymbol{ax + by = a^2,\ bx + ay = b^2.}$

Sol. We have, $ax + by - a^2 = 0$ ...(i)

$bx + ay - b^2 = 0$ ...(ii)

On cross multiplication we get,

$$\begin{array}{cccccc} & x & & y & & 1 \\ b & & -a^2 & & a & & b \\ a & & -b^2 & & b & & a \end{array}$$

$$\frac{x}{-b^3+a^3} = \frac{y}{-a^2b+b^2a} = \frac{1}{a^2-b^2}$$

$$x = \frac{a^3-b^3}{a^2-b^2} = \frac{(a^2+ab+b^2)(a-b)}{(a+b)(a-b)}$$

$$x = \frac{a^2+ab+b^2}{a+b}$$

$$y = \frac{-a^2b+b^2a}{a^2-b^2} = \frac{-ab(a-b)}{(a+b)(a-b)}$$

$$y = -\frac{ab}{a+b}.$$ **Ans.**

8. $\boldsymbol{\frac{5}{x+y} - \frac{2}{x-y} = -1,\ \frac{15}{x+y} + \frac{7}{x-y} = 10;\ x + y \neq 0,\ x - y \neq 0.}$

Sol. We have, $\frac{5}{x+y} - \frac{2}{x-y} + 1 = 0$ ...(i)

$$\frac{15}{x+y} + \frac{7}{x-y} - 10 = 0 \ \text{...(ii)}$$

On cross multiplication we get,

$$\begin{array}{cccccc} & x & & y & & 1 \\ -2 & & 1 & & 5 & & -2 \\ 7 & & -10 & & 15 & & 7 \end{array}$$

$$\frac{x}{20-7} = \frac{y}{15+50} = \frac{1}{35+30}$$

$$\frac{x}{13} = \frac{y}{65} = \frac{1}{65}$$

$$x = \frac{1}{5} \ y = 1.$$ **Ans.**

9. $\boldsymbol{x - y = a + b;\ ax + by = a^2 - b^2.}$

Sol. We have,

$x - y = a + b;$

$x - y - (a + b) = 0$ ...(i)

$ax + by = a^2 - b^2;$

$ax + by - (a^2 - b^2) = 0$ ...(ii)

On cross multiplication, we get

$$\begin{array}{cccccc} & x & & y & & 1 \\ -1 & & -(a+b) & & 1 & & -1 \\ b & & -(a^2-b^2) & & a & & b \end{array}$$

$$\frac{x}{a^2-b^2+ab+b^2}$$

$$= \frac{y}{-a^2-ab+a^2-b^2}$$

$$= \frac{1}{b+a}$$

$$x = \frac{a^2+ab}{a+b} = \frac{a(a+b)}{a+b} = a$$

$$y = \frac{-ab-b^2}{a+b} = \frac{-b(a+b)}{a+b} = -b.$$

10. $\boldsymbol{\frac{x}{a} + \frac{y}{b} = 2,\ ax - by = a^2 - b^2;\ a \neq 0,\ b \neq 0.}$

Sol. We have, $\frac{x}{a} + \frac{y}{b} = 2$

$bx + ay - 2ab = 0$

$ax - by - (a^2 - b^2) = 0$

On cross multiplying, we get

$$\begin{array}{cccccc} & x & & y & & 1 \\ a & & -2ab & & b & & a \\ -b & & -(a^2-b^2) & & a & & -b \end{array}$$

$$\frac{x}{-a^3+ab^2-2ab^2} = \frac{y}{-2a^2b+a^2b-b^3}$$

$$= \frac{1}{-b^2 - a^2}$$

$$x = \frac{-a^3 + ab^2 - 2ab^2}{-b^2 - a^2}$$

$$= \frac{-a^3 - ab^2}{-b^2 - a^2} = \frac{-a(a^2 + b^2)}{-(a^2 + b^2)}$$

$$x = a$$

$$y = \frac{-2a^2b + a^2b - b^3}{-b^2 - a^2}$$

$$= \frac{-a^2b - b^3}{-b^2 - a^2} = \frac{-b(a^2 + b^2)}{-(b^2 + a^2)}$$

$$y = b$$

11. $ax - ay = 2$; $(a - 1)x + (a + 1)y = 2(a^2 + 1)$

Sol. We have,

$$ax - ay - 2 = 0 \qquad ...(i)$$

$$(a - 1)x + (a + 1)y - 2(a^2 + 1) = 0 \quad ...(ii)$$

On cross multiplying, we get

	x		y		1
$-a$		-2		a	$-a$
$a + 1$		$-2(a^2 + 1)$		$a-1$	$a + 1$

$$\frac{x}{2a(a^2 + 1) + 2(a + 1)}$$

$$= \frac{y}{-2(a - 1) + 2a(a^2 + 1)}$$

$$= \frac{1}{a(a + 1) + a(a - 1)}$$

$$x = \frac{2a^3 + 2a + 2a + 2}{a^2 + a + a^2 - a}$$

$$= \frac{2(a^3 + 2a + 1)}{2a^2} = \frac{a^3 + 2a + 1}{a^2}.$$

$$y = \frac{-2a + 2 + 2a^3 + 2a}{a^2 + a + a^2 - a}$$

$$= \frac{2(1 + a^3)}{2a^2} = \frac{a^3 + 1}{a^2}$$ **Ans.**

EXERCISE 3.5

1. Find k for unique solution

$2x + y = 3$

$ky + x = 8$

Sol. The given system is of the form

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

$a_1 = 2$, $b_1 = 1$ and $a_2 = 1$, $b_2 = k$ and $c_1 = -3$ and $c_2 = -8$

Clearly, for unique solution $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\frac{2}{1} \neq \frac{1}{k}$$

$$k \neq \frac{1}{2}$$

Hence, k can take any value except $\frac{1}{2}$.

2. Find k for unique solution :

$kx + y = 10$

$ky - x = 7$.

Sol. The given system is of the form

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

$a_1 = k$, $a_2 = -1$ and $b_1 = 1$, $b_2 = k$ and $c_1 = -10$ and $c_2 = -7$

Clearly, for unique solution $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\frac{k}{1} = \frac{1}{k}$$

$\therefore$ k is possible for any real value.

3. Show that the system of equations $5x - 10y = 0$ and $x + y = 3$ has unique solution. Also find the value of x and y.

Sol. The given system of equations can be written as

$$5x - 10y = 0$$
$$x + y - 3 = 0$$

The given system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_{2y} + c_2 = 0$$

Where,

$a_1 = 5$, $b_1 = -10$, $c_1 = 0$

$a_2 = 1, \quad b_2 = 1, \quad c_2 = -3$

Clearly, for unique solution $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\frac{5}{1} \neq \frac{-10}{1}$$

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

$$= \frac{30-0}{5+10}$$

$$= 2$$

$$y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

$$= \frac{0+15}{5+10}$$

$= 1.$ **Ans.**

4. Find k for unique solution :

$x + y = 2$

$3x - 2y = k$

Sol. The given system is in the form of

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where,

$a_1 = 1 \quad b_1 = 1 \quad c_1 = -2$

$a_2 = 3 \quad b_2 = -2 \quad c_2 = -k$

Clearly, for unique solution $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\frac{1}{3} \neq \frac{1}{-2}$$

$\therefore$ It is possible for every real value of k.

5. Show that the system of equations $x - y = 3$ and $2x + 3y = 7$ has unique solution.

Sol. We have,

$x - y = 3; \quad x - y - 3 = 0$

$2x + 3y = 7; \quad 2x + 3y - 7 = 0$

The given system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where,

$a_1 = 1 \quad b_1 = -1 \quad c_1 = -3$

$a_2 = 2 \quad b_2 = 3 \quad c_2 = -7$

For unique solution

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{1}{2} \neq \frac{-1}{3}$$

$\therefore$ Hence it is proved that equations has unique solution.

6. Show that the system of equations $ax + by = c$ and $bx + ay = c$ can have infinitely many solution. Find the relations between a and b in this condition.

Sol. We have,

$ax + by = c; \quad ax + by - c = 0$

$bx + ay = c; \quad bx + ay - c = 0$

The given system of equations in the form of

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where,

$a_1 = a \quad b_1 = b \quad c_1 = -c$

$a_2 = b \quad b_2 = a \quad c_2 = -c$

For infinitely many solutions :

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{a}{b} = \frac{b}{a} = \frac{-c}{-c}$$

$\therefore \quad a = b.$

7. For which values of a and b, will the following pair of linear equations have infinitely many solutions ?

$x + 2y = 1$

$(a - b)x + (a + b)y = a + b - 2$

Sol. We have, $x + 2y = 1$ and $(a - b)x + (a + b)y = a + b - 2$

The given system of equations in the form of

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Here,

$a_1 = 1 \quad b_1 = 2 \quad c_1 = -1$

$a_2 = a-b \quad b_2 = a+b \quad c_2 = -a-b+2$

For infinitely many Solutions :

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{1}{a-b} = \frac{2}{a+b} = \frac{-1}{-a-b+2}$$

$\therefore \quad -a-b+2 = -a+b$

$b = 1 \quad a = 3.$

8. Find the value(s) of p in the given pair of equations :

$$\mathbf{3x - y - 5 = 0}$$

and $\mathbf{6x - 2y - p = 0}$

if the lines represented by these equations are parallel.

Sol. We have, $3x - y - 5 = 0$

$$6x - 2y - p = 0$$

The given system of equations in the form of

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

Here,

$$a_1 = 3, b_1 = -1$$
$$c_1 = -5$$
$$a_2 = 6, b_2 = -2$$
$$c_2 = -p$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{3}{6} = \frac{-1}{-2} = \frac{-5}{-p}$$

$p = 10.$ **Ans.**

EXERCISE 3.6

Solve for x and y by using method of substitution :

1. $\mathbf{\frac{a}{x} - \frac{b}{y} = 0,\ \frac{ab^2}{x} + \frac{a^2b}{y} = a^2 + b^2,}$

$\mathbf{x \neq 0, y \neq 0}$

Sol. Let, $\frac{1}{x} = u$

$$\frac{1}{y} = v$$

$au - bv = 0$ ...(1)

$ab^2u + a^2bv = a^2 + b^2$ ...(2)

From equation (1)

$v = \frac{au}{b}$ ...(3)

Substituting the value of v from equation (3) in equation (2)

$$ab^2u + a^2b\left(\frac{au}{b}\right) = a^2 + b^2$$

$$ab^2u + a^3u = a^2 + b^2$$
$$au(b^2 + a^2) = a^2 + b^2$$

$$a = \frac{1}{u}$$
$$= x.$$

On substituting the value of a in equation (1)

$$xu - bv = 0$$

$$x \times \frac{1}{x} - bv = 0$$

$$b = \frac{1}{v}$$
$$= y.$$

2. $\mathbf{5x + \frac{4}{y} = 9,}$

$\mathbf{7x - \frac{2}{y} = 5,}$

$\mathbf{y \neq 0.}$

Sol. We have, $5x + \frac{4}{y} = 9$

$$7x - \frac{2}{y} = 5$$

Let $\frac{1}{y} = a$

$5x + 4a = 9$ ...(1)

$7x - 2a = 5$ ...(2)

from equation (i)

$x = \frac{9-4a}{5}$ ...(3)

Substituting the value of x in equation (2)

$$7\left[\frac{9-4a}{5}\right] - 2a = 5$$

$$\frac{63-28a}{5} - 2a = 5$$

$$63 - 28a - 10a = 25$$
$$63 - 38a = 25$$
$$-38a = 25 - 63$$
$$-38a = -38$$
$$a = 1 = y$$

Putting the value of y in equation (1)

$$5x + 4 \times 1 = 9$$
$$5x + y = 9$$

$$5x = 9 - 4$$
$$5x = 5$$
$$x = 1.$$

3. $\boldsymbol{\frac{m}{x} - \frac{n}{y} = a}$

$\boldsymbol{px - qy = 0}$

$\boldsymbol{x \neq 0, y \neq 0}$

Sol. We have,

$$\frac{m}{x} - \frac{n}{y} = a; \quad \frac{m}{x} - \frac{n}{y} - a = 0 \quad ...(1)$$

$$px - qy = 0; \quad px - qy = 0 \quad ...(2)$$

From equation (2) we get

$$x = \frac{qy}{p}$$

On substituting the value of x in equation (1)

$$m \times \frac{p}{qy} - \frac{n}{y} = a$$

$$\frac{mp - nq}{qy} = a$$

$$mp - nq = aqy$$

$$y = \frac{mp - nq}{aq}$$

On substituting the value of y in equation (2)

$$px - q\left[\frac{mp - nq}{aq}\right] = 0$$

$$px - \left(\frac{mp - nq}{a}\right) = 0$$

$$apx - mp + nq = 0$$

$$apx = mp - nq$$

$$x = \frac{mp - nq}{ap}. \textbf{ Ans.}$$

4. $\boldsymbol{\frac{2}{y} + \frac{3}{x} = \frac{7}{xy}, \frac{1}{y} + \frac{9}{x} = \frac{11}{xy}, x \neq 0, y \neq 0}$

Sol. We have $\frac{2}{y} + \frac{3}{x} = \frac{7}{xy}$

$$2x + 3y = 7 \quad ...(1)$$

$$\frac{1}{y} + \frac{9}{x} = \frac{11}{xy}$$

$$x + 9y = 11 \quad ...(2)$$

From equation (2) we get

$$x = 11 - 9y \quad ...(3)$$

On substituting the value of x in the equation (1) we get,

$$2(11 - 9y) + 3y = 7$$
$$22 - 18y + 3y = 7$$
$$22 - 15y = 7$$
$$-15y = 7 - 22$$
$$-15y = -15$$
$$y = 1.$$

On substituting the value of y in equation (3) we get

$$x = 11 - 9 \times 1$$
$$x = 2.$$

5. $\boldsymbol{\frac{1}{2x} - \frac{1}{y} = -1, \frac{1}{x} + \frac{1}{2y} = 8, x \neq 0,}$ $\boldsymbol{y} \neq 0$

Sol. We have, $\frac{1}{2x} - \frac{1}{y} = -1 \quad ...(1)$

$$\frac{1}{x} + \frac{1}{2y} = 8 \quad ...(2)$$

From equation (1) we have,

$$\frac{1}{y} = 1 + \frac{1}{2x} \quad ...(3)$$

Substituting the value of $\frac{1}{y}$ in equation (2)

$$\frac{1}{x} + \frac{1}{2}\left[1 + \frac{1}{2x}\right] = 8$$

$$\frac{1}{x} + \frac{1}{2}\left[\frac{2x+1}{2x}\right] = 8$$

$$\frac{1}{x} + \frac{2x+1}{4x} = 8$$

$$\frac{4 + 2x + 1}{4x} = 8$$

$$4 + 2x + 1 = 32x$$
$$5 = 32x - 2x$$
$$5 = 30x$$
$$\frac{5}{30} = x$$
$$x = \frac{1}{6}$$

On substituting the value of x in equation (3)

$$\frac{1}{y} = 1 + \frac{1}{2 \times \frac{1}{6}}$$

$$\frac{1}{y} = 1 + \frac{1}{\frac{2}{6}}$$

$$\frac{1}{y} = 1 + \frac{6}{2}$$

$$\frac{1}{y} = 1 + 3$$

$$\frac{1}{y} = 4$$

$\therefore \quad y = \frac{1}{4}$. **Ans.**

6. $\mathbf{\frac{2}{x} + \frac{3}{y} = \frac{9}{xy}, \frac{4}{x} + \frac{9}{y} = \frac{21}{xy}, x \neq 0, y \neq 0}$

Sol. We have, $\frac{2}{x} + \frac{3}{y} = \frac{9}{xy}$

$$2y + 3x = 9 \quad ...(1)$$

$$\frac{4}{x} + \frac{9}{y} = \frac{21}{xy}$$

$$4y + 9x = 21 \quad ...(2)$$

From equation (2) we get

$$x = \frac{21 - 4y}{9} \quad ...(3)$$

Substituting the value of x in equation (1)

$$2y + 3\left[\frac{21 - 4y}{9}\right] = 9$$

$$y = 3$$

$$18y + 63 - 12y = 81$$

$$6y = 81 - 63$$

$$6y = 18$$

$$y = 3.$$

Substituting the value of y in equation (3)

$$x = \frac{21 - 4 \times 3}{9}$$

$$x = \frac{21 - 12}{9}$$

$x = 1$. **Ans.**

7. $\mathbf{x + y = 5xy, 3x + 2y = 13xy}$

Sol. We have, $x + y = 5xy \quad ...(1)$

$$3x + 2y = 13xy \quad ...(2)$$

From equation (1) we get

$$x = 5xy - y \quad ...(3)$$

Substituting the value of x in equation (2)

$$3[5xy - y] + 2y = 13xy$$

$$15xy - 3y + 2y = 13xy$$

$$y = 2xy$$

$$x = \frac{1}{2}.$$

Substituting the value of x in equation (3)

$$\frac{1}{2} = 5 \times \frac{1}{2} \times y - y$$

$$\frac{1}{2} = y\left[\frac{5}{2} - 1\right]$$

$$\frac{1}{2} = y\left[\frac{5 - 2}{2}\right]$$

$$\frac{1}{2} = y\left(\frac{3}{2}\right)$$

$y = \frac{1}{3}$. **Ans.**

8. $\mathbf{\frac{5}{x + y} - \frac{2}{x - y} = -1, \frac{15}{x + y} + \frac{7}{x - y} = 10, x + y \neq 0, x - y \neq 0}$

Sol. Let $\frac{1}{x + y} = a, \frac{1}{x - y} = b$

We have $5a - 2b = -1 \quad ...(1)$

$$15a + 7b = 10 \quad ...(2)$$

From equation (1) we get

$$a = \frac{-1 + 2b}{5} \quad ...(2)$$

Substituting the value of a in equation (2)

$$15\left[\frac{-1 + 2b}{5}\right] + 7b = 10$$

$$\frac{-15 + 30b}{5} + \frac{7b}{1} = 10$$

$$\frac{-15 + 30b + 35b}{5} = 10$$

$$-15 + 30b + 35b = 50$$

$$65b = 50 + 15$$

$$65b = 65$$

$$b = 1$$

$\because \quad b = \frac{1}{x - y}$

$\therefore \quad \frac{1}{x - y} = 1$

Putting the value of b in equation (3)

$$a = \frac{-1+2\times 1}{5}$$

$$= \frac{1}{5}$$

$$\therefore \quad \frac{1}{x+y} = \frac{1}{5}$$

Putting the value of x in equ. (4)

$$3 + y = 5$$
$$y = 5 - 3$$
$$y = 2.$$

$$\frac{1}{x+y} = \frac{1}{5} \qquad \frac{1}{x-y} = 1$$

$$x + y = 5 \quad ...(4)$$
$$x - y = 1 \quad ...(5)$$

Adding the both equ. (4) and (5) we get

$$2x = 6$$
$$x = 3$$

9. $27x + 31y = 85$, $31x + 27y = 89$

Sol. We have, $27x + 31y = 85$ ...(1)

$31x + 27y = 89$ ...(2)

From equation (1) we get

$$x = \frac{85-31y}{27} \quad ...(3)$$

Substituting the value of x in equation (2)

$$31\left[\frac{85-31y}{27}\right] + 27y = 89$$

$$2635 - 961y + 729y = 2403$$
$$232\,y = 232$$
$$y = 1.$$

Substituting the value of y in equation (3)

$$x = \frac{85-31\times 1}{27}$$

$x = 2.$ **Ans.**

10. $37x - 39y = 150$, $39x - 37y = 154$.

Sol. We have, $37x - 39y = 150$ ...(1)

$39x - 37y = 154$ ...(2)

From equation (1)

$$x = \frac{150+39y}{37} \quad ...(3)$$

Substituting the value of x in equation (2)

$$39\left[\frac{150+39y}{37}\right] - 37y = 154$$

$$5850 + 1521y - 1369y = 5698$$
$$152y = -152$$
$$y = -1.$$

Substituting the value of y in equation (3)

$$x = \frac{150+39\times -1}{37}$$

$= 3.$ **Ans.**

EXERCISE 3.7

1. 5 chairs and 4 tables together cost ₹ 2,800 while 4 chairs and 3 tables together cost ₹ 2170. Find the cost of a chair and that of a table.

Sol. Let cost of chair = x

cost of tables = y

$$\therefore \quad 5x + 4y = 2{,}800 \quad ...(1)$$
$$4x + 3y = 2170 \quad ...(2)$$

From equation (1) we get

$$x = \frac{2{,}800-4y}{5} \quad ...(3)$$

Substituting the value of x in equation (2)

$$4\left[\frac{2{,}800-4y}{5}\right] + 3y = 2{,}170$$

$$11{,}200 - 16y + 15y = 10{,}850$$
$$-y = 10{,}850 - 11200$$
$$y = 350.$$

Substituting the value of y in equation (3)

$$x = \frac{2800-4\times 350}{5}$$

$= 280.$

Hence, cost of chair = ₹ 280 and table = ₹ 350.

2. 37 pens and 53 pencils together cost ₹ 820 while 53 pens and 37 pencils together cost ₹ 980. Find the cost of a pen and that of a pencil.

Sol. Let cost of pens = x

cost of pencil = y

$$\therefore \quad 37x + 53y = 820$$
$$53x + 37y = 980$$

On cross multiplication, we get

$$\begin{array}{cccccc} & x & & y & & 1 \\ 53 & & -820 & & 37 & & 53 \\ 37 & & -980 & & 53 & & 37 \end{array}$$

$$= \frac{x}{(-980 \times 53) - (-820 \times 37)}$$

$$= \frac{y}{(-820 \times 53) - (-980 \times 37)}$$

$$= \frac{1}{(37 \times 37) - (53 \times 53)}$$

$$\frac{x}{-21,600} = \frac{y}{-7,200} = \frac{1}{-1,440}$$

$$x = \frac{-21,600}{-1,440} = 15.$$

$$y = \frac{-7,200}{-1,440} = 5.$$

$\therefore$ Cost of a pen = ₹ 15 and pencil = ₹ 5.

3. A lady has only 20-paisa coins and 25-paisa coins in her purse. If she has 50 coins in all totalling ₹ 11·50, how many coins of each kind does she have ?

Sol. Let No. of 20 paisa coins = x

No. of 25 paisa coins = y

Hence, $\cdot 20x + \cdot 25\, y = 11\cdot 50$ ...(1)

$x + y = 50$ ...(2)

From equation (2)

$y = 50 - x$ ...(3)

Substituting y in equation (1)

$\cdot 20x + \cdot 25(50 - x) = 11\cdot 50$

$\cdot 20x + 12\cdot 5 - \cdot 25x = 11\cdot 50$

$x = 20.$

Substituting the value of x in equation (3)

$y = 50 - 20 = 30$

Hence, she has 20 of 20 paisa coins and 30 of 25 paisa coins.

4. The sum of two numbers is 137 and their difference is 43. Find the numbers.

Sol. Let the two number = x and y

$x + y = 137$ ...(1)

$x - y = 43$ ...(2)

From equation (1) we get :

$x = 137 - y$ ...(3)

Substituting the value of x in equation (2)

$137 - y - y = 43$

$-2y = 43 - 137$

$-2y = -94$

$$y = \frac{94}{2}$$

$y = 47.$

Substituting the value of y in equation (3).

$x = 137 - 47$

$= 90$

Hence, two numbers is 90 and 47.

5. Find two numbers such that the sum of twice the first and thrice the second is 92, and four times the first exceeds seven times the second the sum of by 2.

Sol. Let the two numbers = x and y

$2x + 3y = 92$ ...(1)

$4x - 7y = 2$ ...(2)

Multiply equation (1) by 2 and substracting equ. (2) we get :

$$\begin{array}{rcl} 4x + 6y &=& 184 \\ 4x - 7y &=& 2 \\ - \quad + & & - \\ \hline 13y &=& 182 \end{array}$$

$y = 14.$

Substituting the value of y in equation (1)

$2x + 3 \times 14 = 92$

$2x + 42 = 92$

$2x = 92 - 42$

$2x = 50$

$$x = \frac{50}{2}$$

$x = 25.$

Hence, the two numbers = 25 and 14.

6. Find two numbers such that the sum of thrice the first and second is 142, and four times the first exceeds the second by 138.

Sol. Let the two numbers be = x and y

$3x + y = 142$ ...(1)

$4x - y = 138$ ...(2)

By adding both the equations we get :

$$7x = 280$$

$$x = \frac{280}{7}$$

$$x = 40.$$

Substituting the value of x in equation (1) we get :

$$3 \times 40 + y = 142$$

$$y = 142 - 120$$

$$y = 22.$$

Hence, both the numbers = 40,22.

7. If 45 is subtracted from twice the greater of two numbers, it results in the other number. If 21 is subtracted from twice the smaller number. It results in the greater number. Find the no.

Sol. Let the greater no. = x and smaller no. = y

$\therefore$ $2x - 45 = y$ ...(1)

$2y - 21 = x$ ...(2)

Substituting x from equation (2) in equation (1) we get :

$$2[2y - 21] - 45 = y$$

$$4y - 42 - 45 = y$$

$$-87 = y - 4y$$

$$-87 = -3y$$

$$\frac{87}{3} = y$$

$$y = 29.$$

Substituting y in equation (1) we get :

$$2x - 45 = 29$$

$$2x = 29 + 45$$

$$2x = 74$$

$$x = \frac{74}{2}$$

$$x = 37.$$

Hence, greater no. = 37 and smaller no, = 29.

8. If three times the larger of two numbers is divided by the smaller, we get 4 as the quotient and 8 as the remainder. If five times the smaller is divided by the larger. We get 3 as the quotient and 5 as the remainder. Find the numbers.

Sol. Let the larger number = x and smaller numbers = y

$3x = y \times 4 + 8$ ...(1)

$5y = x \times 3 + 5$ ...(2)

From equation (1) we get :

$x = \frac{4y+8}{3}$ ...(3)

Substituting x in equation (1)

$$5y = \frac{4y+8}{3} \times 3 + 5$$

$$15y = 12y + 24 + 15$$

$$15y - 12y = 39$$

$$3y = 39$$

$$y = \frac{39}{3}$$

$$y = 13.$$

Substituting the value of y in equation (3) we get :

$$x = \frac{4 \times 13 + 8}{3}$$

$$x = \frac{52 + 8}{3}$$

$$x = \frac{60}{3}$$

$$x = 20$$

Hence, the largest number = 20 and smallers number = 13.

9. If 2 is added to each of two given numbers, their ratio becomes 1 : 2. Howevers, if 4 is subtracted from each of the given numbers, the ratio becomes 5 : 11. Find the numbers.

Sol. Let the two given numbers = x and y

$$\frac{x+2}{y+2} = \frac{1}{2}$$

$$\frac{x-4}{y-4} = \frac{5}{11}$$

$$2x + 4 = y + 2$$

$2x - y = -2$ ...(1)

$$11x - 44 = 5y - 20$$

$11x - 5y = 24$ ...(2)

From equation (1) we get :

$y = 2x + 2$ ...(3)

Substituting the value of y in equation (2)

$11x - 5 \times (2x + 2) = 24$

$11x - 10x - 10 = 24$

$x = 34.$

Substituting the value of x in equation (3)

$y = 2 \times 34 + 2$

$y = 70.$

Both the numbers = 34, 70.

10. The difference between two numbers is 14 and the difference between their squares is 448. Find the numbers.

Sol. Let the two numbers = x and y

$x - y = 14$ ...(1)

$x^2 - y^2 = 448$ ...(2)

From equation (1) we get :

$x = 14 + y$ ...(3)

$(14 + y)^2 - y^2 = 448$

$196 + y^2 + 28y - y^2 = 448$

$196 + 28y = 448$

$28y = 448 - 196$

$28y = 252$

$y = \frac{252}{28}$

$y = 9.$

Substituting the value of y in equation (3)

$14 + 9 = x$

$x = 23.$

Hence, both number = 23, 9.

11. The sum of the digits of a two digit number is 12. The number obtained by interchanging its digits exceeds the given number by 18. Find the number.

Sol. Let us assume x and y are the two digits of no. Therefore, two-digit number is $= 10x + y$ and the reversal number is $= 10y + x$

Given $x + y = 12$

$\therefore$ $y = 12 - x$ ...(1)

Also given

$10y + x - 10x - y = 18$

$9y - 9x = 18$

$y - x = 2$ ...(2)

Substituting the value of y from equation (1) in (2)

$12 - x - x = 2$

$12 - 2x = 2$

$2x = 10$

$x = 5$

Substituting the value of x in equation (2)

$y - 5 = 2$

$y = 7.$

Therefore, two digit number

$= 10x + y$

$= 10 \times 5 + 7$

$= 57.$

12. A number consisting of two digits is seven times the sum of its digits. When 27 is subtracted from the number, the digits are reversed. Find the no.

Sol. Let the two digits = x and y

$\therefore$ The number = $10x + y$

Given $10x + y = 7(x + y)$

$\Rightarrow$ $3x - 6y = 0$ ...(1)

$10x + y - 27 = 10y + x$

$\Rightarrow$ $9x - 9y = 27$ ...(2)

On multiplying equation (1) by 3 and with (2) subtracting

$$\begin{array}{r} 9x - 18y = 0 \\ 9x - 9y = 27 \\ - \quad + \quad - \\ \hline -9y = -27 \end{array}$$

$y = 3.$

On substituting the value of y in equation (1)

$3x - 6 \times 3 = 0$

$3x - 18 = 0$

$3x = 18$

$x = \frac{18}{3}$

$x = 6$

Then the number $= 10 \times 6 + 3 = 63$

13. The sum of the digits of a two digit number is 15. The number obtained by interchanging the digits exceeds the given number by 9. Find the number.

Sol. Let the two digits = x and y

$\therefore$ The number = $10x + y$

Given $x + y = 15$ ...(1)

$10y + x = 10x + y + 9$

$\Rightarrow$ $-9x + 9y = 9$

From equation (1) we get :

$y = 15 - x$

On substituting the value of y in equation (2)

$-9x + 9(15 - x) = 9$

$-9x + 135 - 9x = 9$

$$-18x = 9 - 135$$
$$-18x = -125$$
$$x = \frac{126}{18}$$
$$x = 7.$$

On substituting the value of x in equation (1) we get :

$$\Rightarrow \quad 7 + y = 15$$
$$y = 8.$$

$\therefore$ The number = $10 \times 7 + 8 = 78$.

14. A two-digit number is 3 more than 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the numbers.

Sol. Let the two digits $= x$ and y

$\therefore$ The number is $= 10x + y$

Given $\quad 10x + y = 3 + 4(x + y)$

$$10x + y = 3 + 4x + 4y$$
$$10x - 4x + y - 4y = 4$$
$$\Rightarrow \quad 6x - 3y = 3 \quad ...(1)$$
$$10x + y + 18 = 10y + x$$
$$\Rightarrow \quad 9x - 9y = -18 \quad ...(2)$$

On multiplying the equation (1) by 3 and subtracting with equation (2) we get :

$$18x - 9y = 9$$
$$9x - 9y = -18$$
$$- \quad + \quad +$$
$$9x = 27$$
$$x = 3.$$

On substituting the value of x in equation (1) we get :

$$6 \times 3 - 3y = 3$$
$$18 - 3y = 3$$
$$-3y = 3 - 18$$
$$-3y = -15$$
$$y = \frac{15}{3}$$
$$y = 5.$$

Hence, the number

$$= 10 \times 3 + 5 = 35.$$

15. A number consists of two digits when it is divided by the sum of its digits the quotient is 6 with no remainder when the number is diminished by 9, the digits are reversed. Find the number.

Sol. Let the two digits $= x$ and y

$\therefore$ The number $= 10x + y$

Given $\quad \dfrac{10x + y}{x + y} = 6$

$$10x + y = 6x + 6y$$
$$10x - 6x + y - 6y = 0$$
$$\Rightarrow \quad 4x - 5y = 0 \quad ...(1)$$
$$10x + y - 9 = 10y + x$$
$$\Rightarrow \quad 9x - 9y = 9 \quad ...(2)$$

From equation (1) we get :

$$\Rightarrow \quad x = \frac{5y}{4}$$

On substituting the value of x in equation (2)

$$\frac{9 \times 5y}{4} - 9y = 9$$
$$\frac{45}{4}y - \frac{9}{1}y = 9$$
$$\frac{45y - 36y}{4} = 9$$
$$45y - 36y = 36$$
$$9y = 36$$
$$y = \frac{36}{9}$$
$$y = 4.$$

On substituting the value of y in equation (1) we get :

$$4x - 5 \times 4 = 0$$
$$x = 5.$$

Hence, the number $= 10 \times 5 + 4 = 54$.

16. A two digit number is such that the product of its digits is 35. It 18 is added to the number, the digits interchange their places. Find the number.

Sol. Let the two digits of number $= x$ and y

$\therefore$ The number $= 10x + y$

Given $\quad xy = 35 \quad ...(1)$

$$10x + y + 18 = 10y + x$$
$$\Rightarrow \quad 9x - 9y = -18 \quad ...(2)$$

From equation (1) we get :

$$x = \frac{35}{y}$$

Substituting the value of x in equation (2) we get :

$$9 \times \frac{35}{y} - 9y = -18$$
$$315 - 9y^2 = -18y$$
$$9y^2 - 18y - 315 = 0 \quad ...(3)$$

Dividing whole equation (3) by 9

$$y^2 - 2y - 35 = 0$$
$$y^2 - 7y + 5y - 35 = 0$$
$$y(y - 7) + 5(y - 7) = 0$$
$$(y + 5)(y - 7)\ y = -5 \text{ or } 7$$

Digits can not be negative.

Therefore $y = 7$

Substituting y in equation (1) we get :

$$x \times 7 = 35$$
$$x = 5$$

Their fix no. = $10 \times 5 + 7 = 57$.

17. A two-digit number is such that the product of its digits is 14. It 45 is added to the number, the digits interchange, their places. Find the number.

Sol. Let the two digits = x and y

$\therefore$ The number = $10x + y$

Given : $xy = 14$ ...(1)

$$10x + y + 45 = 10y + x$$

$\Rightarrow$ $9x - 9y = -45$ ...(2)

Dividing equation (2) by 9

$\Rightarrow$ $x - y = -5$ ...(3)

From equation (3) we get :

$$x = -5 + y$$

Substituting the value of x in equation (1)

$$(-5 + y)y = 14$$
$$-5y + y^2 - 14 = 0$$
$$\Rightarrow \quad y^2 - 5y - 14 = 0$$
$$y^2 - 7y + 2y - 14 = 0$$
$$y(y - 7) + 2(y - 7) = 0$$
$$(y + 2)(y - 7) = 0$$
$$y = -2 \text{ or } 7$$

digits can not be negative hence

$$y = 7.$$

Substituting the value of y in equation (1) we get :

$$x \times 7 = 14$$
$$x = 2.$$

$\therefore$ The number = $10 \times 2 + 7 = 27$.

18. A two-digit number is such that the product of its digits is 18. When 63 is subtracted from the number the digits interchange their places. Find the number.

Sol. Let the digits = x and y

$\therefore$ The number = $10x + y$

Given : $xy = 18$ ...(1)

$$10x + y - 63 = 10y + x$$

$\Rightarrow$ $9x - 9y = 63$ ...(2)

dividing equation (2) by 9

$x - y = 7$ ...(3)

from equation (2), we get :

$$x = 7 + y$$

substituting the value of x in equation (1)

$$(7 + y)y = 18$$
$$y^2 + 7y - 18 = 0$$
$$y^2 + 9y - 2y - 18 = 0$$
$$y(y + 9) - 2(y + 9) = 0$$
$$(y - 2)(y + 9);\ y = 2 \text{ or } -9$$

digits can not be negative. Hence

$$y = 2.$$

substituting y in equation (1) we get :

$$x \times 2 = 18$$
$$x = 9.$$

$\therefore$The number= $10 \times 9 + 2 = 92$.

19. A two-digit number is four times the sum of its digits and twice the product of its digits. Find the number.

Sol. Let the two digits = a and b

$\therefore$ The number = $10a + b$

Given : $10a + b = 4(a + b)$

$\Rightarrow$ $6a - 3b = 0$ ...(1)

$10a + b = 2(ab)$

$\Rightarrow$ $10a + b = 2ab$ ...(2)

From equation (1) we get :

$$a = \frac{b}{2} \quad ...(3)$$

On substituting the value of a in equation (2)

$$10 \times \frac{b}{2} + b = 2 \times \frac{b}{2} \times b$$
$$\frac{10b}{2} + \frac{b}{1} = \frac{2b^2}{2}$$
$$\frac{10b + 2b}{2} = \frac{2b^2}{2}$$
$$20b + 4b = 4b^2$$
$$24b = 4b^2$$
$$\frac{24b}{4} = b^2$$
$$6b = b^2$$
$$b = 6.$$

substituting the value of b in equation (3) we get :

$$a = \frac{6}{2}$$

$a = 3.$

Hence, the number

$= 10 \times 3 + 6 = 36.$

20. The sum of the numerator and the denominator of a fraction is 8. If 3 is added to both of the numerator and the denominator, the fraction becomes $\frac{3}{4}$. Find the fraction.

Sol. Let the fraction $= \frac{a}{b}$

Given : $a + b = 8$ ...(1)

$$\frac{a+3}{b+3} = \frac{3}{4}$$
$$4a + 12 = 3b + 9$$
$$4a - 3b = 9 - 12$$
$$4a - 3b = -3$$

$\Rightarrow$ $4a - 3b + 3 = 0$ ...(2)

from equation (1) we get :

$$a = 8 - b$$

On substituting the value of a in equation (2) we get :

$$4(8 - b) - 3b + 3 = 0$$
$$32 - 4b - 3b + 3 = 0$$
$$32 - 7b + 3 = 0$$
$$-7b + 35 = 0$$
$$-7b = -35$$
$$b = \frac{35}{7}$$
$$b = 5.$$

On substituting the value of b in equation (1) we get :

$$a + 5 = 8$$
$$a = 8 - 5$$
$$a = 3.$$

Hence, the fraction $= \frac{a}{b} = \frac{3}{5}.$

21. The sum of numerator and denominator of a fraction is 3 less than twice the denominator. If each of the numerator and denominator is decreased by 1, the fraction becomes $\frac{1}{2}$. Find the fraction.

Sol. Let the fraction $= \frac{a}{b}$

Given : $a + b = 2b - 3$

$\Rightarrow$ $a - b + 3 = 0$ ...(1)

$$\frac{a-1}{b-1} = \frac{1}{2}$$
$$2a - 2 = b - 1$$
$$2a - b = -1 + 2$$
$$2a - b = 1$$

$\Rightarrow$ $2a - b - 1 = 0$ ...(2)

from equation (1), we get :

$a = b - 3$ ...(3)

On substituting the value of a in equation (2) we get :

$$2[b - 3] - b - 1 = 0$$
$$2b - 6 - b - 1 = 0$$
$$b - 7 = 0$$
$$b = 7.$$

On substituting the value of b in equation (3) we get :

$$a = 7 - 3$$
$$a = 4.$$

Hence, the fraction$= \frac{a}{b} = \frac{4}{7}.$

22. Find a fraction which becomes $\left(\frac{1}{2}\right)$ when 1 is subtrac-ted from the numerator and 2 is added to the denominator, and the fraction becomes $\left(\frac{1}{3}\right)$ when 7 is subtracted from the numerator and 2 is subtracted from the denominator.

Sol. Let the fraction $= \frac{a}{b}$

Given : $\frac{a-1}{b+2} = \frac{1}{2}$

$$2a - 2 = b + 2$$
$$2a - b = 2 + 2$$

$\Rightarrow$ $2a - b = 4$ ...(1)

$$\frac{a-7}{b-2} = \frac{1}{3}$$
$$3a - 21 = b - 2$$
$$3a - b = -2 + 21$$

$\Rightarrow$ $3a - b = 19$ ...(2)

On subtracting equation (2) from equation (1) we get :

$$2a - b = 4$$

$$\begin{array}{r} 3a - b = 19 \\ -\quad +\quad - \\ \hline -a = -15 \\ a = 15. \end{array}$$

On substituting the value of a in equation (1) we get :

$$2 \times 15 - b = 4$$
$$30 - b = 4$$
$$-b = 4 - 30$$
$$-b = -26$$
$$b = 26.$$

Hence, the fraction $= \frac{a}{b} = \frac{15}{26}$.

23. The denominator of a fraction is greater than its numerator by 11. If 8 is added to both its numerator and denominator, it becomes $\frac{3}{4}$. Find the fraction.

Sol. Let the fraction $= \frac{a}{b}$

Given : $a + 11 = b$

$\Rightarrow \quad a - b + 11 = 0 \quad$ ...(1)

$$\frac{a+8}{b+8} = \frac{3}{4}$$
$$4a + 32 = 3b + 24$$
$$4a - 3b = 24 - 32$$
$$4a - 3b = -8$$

$\Rightarrow \quad 4a - 3b + 8 = 0 \quad$ ...(2)

from equation (1), we get :

$a = b - 11 \quad$ ...(3)

On substituting the value of a in equation (2) we get :

$$4(b - 11) - 3b + 8 = 0$$
$$4b - 44 - 3b + 8 = 0$$
$$b = 36$$

On substituting the value of b in equation (3), we get :

$$a = 36 - 11$$
$$a = 25$$

Hence,the fraction $= \frac{a}{b} = \frac{25}{36}$.

24. If 2 is added to the numerator of a fraction, it reduces to $\left(\frac{1}{2}\right)$ and if 1 is subtracted from the denominator, it reduces to $\left(\frac{1}{3}\right)$. Find the fraction.

Sol. Let the fraction $= \frac{a}{b}$

Given : $\frac{a+2}{b} = \frac{1}{2}$

$\Rightarrow \quad 2a - b = -4 \quad$ ...(1)

$$\frac{a}{b-1} = \frac{1}{3}$$

$\Rightarrow \quad 3a - b = -1 \quad$ ...(2)

On subtracting equation (1) by equation (2), we get :

$$\begin{array}{r} 2a - b = -4 \\ 3a - b = -1 \\ -\quad +\quad + \\ \hline -a = -3 \\ a = 3. \end{array}$$

On substituting the value of a in equation (1) we get :

$$2 \times 3 - b = -4$$
$$b = 10.$$

Hence,the fraction $= \frac{a}{b} = \frac{3}{10}$.

25. A fraction becomes $\frac{1}{3}$, if 2 is added to both of its numerator and denominator. If 3 is added to both of its numerator and denominator, then it becomes $\frac{2}{5}$. Find the fraction.

Sol. Let the fraction $= \frac{x}{y}$

Given : $\frac{x+2}{y+2} = \frac{1}{3}$

$$3x + 6 = y + 2$$
$$3x - y + 6 - 2 = 0$$

$\Rightarrow \quad 3x - y + 4 = 0 \quad$ ...(1)

$$\frac{x+3}{y+3} = \frac{2}{5}$$
$$5x + 15 = 2y + 6$$
$$5x - 2y + 15 - 6 = 0$$

$\Rightarrow \quad 5x - 2y + 9 = 0 \quad$ ...(2)

from equation (1) we get :

$$y = 3x + 4$$

On substituting the value of y in equation (2) we get :

$$5x - 2[3x + 4] + 9 = 0$$
$$5x - 6x - 8 + 9 = 0$$
$$-x + 1 = 0$$
$$x = 1$$

On substituting the value of x in equation (1), we get :

$$3 \times 1 - y + 4 = 0$$
$$3 - y + 4 = 0$$
$$-y + 7 = 0$$
$$-y = -7$$
$$\boxed{y = 7}$$

Hence, the fraction = $\frac{x}{y} = \frac{1}{7}$

26. The sum of two numbers is 16 and the sum of their reciprocals is $\frac{1}{3}$. Find the numbers.

Sol. Let the two numbers = a and b

Given : $a + b = 16$ ...(1)

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{3} \quad ...(2)$$

from equation (1) we get, $a = 16 - b$

On substitution of the value of a in equation (2) we get,

$$\frac{1}{16-b} + \frac{1}{b} = \frac{1}{3}$$
$$3(b + 16 - b) = (16 - b)b$$
$$3b + 48 - 3b = 16b - b^2$$
$$b^2 - 16b + 48 = 0$$
$$b^2 - 12b - 4b + 48 = 0$$
$$b(b - 12) - 4(b - 12) = 0$$
$$(b - 4)(b - 12) = 0$$
$$b = 12, 4$$

Hence, the numbers = 12 and 4.

27. Two years ago, a man was five times as old as his son. Two years later, his age will be 8 more than three times the age of the son. Find the present ages of the man and his son.

Sol. Let the age of the man = x

age of son = y

Given : $x - 2 = 5(y - 2)$

$$x - 2 = 5y - 10$$
$$x - 5y - 2 + 10 = 0$$
$$x - 5y + 8 = 0$$
$$\Rightarrow \quad x - 5y = -8 \quad ...(1)$$
$$x + 2 = 3(y + 2) + 8$$
$$x + 2 = 3y + 6 + 8$$
$$x - 3y = 14 - 2$$
$$\Rightarrow \quad x - 3y = 12 \quad ...(2)$$

On subtracting the equation (2) from equation (1), we get

$$x - 5y = -8$$
$$x - 3y = 12$$
$$- \quad + \quad\quad -$$
$$-2y = -20; \quad y = 10.$$

On substituting the value of y in equation (1), we get

$x - 5 \times 10 = -8;$ $\quad x = 42.$

Hence, Age of man = 42 years

Age of son = 10 years

28. Five years ago, A was thrice as old as B and 10 years later. A shall be twice as old as B. What are the present ages of A and B.

Sol. Let the age of A = x and age of B = y

Given : $x - 5 = 3(y - 5)$

$$\Rightarrow \quad x - 3y = -10 \quad ...(1)$$
$$x + 10 = 2(y + 10)$$
$$\Rightarrow \quad x - 2y = 10 \quad ...(2)$$

On subtracting equation (2) from equation (1), we get

$$-y = -20; \quad y = 20.$$

On substituting the value of y in equation (1), we get

$$x - 3 \times 20 = -10 \quad x = 50.$$

$\therefore$ Age of A = 50 years, Age of B = 20 years.

29. The present age of a woman is 3 years more than 3 times the age of her daughter. Three years hence, the woman's age will be 10 years more than twice the age of her daughter. Find their present ages.

Sol. Let the age of woman = x

and age of her daughter = y

Given : $x = 3y + 3$

$$\Rightarrow \quad x - 3y = 3 \quad ...(1)$$
$$x + 3 = 2(y + 3) + 10$$
$$x + 3 = 2y + 6 + 10$$
$$x + 3 = 2y + 16$$
$$x - 2y = 16 - 3$$
$$\Rightarrow \quad x - 2y = 13 \quad ...(2)$$

On subtracting (2) from equation (1), we get

$$-y = -10; \quad y = 10$$

On substituting the value of y in

equation (1), we get

$x - 3 \times 10 = 3; \quad x = 33.$

Hence, age of woman = 33 years and age of daughter = 10 years.

30. If twice the son's age in years is added to the mother's age, the sum is 70 years. But, if twice the mother's age is added to the son's age, the sum is 95 years. Find the age of the mother and that of the son.

Sol. Let the age of mother = x and age of son = y

Given : $x + 2y = 70 \quad ...(1)$

$2x + y = 95 \quad ...(2)$

From equation (1) we get,

$x = 70 - 2y \quad ...(3)$

Substituting the value of x in equation (2), we get

$$2[70 - 2y] + y = 95$$
$$140 - 4y + y = 95$$
$$-3y = 95 - 140$$
$$-3y = -45$$
$$y = \frac{45}{3}$$
$$y = 15$$

Substituting the value of y in equation (3), we get

$x = 70 - 2 \times 15; x = 40$

Hence, the age of mother = 40 years and the age of son = 15 years.

31. A man's age is three times the sum of the ages of his two sons. After 5 years, his age will be twice the sum of the ages of his two sons. Find the age of the man.

Sol. Let the father's age = y

Sum of ages of children = x

$y = 3x \quad ...(i)$

After five year's, sum of ages of children will be $(x + 5 + 5)$

$$y + 5 = 2(x + 10)$$
$$3x + 5 = 2x + 20$$
$$3x - 2x = 20 - 5$$
$$x = 15$$

Put the value of x in equation (i)

$$y = 3 \times 15$$
$$= 45$$

Therefore, the father's age will be 45 years.

32. Ten years hence, a man's age will be twice the age of his son. Ten years ago, the man was four times as old as his son. Find their presents ages.

Sol. Let the age of man = x and age of son = y

Given :

$x + 10 = 2(y + 10)$

$\Rightarrow \quad x - 2y = 10 \quad ...(1)$

$x - 10 = 4(y - 10)$

$x - 4y = -30 \quad ...(2)$

On subtracting equation (2) from equation (1), we get

$2y = 40; \quad y = 20$

On substituting the value of y in equation (1), we get

$x - 2 \times 20 = 10 \quad x = 50$

$\therefore$ Hence, age of man = 50 years and age of son = 20 years.

33. The monthly incomes of A and B are in the ratio of 5 : 4 and their monthly expenditures are in the ratio of 7 : 5. If each saves ₹ 3,000 per month, find the monthly income of each.

Sol. Let the income = ₹ x and expenditure = ₹ y

Given : Income = $5x$ and $4x$

Expenditure = $7y$ and $5y$

then saving

$5x - 7y = 3{,}000 \quad ...(1)$

$4x - 5y = 3{,}000 \quad ...(2)$

From equation (1) we get

$$x = \frac{3{,}000 + 7y}{5} \quad ...(3)$$

Substituting the value of x in equation (2) we get

$$4\left[\frac{3{,}000 + 7y}{5}\right] - 5y = 3{,}000$$
$$12{,}000 + 28y - 25y = 15{,}000$$
$$3y = 15{,}000 - 12{,}000$$
$$3y = 3{,}000$$
$$y = \frac{3{,}000}{3}$$
$$y = 1{,}000.$$

On substituting the value of y in equation (3) we get,

$$x = \frac{3{,}000 + 7 \times 1{,}000}{5} = 2{,}000.$$

Hence, Income of A = 5 × 2,000 = ₹ 10,000 and B = 4 × 2,000 = ₹ 8,000.

34. A man sold a chair and a table together for ₹ 760, thereby making a profit of 25% on chair and 10% on table. By selling them together for ₹ 767.50, he would have made a profit of 10% on the chair and 25% on the table Find the cost price of each.

Sol. Let the cost price of chair is = ₹x and cost price of table is = ₹ y

Given :

$$1{\cdot}25x + 1{\cdot}1y = 760 \quad ...(1)$$
$$1{\cdot}1x + 1{\cdot}25y = 767{\cdot}50 \quad ...(2)$$

From equation (1) we get

$$x = \frac{760 - 1{\cdot}1y}{1{\cdot}25} \quad ...(3)$$

On substituting the value of x in equation (2) we get

$$1{\cdot}1\left[\frac{760 - 1{\cdot}1y}{1{\cdot}25}\right] + 1{\cdot}25y = 767{\cdot}50$$
$$836 - 1{\cdot}21y + 1{\cdot}5625y = 959{\cdot}375$$
$$0{\cdot}3525y = 123{\cdot}375$$
$$y = 350.$$

Substituting the value of y in equation (3) we get

$$x = \frac{760 - 1{\cdot}1 \times 350}{1{\cdot}25} = 300.$$

∴ Hence, cost price of chair = 300 and table = 350.

35. On selling a TV at 5% gain and a fridge at 10% gain, a shopkeeper gains ₹ 3,250. But, if he sells the TV at 10% gain and the fridge at 5% loss, he gains ₹ 1,500. Find the actual cost price of TV and that of the fridge.

Sol. Let the cost price of TV = x and fridge = y

Given : $0{\cdot}05x + 0{\cdot}1y = 3{,}250$ ...(1)

$0{\cdot}1x - 0{\cdot}05y = 1{,}500$ ...(2)

from equation (2) we get

$$x = \frac{1{,}500 + 0{\cdot}05y}{0{\cdot}1} \quad ...(3)$$

On substituting the value of x in equation (1) we get

$$0{\cdot}05\left[\frac{1{,}500 + 0{\cdot}05y}{0{\cdot}1}\right] + 0{\cdot}1y = 3{,}250$$
$$75 + 0{\cdot}0025y + 0{\cdot}01y = 325$$
$$0{\cdot}0125y = 250$$
$$y = 20{,}000$$

On substituting the value of y in equation (3) we get

$$x = \frac{1{,}500 + 0{\cdot}05 \times 20{,}000}{0{\cdot}1} = 25{,}000.$$

∴ Hence, cost of TV = 25,000 and fridge = 2,000.

36. A man invested an amount at 12% per annum simple interest and another amount at 10% per annum simple interest. He received an annual interest of ₹ 1,145. But, if he had interchanged the amounts invested, he would have received ₹ 90 less. What amounts did he invest at the different rate ?

Sol. Let the amounts invested by man = x & y

Given : $0{\cdot}12x + 0{\cdot}1y = 1145$ ...(1)

$0{\cdot}1x + 0{\cdot}12y = 1055$ ...(2)

From equation (1), we get

$$y = \frac{1145 - 0{\cdot}12x}{0{\cdot}1} \quad ...(3)$$

On substituting the value of y in equation (2) we get

$$0{\cdot}1x + 0{\cdot}12\left[\frac{1145 - 0{\cdot}12x}{0{\cdot}1}\right] = 1055;$$
$$0.01x + 137.4 - 0.0144x = 105.5$$
$$-0.0044x = 105.5 - 137.4$$
$$-0.0044x = -31.9$$
$$x = \frac{31.9}{0.0044}$$
$$x = 7250.$$

On substituting the value of x in equation (3) we get,

$$y = \frac{1{,}145 - 0{\cdot}12 \times 7{,}250}{0{\cdot}1}$$
$$= 2750.$$

Hence, the amount invested by man = ₹ 7250 and ₹ 2750.

37. There are two classrooms A and B. If 10 students are sent from A to B, the number of students in each room becomes the same. If 20 students are sent from B to A, the number of students in

A becomes double the number of students in B. Find the number of students in each room.

Sol. Let the number of students in A = x and in class B = y

Given : $x - 10 = y + 10$

$\Rightarrow \quad x - y = 20 \quad ...(1)$

$x + 20 = 2(y - 20)$

$x + 20 = 2y - 40$

$x - 2y = - 40 - 20$

$\Rightarrow \quad x - 2y = - 60 \quad ...(2)$

On subtracting the equation (2) from equation (1), we get

$$\begin{array}{rcl} x - \ y &=& 20 \\ x - 2y &=& -60 \\ - \quad + & & + \\ \hline y &=& 80. \end{array}$$

On substituting the value of y in equation (1), we get

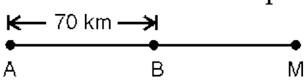
$x - 80 = 20; \quad x = 100.$

Hence, students in class A = 100 and in B = 80.

38. Points A and B are 70 km apart on a highway. A car starts from A and another car starts from B simultaneously. If they travel in the same direction, they meet in 7 hours. But, if they travel towards each other, they meet in 1 hour. Find the speed of each car.

Sol. Let x and y be the cars starting from A and B respectively and let their speeds be x km/h and y km/h respectively. Then, AB = 70 km

Case I : When the two cars move in the same direction. In this case, let the two cars meet at point M



Distance covered by x in 7 hours = $7x$ km

Distance covered by y in 7 hours = $7y$ km

$\therefore$ AM = $(7x)$ km and BM = $(7y)$km

$\Rightarrow \quad (AM - BM) = AB$

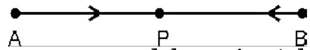
$\Rightarrow \quad (7x - 7y) = 70$

$\Rightarrow \quad 7(x - y) = 70$

$\Rightarrow \quad x - y = 10 \quad ...(1)$

Case II : When the two cars move in opposite direction.

In this case, let the two cars meet at point P.



Distance covered by x in 1 hours = $(1x)$ km

Distance covered by y in 1 hours = $(1y)$ km

$\therefore \quad AP = (x)$ km

and $\quad BP = (y)$ km

$\Rightarrow$ (AP + BP) = AB

$\Rightarrow \quad (x + y) = 70 \quad ...(2)$

On adding (i) and (ii) we get

$$\begin{array}{rcl} x - y &=& 10 \\ x + y &=& 70 \\ \hline 2x &=& 80 \end{array}$$

$$x = \frac{80}{2}$$

$x = 40.$

Putting the value x in equation (1) we get

$40 - y = 10; \quad y = 30.$

Hence, speed of cars = 40 km/h and 30 km/h.

39. A train covered a certain distance at a uniform speed. If the train had been 5 kmph faster, it would have taken 3 hours less than the scheduled time. And, if the train were slower by 4 kmph, it would have taken 3 hours more than the scheduled time. Find the length of the journey.

Sol. Let the speed of train = x km/h

time taken = y hours

We know that,

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$\therefore \quad$ Distance = $xy \quad ...(1)$

If the train would have been 5 km/h faster *i.e.*

Speed = $x + 5$

It would have time taken 3 hours less *i.e.*

Time = $y - 3$

Now, Distance = Speed × Time

Distance = $(x + 5)(y - 3)$

Putting distance = xy from equation (i)

$xy = (x + 5)(y - 3)$

$xy = xy - 3x + 5y - 15$

$3x - 5y + 15 = 0$ ...(2)

Also, if the train slower by 4 km/h

Speed $= x - 4$

It would have time taken 3 hours more

Time $= y + 3$

Now, Distance = Speed × Time

Distance $= (x - 4)(y + 3)$

Putting distance $= xy$ from equation (1)

$xy = xy + 3x - 4y - 12$

$-3x + 4y + 12 = 0$ ...(3)

On adding equation (2) and equation (3) we get

$3x - 5y + 15 = 0$

$-3x + 4y + 12 = 0$

$-y + 27 = 0$

$-y = -27$

$y = 27.$

On substituting the value of y in equation (3) we get

$-3x + 4 \times 27 + 12 = 0; \; x = 40.$

Length of journey (Distance) = 27 × 40 = 1080 km.

40. A man travels 370 km, partly by train and partly by car. If he covers 250 km by train and rest by car, it takes him 4 hours. But, if he travels 130 km by train and the rest by car, he takes 18 minutes longer. Find the speed of the train and that of the car.

Sol. Let the speed of a train = x km/hr.

and the speed of car = y km/hr.

So, Time taken in train journey in first case = $\frac{250}{x}$ hrs.

and time taken in car's journey = $\frac{120}{y}$ hrs.

∴ The total time taken to cover 370 km is = 4 hrs.

or $\frac{250}{x} + \frac{120}{y} = 4$; or $\frac{125}{x} + \frac{60}{y} = 2$

In IInd case, the time taken in train, journey = $\frac{130}{x}$ hrs.

and the time taken in car's journey = $\frac{240}{y}$ hrs.

Total time taken is 4 hrs. 18 minutes.

or $\frac{130}{x} + \frac{240}{y}$ = 4 hrs. 18 minutes

Thus, we obtain the following system of linear equation.

$\frac{125}{x} + \frac{60}{y} = 2$ ...(i)

$\frac{130}{x} + \frac{240}{y} = \frac{43}{10}$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, the equation (1) and (2) reduces to

$125u + 60v = 2$ ...(3)

$130u + 240v = \frac{43}{10}$ ...(4)

Multiplying equation (3) by 4 and subtract the equation (4) we get :

$500u + 240v = 8$

$130u + 240v = 43/10$

$370u = \frac{37}{10}$

$u = \frac{37}{10 \times 370}$

$u = \frac{1}{100}$

Putting the value of u in equation (3) we get

$125 \times \frac{1}{100} + 60v = 2$

$\frac{5}{4} + 60v = 2$

$60v = \frac{2}{1} - \frac{5}{4}$

$60v = \frac{8 - 5}{4}$

$$60v = \frac{3}{4}$$

$$v = \frac{3}{4 \times 60_{20}}$$

$$v = \frac{1}{80}$$

But $\frac{1}{x} = u$, and $\frac{1}{y} = v$

Hence $x = 100$; and $y = 80$.

Hence, Speed of train is 100 km/hr.

and Speed of the car is 80 km/hr.

41. A boat goes 12 km upstream and 40 km downstream in 8 hours. It can go 16 km upstream and 32 km downstream in the same time. Find the speed of boat in still water and the speed of the stream.

Sol. Given : Speed of boat in upstream = 12 km in 8 hours

Speed of boat in downstream = 40 km in 8 hours

To find : The speed of boat and speed of stream

Sol. : Consider that speed of boat = u km/h

and speed of stream = vkm/h

Downstream speed $= (u + v)$ km/h

Upstream speed $= (u - v)$ km/h

$$\therefore \quad \frac{12}{u-v} + \frac{40}{u+v} = 8 \quad ...(1)$$

$$\frac{16}{u-v} + \frac{32}{u+v} = 8 \quad ...(2)$$

Let $\frac{1}{u-v} = x \quad \frac{1}{u+v} = y$

$$\therefore \quad 12x + 40y = 8 \quad ...(3)$$

$$16x + 32y = 8 \quad ...(4)$$

From equation (4) we get

$$x = \frac{8-32y}{16}$$

On substituting the value of x in equation (3) we get,

$$\Rightarrow \quad 12\left(\frac{8-32y}{16}\right) + 40y = 8$$

$$\Rightarrow \quad 96 - 384y + 640y = 128$$

$$\Rightarrow \quad 256y = 128 - 96$$

$$\Rightarrow \quad 256y = 32$$

$$y = \frac{32}{256}$$

$$y = \frac{1}{8}.$$

On substituting the value of y in equation (4) we get,

$$16x + 32 \times \frac{1}{8} = 8$$

$$16x + 4 = 8$$

$$16x = 8 - 4$$

$$16x = 4$$

$$x = \frac{4}{16}$$

$$x = \frac{1}{4}$$

But, $\frac{1}{u-v} = x; \quad \frac{1}{u-v} = \frac{1}{4};$

$$4 = u - v \quad ...(5)$$

And, $\frac{1}{u+v} = y; \quad \frac{1}{u+v} = \frac{1}{8};$

$$8 = u + v \quad ...(6)$$

Adding the both equation (5) and (6)

$$u - v = 4$$
$$u + v = 8$$
$$2u = 12$$
$$u = 6.$$

Putting the value of u in equation (5)

$$6 - v = 4$$
$$-v = 4 - 6$$
$$v = 2.$$

Hence, the speed of boat = 6 km/hr.

the speed of stream = 2 km/hr.

42. Taxi charges in a city consist of fixed charges per day and the remaining depending upon the distance travelled in kilometres. If a person travels 110 km, he pays ₹ 1130 and for travelling 200 km he pays ₹ 1850. Find the fixed charges per day and the rate per km.

Sol. Given : For 110 km = ₹ 1,130

200 km = ₹ 1,850

To Find : Fixed taxi charges per day and rate per km

Sol. : Let fixed charges and rate per km be x and y respectively, then

$x + 110y = 1130$ ...(1)

$x + 200y = 1850$ ...(2)

On subtracting equation (1) from equation (2) we get

$90y = 720 \quad y = 8$

On substituting the value of y in equation (1) we get

$x + 110 \times 8 = 1130 \quad x = 250.$

Hence, fixed taxi charges = ₹ 250 and rate per km = 8.

43. A part of monthly hostel charges in a college are fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 25 days, he has to pay ₹ 3,500 as hostel charges whereas a student B, who takes food for 28 days, pays ₹ 3,800 as hostel charges. Find the fixed charges and the cost of the food per day.

Sol. Given : For 25 days hostel charges = ₹ 3,500

For 28 days hostel charges = ₹ 3,800

To Find : Fixed charges and cost of food per day.

Sol. : Let the fixed charges = y

and cost of food per day = x

Now, A takes food per 25 days and pays ₹ 3,500

that is, $25x + y = 3{,}500$ ...(1)

B takes food per 28 days and pays ₹ 3,800

That is $28x + y = 3{,}800$ ...(2)

Subtracting equation (1) from equation (2) we get

$3x = 300 \; x = 100.$

Substituting the value of x in equation (1) we get

$25 \times 100 + y = 3500; y = 1000$

Hence, fixed charges of hostel = ₹ 1000

and cost of food per day = ₹ 100.

44. The length of a room exceeds its breadth by 3 metres. If the length is increased by 3 metres and the breadth is decreased by 2 metres, the area remains the same. Find the length and the breadth of the room.

Sol. Let the breadth be = x

Length = $x + 3$ m

Area = Length × breadth

$A = (x + 3)x$

$A = x^2 + 3x$

Length is increased by $3m$ and breadth is decreased by $2m$ area remains the same.

Length = $(x + 3 + 3) \Rightarrow x + 6$

breadth = $x - 2$

$x^2 + 3x = (x + 6)(x - 2)$

$x^2 + 3x = x^2 - 2x + 6x - 12$

$x^2 + 3x = x^2 + 4x - 12$

$x^2 - x^2 + 3x - 4x = -12$

$-x = -12$

$x = 12m$ breadth of the room.

Length = $x + 3$

= $12 + 3$

= 15 m length of the room.

45. The area of a rectangle gets reduced 8 m^2, when its length is reduced by 5 m and its breadth is increased by 3 m. If we increase the length by 3 m and breadth by 2 m, the rea is increased by 74 m^2. Find the length and the breadth of the rectangle.

Sol. Let the length and the breadth of the rectangle be xm and ym respectively.

∴ Area of the rectangle = (xy) sq.m

Case 1 : When the length is reduced by 5 m and the breadth is increased by 3m :

New lenght = $(x - 5)$m

New breadth = $(y + 3)$m

∴ New area = $(x-5)(y+3)$ sqm.

∴ $xy - (x - 5)(y + 3) = 8$

$xy - [xy - 5y + 3x - 15] = 8$

$xy - xy + 5y - 3x + 15 = 8$

$3x - 5y = 7$ ...(1)

Case 2 : When the length is incresed by 3m and the breadth is increased by 2m :

New length = $(x + 3)$m

New breadth = $(y + 2)$m

$(x + 3)(y + 2) - xy = 74$

$[xy + 3y + 2x + 6] - xy = 74$

$2x + 3y = 68$ ...(2)

on multiplying equation (1) by 3 (2) by 5 we get :

$9x - 15y = 21$ ...(3)

$10x + 15y = 340$ ...(4)

on adding equation (3) and (4) we get :

$19x = 361; \quad x = 19$

on substituting $x = 19$ in equation (3) we get :

$$9 \times 19 - 15y = 21$$
$$171 - 15y = 21$$
$$-15y = 21 - 171$$
$$-15y = -150$$
$$y = \frac{150}{15}$$
$$y = 10.$$

Hence, the length is 19 m and the breadth is 10 m.

46. 2 men and 5 boys can finish a piece of work in 4 days, while 3 men and 6 boys can finish it in 3 days. Find the time taken by one man alone to finish the work and that taken by one boy alone to finish the work.

Sol. Let one man takes x days to finish the work. and one boy takes y days to finish the work. According to the question,

$$\frac{2}{x} + \frac{5}{y} = \frac{1}{4}$$
$$4(2y + 5x) = 1xy$$
$$8y + 20x = xy$$
$$\frac{3}{x} + \frac{6}{y} = \frac{1}{3} \quad ...(1)$$
$$3(3y + 6x) = xy$$
$$9y + 18x = xy \quad ...(2)$$

On multiplying the equation (1) by 9 and equation (2) by 8 and subtract the equ. (4) from eq. (3) we get,

$$[8y + 20x = xy] \times 9$$
$$\Rightarrow \quad 72y + 180x = 9xy \quad ...(3)$$
$$[9y + 18x = xy] \times 8$$
$$\Rightarrow - 72y + 144x = - 8xy \quad ...(4)$$
$$36x = xy$$
$$y = 36.$$

Put the value of y in equ. (i)

$$8 \times 36 + 20x = 36x$$
$$288 + 20x = 36x$$
$$288 = 36x - 20x$$
$$288 = 16x$$
$$\frac{288}{16} = x$$
$$x = 18$$

$\therefore$ One man takes 18 days and one boy takes 36 days to finish the work.

47. In a ΔABC, $\angle$A = $x°$, $\angle$B = $(3x)°$ and $\angle$C = $y°$. If $3y - 5x = 30$, show that the triangle is right angled.

Sol. The sum of angles of a triangle = 180°

$$\angle A + \angle B + \angle C = 180°$$
$$x° + 3x° + y = 180°$$
$$4x + y = 180° \quad ...(i)$$

Given, $\quad -5x + 3y = 30 \quad ...(ii)$

On multiplying equation (i) by 3 and subtract the equation (ii).

$$12x + 3y = 540$$
$$-5x + 3y = 30$$
$$+ \quad - \quad\quad -$$
$$17x = 510$$
$$x = 30$$
$$\angle A = 30°$$
$$\angle B = 3x$$
$$= 3 \times 30$$
$$= 90°$$

Hence, It is a right angled triangle.

48. Find the four angles of a cyclic quadrilateral ABCD in which $\angle$A = $(x + y + 10)°$, $\angle$B = $(y + 20)°$, $\angle$C = $(x + y - 30)°$ and $\angle$D = $(x + y)°$.

Sol. In a cyclic quadrilateral sum of opposite angles is 180°

A + C = 180 and B + D = 180

$$x + y + 10 + x + y - 30 = 180$$
$$x + y = 100$$
$$y + 20 + x + y = 180$$
$$x + 2y = 160$$

On solving both the equations, we get :

$$x = 40° \quad y = 60°$$

A = 40 + 60 + 10 = 110

B = 60 + 20 = 80

C = 40 + 60 − 30 = 70

D = 40 + 60 = 100

❑

4 Quadratic Equations

POINTS TO REMEMBER

(1) An expression of the from

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n,$$

where $a_0, a_1, a_2......a_n$ are real numbers, n is a non-negative integer and $a_0 \neq 0$, is called a *polynomial* of degree n.

(2) A polynomial of degree 2 is called a *quadratic polynomial*. The general form of a quadratic polynomial is $ax^2 + bx + c$, where a, b, c are real constants, $a \neq 0$ and x is real value. A quadratic polynomial generally denoted by $p(x)$, *i.e.*, $p(x) = ax^2 + bx + c$.

(3) If for $x = \alpha$, where α is a real number, the value of a quadratic polynomial $ax^2 + bx + c$ becomes zero, *i.e.*, $p(\alpha) = 0$ then the real value of α is called a *zero* of the quadratic polynomial.

(4) Every quadratic polynomial can have at the most two zeros. However, there exist quadratic polynomials which do not have any real zero.

(5) An equation $p(x) = 0$, where $p(x)$ is a quadratic polynomial, is called a *quadratic equation*. The general form of a quadratic equation. is $ax^2 + bx + c = 0$, where a, b, c are real constants, $a \neq 0$ and x is a real variable.

(6) In the quadratic equation $ax^2 + bx + c = 0$, a is called the coefficient of x^2, b is called the coefficient of x and c is called constant term.

(7) If the numbers α and β are two zeros of the quadratic polynomial $p(x)$, we say that α and β are the two *roots* of the corresponding quadratic equation $p(x) = 0$. Thus, α is a root of $p(x) = 0$, if and only if $p(\alpha) = 0$ and similarly, β is a root of $p(x) = 0$, if and only if $p(\beta) = 0$.

(8) Finding the roots of a quadratic equation is known as solving the quadratic equation.

(9) **Solving a Quadratic Equation by Factorization Method** :

If we are able to get the factorization

$$ax^2 + bx + c = (rx + s)(fx + g), \quad r \neq 0, f \neq 0$$

for the corresponding quadratic polynomial $ax^2 + bx + c$, then the given equation can be re-written as

$$(rx + s)(fx + g) = 0$$

Therefore, $rx + s = 0$

or $fx + g = 0$

i.e., $x = -\dfrac{s}{r}$ or $x = \dfrac{g}{f}$

Thus, the two roots are $-\dfrac{s}{r}, -\dfrac{g}{f}$.

(10) **Solving a Quadratic Equation by the Method of Completion of Squares.** The roots of the quadratic equation

$$ax^2 + bx + c = 0,$$

where $a, b, c \in R$, $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In words,

$$x = \frac{-\text{coeff. of } x \pm \sqrt{(\text{coeff. of } x)^2 - 4\,(\text{coeff. of } x^2 \times \text{constant term})}}{2 \times \text{coeff. of } x^2}$$

The quantity $b^2 - 4ac$ is called the discriminant of the quadratic equation $ax^2 + bx + c = 0$ and is denoted by D or Δ.

$$\therefore \quad x = \frac{-b \pm \sqrt{D}}{2a},$$

where $D = b^2 - 4ac$.

(11) **Nature of the Roots of** $ax^2 + bx + c = 0$

(i) If $D > 0$, then there are two real and distinct roots given by

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-b + \sqrt{D}}{2a}$$

and $$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-b - \sqrt{D}}{2a}.$$

(ii) If $D = 0$, then the two roots are real and equal, each being equal to $\left(\frac{-b}{2a}\right)$.

(iii) If $D < 0$, then there are no real roots.

(12) **Sum and Product of Roots.** Let α and β be the roots of the quadratic equation $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$. Then by quadratic formula, we have

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

and $$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

(i) Sum of the roots

$$= \alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

(ii) Product of the roots $\alpha.\beta = \frac{c}{a}$

$$= \frac{\text{constant term}}{\text{coefficient of } x^2}.$$

(13) **Formation of Quadratic Equation when its roots are given.** If α and β are the roots, then the required equation is

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

or x^2 – (Sum of roots)x + (Product of roots) = 0

or $\quad x^2 - Sx + P = 0$.

where S = sum of the roots of required equation

and P = Product of the roots of required equation.

(14) An expression in α and β is called *symmetrical* if by interchanging α and β, the expression is not changed.

(15) Some useful relations are given below :

(i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

(ii) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

(iii) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

(iv) $\alpha^3 - \beta^3 = (\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta)$

(v) $\alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta)$

$= (\alpha + \beta)[\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}]$

(vi) $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$

$= [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$

(vii) $\alpha^4 - \beta^4 = (\alpha + \beta)(\alpha^2 + \beta^2)(\alpha - \beta)$

$= (\alpha + \beta)[(\alpha + \beta)^2 - 2\alpha\beta]$

$[\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}]$.

(16) **Factorization of Quadratic Polynomials.** If α and β are the two roots of the quadratic equation $ax^2 + bx + c = 0$, then the quadratic polynomial $ax^2 + bx + c$ can be factorized as $a(x - \alpha)(x - \beta)$.

(i) If the discriminant $D = b^2 - 4ac > 0$, the quadratic equation $ax^2 + bx + c = 0$ has real roots given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} \text{ and } \beta = \frac{-b - \sqrt{D}}{2a}$$

and therefore $ax^2 + bx + c$ can be factorized into real linear factors.

If $D = b^2 - 4ac = 0$, then $x = \frac{b}{2a}$.

$$\therefore ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)$$

(ii) If $D = b^2 - 4ac < 0$, quadratic equation $ax^2 + bx + c = 0$ has no real roots and the quadratic polynomial $ax^2 + bx + c$ can not be factorized into real linear factors.

(17) **Equations Reducible to Quadratic Equations.** Equations, which at

the first side are not quadratic equations, but which can be reduced to quadratic equations by suitable substitutions or simplification are called equations reducible to quadratic equations.

Type 1. *Equation of the form*

$ax^4 + bx^2 + c = 0.$

In such equations, we put $x^2 = y$ so that it is reduced to the quadratic form. Now, solve it for y. Then, find x, using the relation $x = \pm\sqrt{y}$.

Type 2. *Equations of the form*

$$py + \frac{q}{y} = r.$$

Multiplying both sides by y, we get

$py^2 - ry + q = 0,$

which is quadratic equation in y and can easily be solved.

Type 3. *Equations of the form*

$$\sqrt{a - x^2} = bx + c. \qquad \text{...(i)}$$

These involve one radical.

We must seek solutions for which

$a - x^2 \geq 0$ *i.e.*, $x^2 \leq a$

Also, L.H.S. is ≥ 0

$\therefore$ R.H.S. must also be ≥ 0 *i.e.*, $bx + c \geq 0$

$\therefore$ We look for solutions which satisfy both $x^2 \leq a$ and $bx + c \geq 0$.

Squaring both sides of (i), we get

$a - x^2 = (bx + c)^2 = b^2x^2 + 2bcx + c^2$

or $(b^2 + 1)\, x^2 + 2bcx + (c^2 - a) = 0$

which is a quadratic equation in x and can be solved.

Type 4. *Equation of the form*

$$\sqrt{ax + b} + \sqrt{cx + d} = e$$

or $\sqrt{ax + b} - \sqrt{cx + d} = f.$

We find those solutions for which $ax + b \geq 0$ and $cx + d \geq 0$. In such an equation, we transform one of the expressions, with radical sign to the other side and then square both sides. Now, we simplify it and then square again.

Thus, we obtain a quadratic equation, which can be solved easily.

Type 5. *Equations of the form*

$$a\left(x^2 + \frac{1}{x^2}\right) + b\left(x + \frac{1}{x}\right) + c = 0$$

Since, $x^2 + \dfrac{1}{x^2} = \left(x + \dfrac{1}{x}\right)^2 - 2$

$\therefore$ The given equation can be written as

$$a\left[\left(x + \frac{1}{x}\right)^2 - 2\right] + b\left(x + \frac{1}{x}\right) + c = 0$$

or $a\left(x + \dfrac{1}{x}\right)^2 + b\left(x + \dfrac{1}{x}\right) + (c - 2a) = 0$

Now, putting $x + \dfrac{1}{x} = y$, we get

$ay^2 + by + (c - 2a) = 0$

This is a quadratic equation in y.

Let $y = \alpha$ and $y = \beta$ be the roots of this quadratic equation.

$\because \quad y = x + \dfrac{1}{x}$

$\therefore \quad x + \dfrac{1}{x} = \alpha$ or $x + \dfrac{1}{x} = \beta$

$\therefore \quad x^2 - \alpha x + 1 = 0$

or $x^2 - \beta x + 1 = 0$

which are quadratic equations in x and can be solved.

Type 6. *Equations of the form*

$$a\left(x^2 + \frac{1}{x^2}\right) + b\left(x - \frac{1}{x}\right) + c = 0.$$

Since, $x^2 + \dfrac{1}{x^2} = \left(x - \dfrac{1}{x}\right)^2 + 2$

$\therefore$ The given equation can be written as

$$a\left[\left(x - \frac{1}{x}\right)^2 + 2\right] + b\left(x - \frac{1}{x}\right) + c = 0$$

or $a\left(x - \dfrac{1}{x}\right)^2 + b\left(x - \dfrac{1}{x}\right) + (c + 2a) = 0.$

Now, putting $x - \dfrac{1}{x} = y$, we get

$ay^2 + by + (c + 2a) = 0$

This is a quadratic equation in y.

Let $y = \alpha$ and $y = \beta$ be the roots of this quadratic equation.

$$\because \qquad y = x - \frac{1}{x}$$

$$\therefore \qquad x - \frac{1}{x} = \alpha \text{ or } x - \frac{1}{x} = \beta$$

$\therefore$ $x^2 - ax - 1 = 0$

or $x^2 - \beta x - 1 = 0$

which are quadratic equations in x and can be solved.

Type 7. Equations of the form

$(x + a)(x + b)(x + c)(x + d) + k = 0$.

Methods to Solve. Such type of equation is easily solvable if the sum of any two of the constants a, b, c, d is equal to the sum of the other two. We take such pairs together and solve the equation.

(18) **Solution of Problems Involving Quadratic Equation.** There are many word problems which can be solved by means of quadratic equations. Sometimes only one root of the quadratic equation has a meaning for the problem. Any root not satisfying the conditions of the given problem must be rejected. We consider such word problems which involve applications of quadratic equations.

Exercise 4.1

Multiple Choice Type Questions

Q. 1. *The solution of quadratic equation* $x^2 - x - 2 = 0$ *are* :

(a) 1, – 2 (b) – 1, – 2

(c) – 1, 2 (d) 1, 2.

Solution : The given equation is

$$x^2 - x - 2 = 0$$

$$\Rightarrow \quad x^2 - 2x + x - 2 = 0$$

$$\Rightarrow \quad x(x - 2) + 1(x - 2) = 0$$

$$\Rightarrow \quad (x - 2)(x + 1) = 0$$

when, $x - 2 = 0$ and when $x + 1 = 0$

then $x = 2$ then $x = -1$

Hence, the solution of quadratic equation are (– 1, 2).

Q. 2. *The zero of the polynomial* $x^2 + 2x - 3$ *are* :

(a) 1, – 3 (b) – 1, 3

(c) – 1, – 3 (d) 1, 3.

Solution : The given equation is

$$x^2 + 2x - 3 = 0$$

$$\Rightarrow \quad x^2 + 3x - x - 3 = 0$$

$$\Rightarrow \quad x(x + 3) - 1(x + 3) = 0$$

$$\Rightarrow \quad (x + 3)(x - 1) = 0$$

when $x + 3 = 0$ when $x - 1 = 0$

$\therefore$ $x = -3$ $\therefore$ $x = 1$

Hence, the zero of the polynomial are (1, – 3).

Q. 3. *The degree of the polynomial* $x^3 - x + 7$ *is* :

(a) 1 (b) 2

(c) 3 (d) None of these.

Q. 4. *The zero of the polynomial* $p(x) = x^2 + 1$ *are* :

(a) Real

(b) Not real

(c) (a) and (b) both

(d) None of these.

Solution : Let $p(x) = x^2 + 1$

Now, $x^2 \geq 0$ for all real value of x

$\therefore$ $x^2 + 1 \geq 0$ for all real value of x

or $x^2 + 1 \neq 0$ for any real value of x

Thus, there is no real value of x for which $x^2 + 1 = 0$.

Hence, $p(x) = 0$ has no real zeros.

Q. 5 (*i*) *Every quadratic polynomial can have at the most* :

(a) One zero (b) Two zeros

(c) Three zeros (d) None of these.

Q. 5. (*ii*) *From the equation* $4\sqrt{5}x^2 + 7x - 3\sqrt{5} = 0$, *the value of x will be* :

(a) $\frac{\sqrt{5}}{4}, \frac{-3}{\sqrt{5}}$ (b) $\frac{-\sqrt{5}}{4}, \frac{3}{\sqrt{5}}$

(c) $\frac{\sqrt{5}}{4}, \frac{3}{\sqrt{5}}$ (d) $\frac{-\sqrt{5}}{4}, \frac{-3}{\sqrt{5}}$.

[Ans. : 1. (c), **2.** (a), **3.** (c), **4.** (b), **5. (i)** (b), **(ii)** (a).]

Very Short Answer Type Questions

Q. 6. *Find the value of the polynomial $p(x) = x^2 - 5x + 6$ at $x = 4$ and $x = 3$.*

Solution : We have $p(x) = x^2 - 5x + 6$

$\therefore \quad p(4) = (4)^2 - 5(4) + 6$

$= 16 - 20 + 6 = 2$

and $\quad p(3) = (3)^2 - 5(3) + 6$

$= 9 - 15 + 6 = 0.$ **Ans.**

Q. 7. *Examine whether the equation $3x^2 - 4x + 2 = 2x^2 - 2x + 4$ can be put in the form of a quadratic equation.*

Solution : The given equation is

$$3x^2 - 4x + 2 = 2x^2 - 2x + 4$$

Transforming the terms on the R.H.S. to the L.H.S., we get

$$(3x^2 - 4x + 2) - (2x^2 - 2x + 4) = 0$$

or $\quad 3x^2 - 4x + 2 - 2x^2 + 2x - 4 = 0$

or $\quad x^2 - 2x - 2 = 0$

Thus, the given equation is a quadratic equation. **Ans.**

Q. 8. *Which of the following are quadratic equations ?*

(*i*) $x^2 - 6x + 4 = 0$

(*ii*) $x^2 - 2x + 3 = 0$

(*iii*) $x^3 + 6x^2 + 2x - 1 = 0$

(*iv*) $x^2 + \dfrac{1}{x^2} = 2$

(*v*) $\sqrt{5}\,x^2 - \sqrt{3}\,x + \sqrt{2} = 0$

(*vi*) $x^3 - 2x^2 + 4 = 0$

(*vii*) $x - \dfrac{5}{x} = x^2$

(*viii*) $\left(\sqrt{x} - \dfrac{2}{\sqrt{x}}\right)^2 = 1.$

Solution : (i) We have $x^2 - 6x + 4 = 0$

Since, $x^2 - 6x + 4$ is a quadratic polynomial, therefore, $x^2 - 6x + 4 = 0$ is a quadratic equation. **Ans.**

(ii) We have $x^2 - 2x + 3$

Since, $x^2 - 2x + 3$ is a quadratic polynomial, therefore, $x^2 - 2x + 3 = 0$ is a quadratic equation. **Ans.**

(iii) $x^3 + 6x^2 + 2x - 1 = 0$

Being a polynomial of degree 3 it is not a quadratic equation. **Ans.**

(iv) We have $\quad x^2 + \dfrac{1}{x^2} = 2$

or $\quad x^4 + 1 = 2x^2$

or $\quad x^4 - 2x^2 + 1 = 0$

Here, $x^4 - 2x^2 + 1$, being a polynomial of degree 4, therefore it is not quadratic. So, the given equation is not quadratic.

(v) We have $\sqrt{5}\,x^2 - \sqrt{3x} + \sqrt{2} = 0$

Since, $\sqrt{5}\,x^2 - \sqrt{3}x + \sqrt{2}$ is a quadratic polynomial, therefore.

$$\sqrt{5}\,x^2 - \sqrt{3}x + \sqrt{2} = 0$$

is a quadratic equation.

(vi) We have $x^3 - 2x^2 + 4 = 0$

Here, $x^3 - 2x^2 + 4$, being a polynomial of degree 3, is not quadratic. So, the given equation is not quadratic.

(vii) We have $\quad x - \dfrac{5}{x} = x^2$

or $\quad x^2 - 5 = x^3$

or $\quad x^3 - x^2 + 5 = 0$

Here, $x^3 - x^2 + 5$, being a polynomial of degree 3, is not quadratic. So, the given equation is not quadratic.

(viii) We have $\left(\sqrt{x} - \dfrac{2}{\sqrt{x}}\right)^2 = 1$

or $x + \dfrac{2}{x} - 2 \times \sqrt{x} \times \dfrac{2}{\sqrt{x}} = 1$

or $\quad x + \dfrac{2}{x} - 4 = 1$

or $\quad x + \dfrac{2}{x} = 5$

or $\quad x^2 + 2 = 5x$

$\therefore \quad x^2 - 5x + 2 = 0$

Since, $x^2 - 5x + 2$ is a quadratic polynomial, therefore,

$$\left(\sqrt{x} - \frac{2}{\sqrt{x}}\right)^2 = 1$$

is a quadratic equation. **Ans.**

Q. 9. *In each of the following determine whether the given values are solutions of the equation or not :*

(*i*) $3x^2 - 2x - 1 = 0; x = 1$

(*ii*) $2x^2 - 6x + 3 = 0; x = \frac{1}{2}$

(*iii*) $2x^2 - 5x - 3 = 0;$

$x = 3, x = 2, x = -\frac{1}{2}$

(*iv*) $3x^2 - 2x - 5 = 0; x = 1, x = \frac{5}{3}$

(*v*) $x^2 + \sqrt{2}\,x - 4 = 0;$

$x = \sqrt{3}, x = -2\sqrt{2}$

(*vi*) $x^2 + \sqrt{3}\,x - 6 = 0;$

$x = \sqrt{3}, x = -2\sqrt{3}$

(*vii*) $\sqrt{7}x^2 - 6x - 13\sqrt{7} = 0;$

$x = -\sqrt{7}, x = \frac{13}{\sqrt{7}}$

(*viii*) $\frac{1}{4}x^2 + \frac{5}{9}x + \frac{25}{81} = 0; x = \frac{-10}{9}$.

Solution : (i) $3x^2 - 2x - 1 = 0; x = 1$

Substituting $x = 1$ on the L.H.S. of the given equation, we get

L.H.S. $= 3(1)^2 - 2(1) - 1$

$= 3 - 2 - 1 = 0 =$ R.H.S.

Therefore, $x = 1$ is a solution of the given equation. **Ans.**

(ii) $2x^2 - 6x + 3 = 0; x = \frac{1}{2}$

Substituting $x = \frac{1}{2}$ on the L.H.S. of the given equation, we get

$$\text{L.H.S.} = 2\left(\frac{1}{2}\right)^2 - 6\left(\frac{1}{2}\right) + 3$$

$$= \frac{1}{2} - 3 + 3 = \frac{1}{2} \neq \text{R.H.S.}$$

Therefore, $x = \frac{1}{2}$ is not a solution of the given equation. **Ans.**

(iii) $2x^2 - 5x - 3 = 0;$

$x = 3, x = 2, x = -\frac{1}{2}$

Substituting $x = 3$ on the L.H.S. of the given equation, we get

L.H.S. $= 2(3)^2 - 5(3) - 3$

$= 18 - 15 - 3 = 0 =$ R.H.S.

Therefore, $x = 3$ is a solution of the given equation.

Again, substituting $x = 2$ on the L.H.S. of the given equation, we get

L.H.S. $= 2(2)^2 - 5(2) - 3$

$= 8 - 10 - 3 = -5 \neq$ R.H.S.

Therefore, $x = 2$ is not a solution of the given equation.

Now, again substituting $x = -\frac{1}{2}$ on the L.H.S. of the given equation, we get

$$\text{L.H.S.} = 2\left(-\frac{1}{2}\right)^2 - 5\left(-\frac{1}{2}\right) - 3$$

$$= \frac{1}{2} + \frac{5}{2} - 3 = \frac{1 + 5 - 6}{2} = 0$$

$= 0 =$ R.H.S.

Therefore, $x = -\frac{1}{2}$ is a solution of the given equation.

Thus, $x = 3$ and $x = -\frac{1}{2}$ are solutions, but $x = 2$ is not a solution.

(iv) $3x^2 - 2x - 5 = 0; x = 1, x = \frac{5}{3}$

Substituting $x = 1$ on the L.H.S. of the given equation, we get

L.H.S. $= 3(1)^2 - 2(1) - 5$

$= 3 - 2 - 5 = -4 \neq$ R.H.S.

Therefore, $x = 1$ is not a solution of the given equation.

Again, substituting $x = \frac{5}{3}$ one the L.H.S. of the given equation, we get

$$\text{L.H.S.} = 3\left(\frac{5}{3}\right)^2 - 2\left(\frac{5}{3}\right) - 5$$

$$= \frac{25}{3} - \frac{10}{3} - 5$$

$$= \frac{25 - 10 - 15}{3} = 0 = \text{R.H.S}$$

Therefore, $x = \frac{5}{3}$ is a solution of the given equation.

Thus, $x = \frac{5}{3}$ is a solution and $x = 1$ is not a solution of the given equation.

(v) $x^2 + \sqrt{2}\,x - 4 = 0$; $x = \sqrt{3}$, $x = -2\sqrt{2}$

Substituting $x = \sqrt{3}$ on the L.H.S. of the given equation, we get

$$\text{L.H.S.} = \left(\sqrt{3}\right)^2 + \sqrt{2} \times \sqrt{3} - 4$$
$$= 3 + \sqrt{6} - 4$$
$$= \sqrt{6} - 1 \neq 0$$

Therefore, $x = \sqrt{3}$ is not a solution of the given equation.

Again substituting $x = -2\sqrt{2}$ on the L.H.S. of the given equation, we get

$$\text{L.H.S.} = \left(-2\sqrt{2}\right)^2 + \sqrt{2}\left(-2\sqrt{2}\right) - 4$$
$$= 8 - 4 - 4 = 0 = \text{R.H.S.}$$

Therefore, $x = -2\sqrt{2}$ is a solution of the given equation.

Thus, $x = \sqrt{3}$ is not a solution and $x = -2\sqrt{2}$ is a solution.

(vi) $x^2 + \sqrt{3}\,x - 6 = 0$; $x = \sqrt{3}$, $x = -2\sqrt{3}$

Substituting $x = \sqrt{3}$ on the L.H.S. of the given equation, we get

$$\text{L.H.S.} = \left(\sqrt{3}\right)^2 + \sqrt{3}\left(\sqrt{3}\right) - 6$$
$$= 3 + 3 - 6 = 0 = \text{R.H.S.}$$

Therefore, $x = \sqrt{3}$ is a solution of the given equation.

Again, substituting $x = -2\sqrt{3}$ on the L.H.S. of the given equation, we get

$$\text{L.H.S.} = \left(-2\sqrt{3}\right)^2 + \sqrt{3}\left(-2\sqrt{3}\right) - 6$$
$$= 12 - 6 - 6 = 0 = \text{R.H.S.}$$

Therefore, $x = -2\sqrt{3}$ is a solution of the given equation.

Thus, $x = \sqrt{3}$ and $x = -2\sqrt{3}$ are solutions. **Ans.**

(vii) $\sqrt{7}x^2 - 6x - 13\sqrt{7} = 0$; $x = -\sqrt{7}$, $x = \frac{13}{\sqrt{7}}$.

Substituting $x = -\sqrt{7}$ on the L.H.S. of the given equation, we get

L.H.S.

$$= \sqrt{7}\left(-\sqrt{7}\right)^2 - 6\left(-\sqrt{7}\right) - 13\sqrt{7}$$
$$= 7\sqrt{7} + 6\sqrt{7} - 13\sqrt{7} = 0 = \text{R.H.S.}$$

Therefore, $x = -\sqrt{7}$ is a solution of the given equation.

Again, substituting $x = \frac{13}{\sqrt{7}}$ on the L.H.S. of the given equation, we get

L.H.S.

$$= \sqrt{7}\left(\frac{13}{\sqrt{7}}\right)^2 - 6\left(\frac{13}{\sqrt{7}}\right) - 13\sqrt{7}$$
$$= \frac{169}{\sqrt{7}} - \frac{78}{\sqrt{7}} - 13\sqrt{7}$$
$$= \frac{169 - 78 - 91}{\sqrt{7}} = 0$$
$$= \text{R.H.S.}$$

Therefore, $x = \frac{13}{\sqrt{7}}$ is a solution of the given equation.

Thus, $x = -\sqrt{7}$ and $x = \frac{13}{\sqrt{7}}$ are solutions. **Ans.**

(viii) $\frac{1}{4}x^2 + \frac{5}{9}x + \frac{25}{81} = 0$; $x = \frac{-10}{9}$

Substituting $x = -\frac{10}{9}$ on the L.H.S. of the given equation, we get

L.H.S.

$$= \frac{1}{4}\left(\frac{-10}{9}\right)^2 + \frac{5}{9}\left(\frac{-10}{9}\right) + \frac{25}{81}$$
$$= \frac{25}{81} - \frac{50}{81} + \frac{25}{81} = 0 = \text{R.H.S.}$$

Therefore, $x = -\frac{10}{9}$ is a solution of the given equation. **Ans.**

Q. 10. *For the quadratic equation* $a^2x^2 - 3abx + 2b^2 = 0$, $(a \neq 0)$, *determine which of the following are solutions ?*

(*i*) $x = \frac{a}{b}$ (*ii*) $x = \frac{b}{a}$.

Solution : The given quadratic equation is $a^2x^2 - 3abx + 2b^2 = 0$, $(a \neq 0)$

(i) Substituting $x = \frac{a}{b}$ on the L.H.S. of the given equation, we get

$$\text{L.H.S.} = a^2\left(\frac{a}{b}\right)^2 - 3ab\left(\frac{a}{b}\right) + 2b^2$$

$$= \frac{a^4}{b^2} - \frac{3a^2b}{b} + 2b^2$$

$$= \frac{a^4 - 3a^2b^2 + 2b^4}{b^2}$$

$\neq$ R.H.S.

Therefore, $x = \frac{a}{b}$ is not a solution of the given equation.

(ii) Substituting $x = \frac{b}{a}$ on the L.H.S. of the given equation, we get

$$\text{L.H.S.} = a^2\left(\frac{b}{a}\right)^2 - 3ab\left(\frac{b}{a}\right) + 2b^2$$

$$= b^2 - 3b^2 + 2b^2$$

$= 0 =$ R.H.S.

Therefore, $x = \frac{b}{a}$ is a solution of the given equation.

Thus, $x = \frac{b}{a}$ is a solution, but $x = \frac{a}{b}$ is not a solution. **Ans.**

Q. 11. *Prove that the following equation is a quadratic equation*

$$4x^2 + 5x + 3 = 3x^2 + 4x + 2.$$

Solution : $4x^2 - 3x^2 + 5x - 4x + 3 - 2 = 0$

$\Rightarrow \quad x^2 + x + 1 = 0$

which is a quadratic equation .

Hence Proved.

Exercise 4.2

Multiple Choice Type Questions

Q. 1. *If one root quadratic equation* $x^2 + kx + 3 = 0$, *is* 1, *then the value of k will be* :

(a) 1 (b) – 3 (c) – 4 (d) – 5.

Solution : The given quadratic equation is

$$x^2 + kx + 3 = 0$$

Substituting $x = 1$ in the given equation

$\Rightarrow \quad (1)^2 + k \times 1 + 3 = 0$

$\Rightarrow \quad 1 + k + 3 = 0$

$\Rightarrow \quad k + 4 = 0$

$\Rightarrow \quad k = -4.$

Q. 2. *If one root of quadratic equation* $2x^2 + px - 4 = 0$, *is* 2, *then the value of the p will be* :

(a) – 3 (b) – 2

(c) 2 (d) 3.

Solution : The given quadratic equation is $2x^2 + px - 4 = 0$

Substituting the value $x = 2$ in the given equation

$\Rightarrow \quad 2(2)^2 + p \times 2 - 4 = 0$

$\Rightarrow \quad 8 + 2p - 4 = 0$

$\Rightarrow \quad 2p + 4 = 0$

$$p = \frac{-4}{2} = -2.$$

Q. 3. *If one root of quadratic equation* $ax^2 + bx + c = 0$ *is* 1, *then* :

(a) $a = 1$ (b) $b = 1$

(c) $c = 1$ (d) $a + b + c = 0$.

Solution : The quadratic equation is the given

$$ax^2 + bx + c = 0$$

Substituting the value of $x = 1$ in the given equation

$\Rightarrow \quad a(1)^2 + b \times 1 + c = 0$

$\therefore \quad a + b + c = 0.$

Q. 4. *If* $2x^2 + 1 = 33$, *then the value of x will be* :

(a) ± 2 (b) ± 3

(c) ± 4 (d) ± 1.

Solution : We have,

$$2x^2 + 1 = 33$$

$$\Rightarrow \quad 2x^2 = 33 - 1 = 32$$

$$\Rightarrow \quad x^2 = \frac{32}{2} = 16$$

$$\therefore \quad x = \pm 4.$$

Q. 5. *One root of quadratic equation* $x^2 + 3x - 10 = 0$:

(a) -2 (b) $+2$

(c) 0 (d) 1.

Q. 6. *If* $\frac{x}{6} = \frac{6}{x}$, *then the value of* x *will be* :

(a) 6 (b) -6

(c) ± 6 (d) None of these.

Solution : We have, $\frac{x}{6} = \frac{6}{x}$

$$\Rightarrow \quad x^2 = 36$$

$$\therefore \quad x = \pm 6.$$

[Ans. : 1. (c), **2.** (b), **3.** (d), **4.** (c), **5.** (b), **6.** (c).**]**

Very Short Answer Type Questions

Solve the following quadratic equations by factorization :

Q. 7. $x^2 + 6x + 5 = 0$.

Solution : The quadratic polynomial $x^2 + 6x + 5$ can be factorized as

$$x^2 + 6x + 5 = x^2 + 5x + x + 5$$
$$= x(x + 5) + 1(x + 5)$$
$$= (x + 5)(x + 1)$$

$\therefore$ The given quadratic equation can be written as

$$(x + 5)(x + 1) = 0$$

Therefore, $x + 5 = 0$ or $x + 1 = 0$

Therefore, $x = -5$ or $x = -1$

Thus, the two roots of the given quadratic equation are -5 and -1. **Ans.**

Q. 8. $9x^2 + 6x + 1 = 0$.

Solution : The quadratic polynomial $9x^2 + 6x + 1$ can be factorized as

$$9x^2 + 6x + 1 = 9x^2 + 3x + 3x + 1$$
$$= 3x(3x + 1) + 1(3x + 1)$$
$$= (3x + 1)(3x + 1)$$

$\therefore$ The given quadratic equation can be written as

$$(3x + 1)(3x + 1) = 0$$

Therefore, $3x + 1 = 0$

or $3x + 1 = 0$

Therefore, $x = -\frac{1}{3}$ or $x = -\frac{1}{3}$

Thus, the two roots of the given quadratic equation are $-\frac{1}{3}$ and $-\frac{1}{3}\cdot$ **Ans.**

Q. 9. $36x^2 + 60x + 25 = 0$.

Solution : The quadratic polynomial $36x^2 + 60x + 25$ can be factorized as

$$36x^2 + 60x + 25$$
$$= 36x^2 + 30x + 30x + 25$$
$$= 6x(6x + 5) + 5(6x + 5)$$
$$= (6x + 5)(6x + 5)$$

$\therefore$ The given quadratic equation can be written as $(6x + 5)(6x + 5) = 0$

Therefore, $6x + 5 = 0$ or $6x + 5 = 0$

Therefore, $x = \frac{-5}{6}$ or $x = \frac{-5}{6}$

Thus, the two roots of the given quadratic equations are $\frac{-5}{6}$ and $-\frac{5}{6}\cdot$ **Ans.**

Q. 10. $3x^2 + 10 = -11x$.

Solution : We have,

$$3x^2 + 10 = -11x$$

or $3x^2 + 11x + 10 = 0$

The quadratic polynomial $3x^2 + 11x + 10$ can be factorized as

$$3x^2 + 11x + 10 = 3x^2 + 6x + 5x + 10$$
$$= 3x(x + 2) + 5(x + 2)$$
$$= (x + 2)(3x + 5)$$

$\therefore$ The given quadratic equation can be written as

$$(x + 2)(3x + 5) = 0$$

Therefore, $x + 2 = 0$ or $3x + 5 = 0$

Therefore $x = -2$ or $x = \frac{-5}{3}\cdot$

Thus, the two roots of the given quadratic equation are -2 and $\frac{-5}{3}\cdot$ **Ans.**

Q. 11. $9x^2 - 22x + 8 = 0$.

Solution : The quadratic polynomial $9x^2 - 22x + 8$ can be factorized as

$$9x^2 - 22x + 8 = 9x^2 - 18x - 4x + 8$$
$$= 9x(x-2) - 4(x-2)$$
$$= (x-2)(9x-4)$$

$\therefore$ The given quadratic equation can be written as

$$(x-2)(9x-4) = 0$$

Therefore, $x - 2 = 0$ or $9x - 4 = 0$

Therefore, $x = 2$ or $x = \frac{4}{9}$

Thus, the two roots of the given quadratic equation are 2 and $\frac{4}{9}$. **Ans.**

Q. 12. $25x(x+1) = -4$.

Solution : We have,

$$25x(x+1) = -4$$

or $\quad 25x^2 + 25x + 4 = 0$

The quadratic polynomial $25x^2 + 25x + 4$ can be factorized as

$25x^2 + 25x + 4$
$$= 25x^2 + 20x + 5x + 4$$
$$= 5x(5x+4) + 1(5x+4)$$
$$= (5x+4)(5x+1)$$

$\therefore$ The given quadratic equations can be written as

$$(5x+4)(5x+1) = 0$$

Therefore, $\quad 5x + 4 = 0$

or $\quad 5x + 1 = 0$

Therefore, $x = -\frac{4}{5}$ or $x = -\frac{1}{5}$

Thus, the two roots of the given quadratic equation are $\frac{-4}{5}$ and $-\frac{1}{5}$.

Ans.

Q. 13. $(x-4)(x+2) = 0$.

Solution : Given, $(x-4)(x+2) = 0$

Therefore, $x - 4 = 0$ or $x + 2 = 0$

Therefore, $x = 4$ or $x = -2$

Thus, the two roots of the given quadratic equation are 4 and -2. **Ans.**

Q. 14. $(3x-5)(2x+7) = 0$.

Solution : Given,

$$(3x-5)(2x+7) = 0$$

Therefore, $3x - 5 = 0$ or $2x + 7 = 0$

Therefore, $x = \frac{5}{3}$ or $x = -\frac{7}{2}$

Thus, the two roots of the given quadratic equation are $\frac{5}{3}$ and $-\frac{7}{2}$. **Ans.**

Q. 15. $(2x+3)(3x-7) = 0$.

Solution : Given,

$$(2x+3)(3x-7) = 0$$

Therefore, $2x + 3 = 0$ or $3x - 7 = 0$

Therefore, $x = \frac{-3}{2}$ or $x = \frac{7}{3}$.

Thus, the two roots of the given quadratic equation are $-\frac{3}{2}$ and $\frac{7}{3}$. **Ans.**

Q. 16. $x^2 - 3x - 18 = 0$.

Solution : The quadratic polynomial $x^2 - 3x - 18$ can be factorized as

$$x^2 - 3x - 18 = x^2 - 6x + 3x - 18$$
$$= x(x-6) + 3(x-6)$$
$$= (x-6)(x+3)$$

$\therefore$ The given quadratic equation can be written as

$$(x-6)(x+3) = 0$$

Therefore, $x - 6 = 0$ or $x + 3 = 0$

Therefore, $x = 6$ or $x = -3$

Thus, the two roots of the given quadratic equation are 6 and -3. **Ans.**

Q. 17. $x^2 + x - 12 = 0$.

Solution : $x^2 + x - 12 = 0$

$\therefore \quad x^2 + 4x - 3x - 12 = 0$

$\therefore \quad x(x+4) - 3(x+4) = 0$

$\therefore \quad (x+4)(x-3) = 0$

$\therefore \quad x + 4 = 0$ or $x - 3 = 0$

$\therefore \quad x = -4$ or $x = 3$. **Ans.**

Q. 18. $x^2 - 2ax + a^2 = 0$.

Solution : $x^2 - ax - ax + a^2 = 0$

$x(x-a) - a(x-a) = 0$

$\therefore \quad (x-a)(x-a) = 0$

$\therefore \quad x - a = 0$ or $x - a = 0$

$\therefore \quad x = a$ or $x = a$. **Ans.**

Q. 19. $4x^2 - 9x - 100 = 0$.

Solution : The quadratic polynomial $4x^2 - 9x - 100$ can be factorized as $4x^2 - 9x - 100$

$$= 4x^2 - 25x + 16x - 100$$
$$= x(4x - 25) + 4(4x - 25)$$
$$= (4x - 25)(x + 4)$$

$\therefore$ The given quadratic equation can be written as

$$(4x - 25)(x + 4) = 0$$

Therefore, $4x - 25 = 0$ or $x + 4 = 0$

Therefore, $x = \frac{25}{4}$ or $x = -4$

Thus, the two roots of the given quadratic equation are $\frac{25}{4}$ and -4. **Ans.**

Q. 20. $4\sqrt{5}x^2 + 7x - 3\sqrt{5} = 0$.

Solution :

$$4\sqrt{5}x^2 + 12x - 5x - 3\sqrt{5} = 0$$

$\therefore$ $4x(\sqrt{5}x + 3) - \sqrt{5}(\sqrt{5} + 3) = 0$

$\therefore$ $(\sqrt{5}x + 3)(4x - \sqrt{5}) = 0$

$\therefore$ $\sqrt{5}x + 3 = 0$ or $4x - \sqrt{5} = 0$

$\therefore$ $x = \frac{-3}{\sqrt{5}}$ or $x = \frac{\sqrt{5}}{4}$. **Ans.**

Short Answer Type Questions

Q. 21. $\sqrt{3}x^2 + 11x + 6\sqrt{3} = 0$.

Solution : The quadratic polynomial $\sqrt{3}x^2 + 11x + 6\sqrt{3}$ can be factorized as $\sqrt{3}x^2 + 11x + 6\sqrt{3}$

$$= \sqrt{3}x^2 + 9x + 2x + 6\sqrt{3}$$
$$= \sqrt{3}x\left(x + 3\sqrt{3}\right) + 2(x + 3\sqrt{3})$$
$$= \left(x + 3\sqrt{3}\right)\left(\sqrt{3}x + 2\right)$$

$\therefore$ The given quadratic equation can be written as

$$\left(x + 3\sqrt{3}\right)\left(\sqrt{3}x + 2\right) = 0$$

Therefore, $x + 3\sqrt{3} = 0$

or $\sqrt{3}\,x + 2 = 0$

Therefore, $x = -3\sqrt{3}$ or $x = -\frac{2}{\sqrt{3}}$

Thus, the two roots of the given quadratic equation are $-3\sqrt{3}$ and $-\frac{2}{\sqrt{3}}$. **Ans.**

Q. 22. $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$.

Solution : The quadratic polynomial $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ can be factorized as

$$= 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$$
$$= 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3}$$
$$= 4x\left(\sqrt{3}x + 2\right) - \sqrt{3}\left(\sqrt{3}x + 2\right)$$
$$= \left(\sqrt{3}x + 2\right)\left(4x - \sqrt{3}\right)$$

$\therefore$ The given quadratic equation can be written as

$$\left(\sqrt{3}x + 2\right)\left(4x - \sqrt{3}\right) = 0$$

Therefore, $\sqrt{3}x + 2 = 0$

or $4x - \sqrt{3} = 0$

Therefore, $x = -\frac{2}{\sqrt{3}}$ or $x = \frac{\sqrt{3}}{4}$

Thus, the two roots of the given quadratic equation are $-\frac{2}{\sqrt{3}}$ and $\frac{\sqrt{3}}{4}$. **Ans.**

Q. 23. $4x^2 - 4ax + (a^2 - b^2) = 0$.

Solution : The quadratic polynomial $4x^2 - 4ax + (a^2 - b^2)$ can be factorized as

$$4x^2 - 4ax + (a^2 - b^2)$$
$$= 4x^2 - 2(a + b)x - 2(a - b)x + (a^2 - b^2)$$
$$= 2x[2x - (a + b)] - (a - b)[2x - (a + b)]$$
$$= [2x - (a + b)][2x - (a - b)]$$

$\therefore$ The given quadratic equation can be written as

$$[2x - (a - b)][2x - (a - b)] = 0$$

Therefore, $2x - (a + b) = 0$

or $2x - (a - b) = 0$

Therefore, $x = \frac{a + b}{2}$ or $x = \frac{a - b}{2}$

Thus, the two roots of the given quadratic equation are $\frac{a + b}{2}$ and $\frac{a - b}{2}$. **Ans.**

Q. 24. $a^2b^2x^2 - (a^2 + b^2)x + 1 = 0$, $a \neq 0, b \neq 0$.

Solution : The quadratic polynomial

$$a^2b^2x^2 - (a^2 + b^2)x + 1$$

can be factorized as

$$a^2b^2x^2 - (a^2 + b^2)x + 1$$
$$= a^2b^2x^2 - a^2x - b^2x + 1$$
$$= a^2x(b^2x - 1) - 1(b^2x - 1)$$
$$= (b^2x - 1)(a^2x - 1)$$

∴ The given quadratic equation can be written as

$$(b^2x - 1)(a^2x - 1) = 0$$

Therefore, $b^2x - 1 = 0$

or $a^2x - 1 = 0$

Therefore, $x = \frac{1}{b^2}$ or $x = \frac{1}{a^2}$.

Thus, the two roots of the given quadratic equation are $\frac{1}{a^2}$ and $\frac{1}{b^2}$. **Ans.**

Q. 25. $\left(x - \frac{1}{2}\right)^2 = \frac{1}{4}$.

Solution : The given quadratic equation is $\left(x - \frac{1}{2}\right)^2 = \frac{1}{4}$

or $x^2 - 2(x)\left(\frac{1}{2}\right) + \frac{1}{4} = \frac{1}{4}$

or $x^2 - x = 0$

or $x(x - 1) = 0$

Therefore, $x = 0$ or $x - 1 = 0$

Therefore, $x = 0$ or $x = 1$

Thus, the two roots of the given quadratic equation are $x = 0$ and $x = 1$.

Ans.

Q. 26. $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$.

Solution : We have,

$$\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$$

or $\frac{x - 1 + 2(x-2)}{(x-2)(x-1)} = \frac{6}{x}$

or $\frac{x - 1 + 2x - 4}{(x-2)(x-1)} = \frac{6}{x}$

or $\frac{3x - 5}{(x-2)(x-1)} = \frac{6}{x}$

On cross multiplication, we get

$$x(3x - 5) = 6(x - 2)(x - 1)$$

or $3x^2 - 5x = 6(x^2 - 3x + 2)$

or $3x^2 - 5x - 6x^2 + 18x - 12 = 0$

or $-3x^2 + 13x - 12 = 0$

or $3x^2 - 13x + 12 = 0$

The quadratic polynomial $3x^2 - 13x + 12$ can be factorized as

$$3x^2 - 13x + 12 = 3x^2 - 9x - 4x + 12$$
$$= 3x(x - 3) - 4(x - 3)$$
$$= (x - 3)(3x - 4)$$

∴ The given quadratic equation can be written as

$$(x - 3)(3x - 4) = 0$$

Therefore, $x - 3 = 0$

or $3x - 4 = 0$

Therefore, $x = 3$

or $x = \frac{4}{3}$

Thus, the two roots are

$x = 3$ and $x = \frac{4}{3}$. **Ans.**

Q. 27. $\frac{x-3}{x+3} - \frac{x+3}{x-3} = \frac{48}{7}$, $x \neq 3, x \neq -3$.

Solution : We have,

$$\frac{x-3}{x+3} - \frac{x+3}{x-3} = \frac{48}{7}$$

or $\frac{(x-3)^2 - (x+3)^2}{(x+3)(x-3)} = \frac{48}{7}$

One cross multiplication, we get

$$7[x^2 - 6x + 9 - x^2 - 6x - 9] = 48(x^2 - 9)$$

or $-84x = 48x^2 - 432$

or $48x^2 + 84x - 432 = 0$

or $4x^2 + 7x - 36 = 0$

The quadratic polynomial $4x^2 + 7x - 36$ can be factorized as

$$4x^2 + 7x - 36$$
$$= 4x^2 + 16x - 9x - 36$$
$$= 4x(x + 4) - 9(x + 4)$$
$$= (x + 4)(4x - 9).$$

$\therefore$ The given quadratic equation can be written as

$(x + 4)(4x - 9) = 0$

Therefore, $x + 4 = 0$ or $4x - 9 = 0$

Therefore, $x = -4$ or $x = \frac{9}{4}$

Thus, the two roots are $x = -4$ and $x = \frac{9}{4}$. **Ans.**

Exercise 4.3

Multiple Choice Type Questions

Q. 1. *Discriminant of* $ax^2 + bx + c = 0$ *is* :

(a) $\sqrt{b^2 - 4ac}$

(b) $-\sqrt{b^2 - 4ac}$

(c) $-b \pm \sqrt{b^2 - 4ac}$

(d) None of these.

Q. 2. *A quadratic equation* $ax^2 + bx + c = 0, a \neq 0$ *has equal roots if* $b^2 - 4ac$ *is* :

(a) equal to 0

(b) greater than 0

(c) less than 0

(d) None of these.

Q. 3. *The roots of* $ax^2 + bx + c = 0$, $a \neq 0$ *are real and unequal, if* $b^2 - 4ac$ *is* :

(a) equal to 0

(b) ≥ 0

(c) ≤ 0

(d) None of these.

Q. 4. *The roots of equation* $2x^2 - 8x + c = 0, a \neq 0$ *are equal, then the value of* c *is* :

(a) 2 (b) 4

(c) 6 (d) 8.

Q. 5. *If the discriminant* (D) *of a quadratic equation* $ax^2 + bx + c = 0, a \neq 0$ *is greater than zero, the roots are* :

(a) Real and unequal

(b) Real and equal

(c) Not real

(d) None of these.

Q. 6. *An expression in* α *and* β *is called symmetrical expression if by interchanging* α *and* β, *the expression is* :

(a) Changed

(b) Not changed

(c) My be (a) and (b)

(d) None of these.

Q. 7. *If* $x^2 + 5bx + 16 = 0$ *has no real roots, then* :

(a) $b > \frac{8}{5}$ (b) $b < \frac{-8}{5}$

(c) $\frac{-8}{5} < b < \frac{8}{5}$ (d) None of these.

Q. 8. *If the roots of* $5x^2 - px + 1 = 0$ *are real and distinct, then* :

(a) $p > 2\sqrt{5}$

(b) $p < -2\sqrt{5}$

(c) $-2\sqrt{5} < p < 2\sqrt{5}$

(d) $p > 2\sqrt{5}$ or $p < -2\sqrt{5}$.

Q. 9. *The roots of equation* $3x^2 - 7x - 5 = 0$ *are* :

(a) Real and equal

(b) Imaginary

(c) Real, unequal and rational

(d) Real, unequal and irrational.

Solution : The given equation is

$3x^2 - 7x - 5 = 0$

Here, $a = 3, b = -7, c = -5$

$\therefore \quad D = b^2 - 4ac$

$= (-7)^2 - 4 \times 3 \times (-5)$

$= 49 + 60 = 109$

Since, $b^2 - 4ac > 0$, the given quadratic equation has real unequal and irrational roots.

Q. 10. *The value of the discriminant of the equation* $2x^2 + x - 1 = 0$ *is* :

(a) 2 (b) -3

(c) 7 (d) 9.

Solution : The given equation is

$2x^2 + x - 1 = 0$

Here, $a = 2, b = 1, c = -1$

$\therefore \quad D = b^2 - 4ac$

$= 1 - 4 \times 2 \times (-1)$

$= 1 + 8 = 9.$

Q. 11. *The roots of* $2x^2 - 6x + 7 = 0$ *are* :

(a) Imaginary

(b) Real and equal

(c) Real, unequal and rational

(d) Real, unequal and irrational.

Solution : The given equation is

$$2x^2 - 6x + 7$$

Here, $a = 2$, $b = -6$, $c = 7$

$$\therefore \quad D = b^2 - 4ac$$
$$= (-6)^2 - 4 \times 2 \times 7$$
$$= 36 - 56 = -20$$

Since, $b^2 - 4ac < 0$, the given quadratic equation has imaginary roots.

Q. 12. *If the quadratic equation* $x^2 + 2(p+2)x + 9p = 0$ *has equal roots, then the values of p are* :

(a) 1, 4 (b) 1, – 4

(c) – 1, 4 (d) – 1, – 4.

Solution : The given quadratic equation is

$$x^2 + 2(p+2)x + 9p = 0$$

Here, $a = 1$, $b = 2(p+2)$, $c = 9p$

$$\therefore D = b^2 - 4ac$$
$$= [2(p+2)]^2 - 4 \times 1 \times 9p$$
$$= 4(p^2 + 4 + 4p) - 36p$$
$$= 4p^2 + 16 + 16p - 36p$$
$$= 4p^2 + 16 - 20p$$

The given equation will have real roots, if $D = 0$

$$\therefore \quad 4p^2 - 20p + 16 = 0$$
$$\Rightarrow \quad p^2 - 5p + 4 = 0$$
$$\Rightarrow \quad p^2 - 4p - p + 4 = 0$$
$$\Rightarrow p(p-4) - 1(p-4) = 0$$
$$\Rightarrow \quad (p-4)(p-1) = 0$$
$$\therefore \; p - 4 = 0 \text{ or } p - 1 = 0$$
$$p = 4, p = 1.$$

Q. 13. *The nature of the roots of the quadratic equation* $x^2 - 5x - 7 = 0$ *is* :

(a) Imaginary

(b) Real and equal

(c) Real and unequal

(d) None of these.

Solution : The given equation is

$$x^2 - 5x - 7 = 0$$

Here, $a = 1$, $b = -5$, $c = -7$

$$\therefore \quad D = b^2 - 4ac$$
$$= (-5)^2 - 4 \times 1 \times (-7)$$
$$= 25 + 28 = 53$$

Since, $b^2 - 4ac > 0$, the given quadratic equation has real and unequal roots.

Q. 14. *If the quadratic equation* $4x^2 - 3kx + 1 = 0$ *has equal roots, then the value of k is* :

(a) $+\frac{4}{3}$ (b) $\pm\frac{4}{3}$

(c) $\pm\frac{3}{4}$ (d) $\pm\frac{5}{3}$.

Solution : The given quadratic equation is

$$4x^2 - 3kx + 1 = 0$$

Here, $a = 4$, $b = -3k$, $c = 1$

$$\therefore \quad D = b^2 - 4ac$$
$$= (-3k)^2 - 4 \times 4 \times 1$$
$$= 9k^2 - 16$$

The given equation will have equal roots, if $D = 0$

$$\therefore \quad 9k^2 = 16$$
$$k^2 = \frac{16}{9}$$
$$k = \pm\frac{4}{3}.$$

Q. 15. *The equation* $3x^2 + 7x + 8 = 0$ *is true for* :

(a) All real values of x

(b) No real value of x

(c) Positive real values of x

(d) Integral values of x.

Solution : The given equation is

$$3x^2 + 7x + 8 = 0$$

Here, $a = 3$, $b = 7$, $c = 8$

$$\therefore \quad D = b^2 - 4ac$$
$$= (7)^2 - 4 \times 3 \times 8$$
$$= 49 - 96 = -47$$

The given equation has no real values of x.

[Ans. : 1. (d), **2.** (a), **3.** (b), **4.** (d), **5.** (a), **6.** (b), **7.** (c), **8.** (d), **9.** (d), **10.** (d), **11.** (a), **12.** (a), **13.** (c), **14.** (b), **15.** (b).**]**

Very Short Answer Type Questions

Q. 16. *Write the discriminant of the following quadratic equations :*

(*i*) $x^2 + x + 1 = 0$

(*ii*) $2x^2 - 7x + 6 = 0$

(*iii*) $3x^2 + 8x + 4 = 0$

(*iv*) $3x^2 + 2x - 1 = 0$

(*v*) $2x^2 - 5\sqrt{2}\,x + 4 = 0$

(*vi*) $\sqrt{3}x^2 - 2\sqrt{2}\,x - 2\sqrt{3} = 0.$

Solution : (i) The given equation is $x^2 + x + 1 = 0$

Here, $a = 1, b = 1$ and $c = 1$

$\therefore$ Discriminant $(D) = b^2 - 4ac$

$= (1)^2 - 4(1)(1) = 1 - 4 = -3.$ **Ans.**

(ii) The given equation is

$$2x^2 - 7x + 6 = 0$$

Here, $a = 2, b = -7$ and $c = 6$

$\therefore$ Discriminant (D)

$= b^2 - 4ac$

$= (-7)^2 - 4(2)(6)$

$= 49 - 48 = 1.$ **Ans.**

(iii) $3x^2 + 8x + 4 = 0.$

$D = b^2 - 4ac$

$= (8)^2 - 4(3)(4)$

$= 64 - 48$

$= 16.$ **Ans.**

(iv) $3x^2 + 2x - 1 = 0$

$D = b^2 - 4ac$

$= (2)^2 - 4(3)(-1)$

$= 4 + 12$

$= 16.$

(v) The given equation is

$$2x^2 - 5\sqrt{2}\,x + 4 = 0.$$

Here, $a = 2, b = -5\sqrt{2}$ and $c = 4$

$\therefore$ Discriminant (D)

$= b^2 - 4ac$

$= \left(-5\sqrt{2}\right)^2 - 4(2)(4)$

$= 50 - 32 = 18.$ **Ans.**

(vi) The given equation is

$$\sqrt{3}\,x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$$

Here, $a = \sqrt{3}$, $b = -2\sqrt{2}$

and $c = -2\sqrt{3}$

$\therefore$ Discriminant (D)

$= b^2 - 4ac$

$= \left(-2\sqrt{2}\right)^2 - 4\left(\sqrt{3}\right)\left(-2\sqrt{3}\right)$

$= 8 + 24 = 32.$ **Ans.**

Q. 17. *Determine which of the following quadratic equations have real roots :*

(*i*) $2x^2 + x - 1 = 0$

(*ii*) $2x^2 + 5x + 5 = 0$

(*iii*) $25x^2 + 30x + 7 = 0$

(*iv*) $3x^2 + 2x - 1 = 0$

(*v*) $x^2 - 4x + 4 = 0$

(*vi*) $4 - 11x = 3x^2.$

Solution : (i) The given equation is

$$2x^2 + x - 1 = 0.$$

Here, $a = 2, b = 1$ and $c = -1$

$D = b^2 - 4ac = (1)^2 - 4(2)(-1)$

$= 1 + 8 = 9 > 0$

Since, $D > 0$, the given quadratic equation has real (unequal) roots. **Ans.**

(ii) The given equation is

$$2x^2 + 5x + 5 = 0.$$

Here, $a = 2, b = 5$ and $c = 5$

$D = b^2 - 4ac = (5)^2 - 4(2)(5)$

$= 25 - 40 = -15 < 0.$

Since, $D < 0$, the given quadratic equation has no real roots. **Ans.**

(iii) The given equation is

$$25x^2 + 30x + 7 = 0$$

Here, $a = 25, b = 30$ and $c = 7$

$D = b^2 - 4ac = (30)^2 - 4(25)(7)$

$= 900 - 700 = 200 > 0.$

Since, $D > 0$, the given quadratic equation has real (unequal) roots. **Ans.**

(iv) The given equation is

$$3x^2 + 2x - 1 = 0$$

Here, $a = 3, b = 2$ and $c = -1$

$D = b^2 - 4ac = (2)^2 - 4(3)(-1)$

$= 4 + 12 = 16 > 0.$

Since, $D > 0$, the given quadratic equation has real (unequal) roots. **Ans.**

(v) The given equation is

$$x^2 - 4x + 4 = 0$$

Here, $a = 1, b = -4$ and $c = 4$

$$D = b^2 - 4ac = (-4)^2 - 4(1)(4)$$
$$= 16 - 16 = 0$$

Since, $D = 0$, the given quadratic equation has real (equal) roots. **Ans.**

(vi) The given equation is

$$4 - 11x = 3x^2$$

or $3x^2 + 11x - 4 = 0$

Here, $a = 3, b = 11$, and $c = -4$.

$$\therefore \quad D = b^2 - 4ac = (11)^2 - 4(3)(-4)$$
$$= 121 + 48 = 169 > 0$$

Since, $D > 0$, the given quadratic equation has real (unequal) roots. **Ans.**

Q. 18. *In the following, determine the set of values of p for which the given quadratic equation has real roots.*

(*i*) $px^2 - 6x - 2 = 0$

(*ii*) $x^2 + 2x + p = 0$

(*iii*) $2x^2 + px + 2 = 0$

(*iv*) $2x^2 + 3x + p = 0$.

Solution : (i) the given equation is

$$px^2 - 6x - 2 = 0$$

Here, $a = p, b = -6$ and $c = -2$

$$\therefore \quad D = b^2 - 4ac$$
$$= (-6)^2 - 4(p)(-2)$$
$$= 36 + 8p$$

The given equation will have real roots, if $D \geq 0$.

$\therefore \quad 36 + 8p \geq 0$

or $\quad 8p \geq -36$

or $\quad p \geq \frac{-36}{8}$

i.e., $\quad p \geq -\frac{9}{2}$. **Ans.**

(ii) The given equation is

$$x^2 + 2x + p = 0$$

Here, $a = 1, b = 2$ and $c = p$.

$$D = b^2 - 4ac$$
$$= (2)^2 - 4(1)(p) = 4 - 4p$$

The given equation will have real roots, if $D \geq 0$.

$\therefore \quad 4 - 4p \geq 0$

or $\quad 4 \geq 4p$

or $\quad 4p \leq 4$

i.e., $\quad p \leq 1$. **Ans.**

(iii) The given equation is

$$2x^2 + px + 2 = 0$$

Here, $a = 2, b = p$ and $c = 2$

$$\therefore \quad D = b^2 - 4ac$$
$$= (p)^2 - 4(2)(2) = p^2 - 16$$

The given equation will have real roots, if $D \geq 0$.

$\therefore \quad p^2 - 16 \geq 0$

$\Rightarrow \quad p^2 \geq 16$

i.e., $\quad p \geq \sqrt{16}$ or $p \leq -\sqrt{16}$

$\therefore \quad p \geq 4 \quad$ or $p \leq -4$. **Ans.**

(iv) The given equation is

$$2x^2 + 3x + p = 0.$$

Here, $a = 2, b = 3$ and $c = p$

$$D = b^2 - 4ac$$
$$= (3)^2 - 4(2)(p) = 9 - 8p$$

The given equation will have real roots, if $D \geq 0$

$\therefore \quad 9 - 8p \geq 0$

or $\quad 9 \geq 8p$

or $\quad 8p \leq 9$

i.e., $\quad p \leq \frac{9}{8}$. **Ans.**

Short Answer Type Questions

Q. 19. *In the following, determine whether the given quadratic equations have real roots and if so, find the roots :*

(*i*) $x^2 + 2x + 4 = 0$

(*ii*) $\frac{3}{4}x^2 - 8x + 3 = 0$

(*iii*) $25x^2 + 20x + 7 = 0$

(*iv*) $2x^2 + 5\sqrt{3}\,x + 6 = 0$

(*v*) $16x^2 = 24x + 1$

(*vi*) $x^2 - \frac{1}{3}x + \frac{3}{2} = 0$.

Solution : (i) The given equation is

$$x^2 + 2x + 4 = 0$$

Here, $a = 1, b = 2$ and $c = 4$

$$D = b^2 - 4ac = (2)^2 - 4(1)(4)$$
$$= 4 - 16 = -12 < 0$$

Since, $b^2 - 4ac < 0$, the given quadratic equation has no real roots. **Ans.**

(ii) The given equation is

$$\frac{3}{4}x^2 - 8x + 3 = 0$$

Here, $a = \frac{3}{4}$, $b = -8$ and $c = 3$

$\therefore$ $D = b^2 - 4ac$

$$= (-8)^2 - 4\left(\frac{3}{4}\right)(3)$$

$$= 64 - 9 = 55 > 0.$$

Since, $D > 0$, the given quadratic equation has two real and distinct roots, given by

$$\alpha = \frac{-b+\sqrt{D}}{2a}$$

$$= \frac{-(-8)+\sqrt{55}}{2\left(\frac{3}{4}\right)}$$

$$= \frac{8+\sqrt{55}}{\left(\frac{3}{2}\right)} = \frac{2}{3}\left(8+\sqrt{55}\right)$$

and $\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(-8)-\sqrt{55}}{2\,(3/4)}$

$$= \frac{8-\sqrt{55}}{(3/2)} = \frac{2}{3}(8-\sqrt{55})$$

Hence, roots are

$\frac{2}{3}\left(8+\sqrt{55}\right)$ and $\frac{2}{3}(8-\sqrt{55})$ **Ans.**

(iii) The given equation is

$$25x^2 + 20x + 7 = 0$$

Here, $a = 25$, $b = 20$ and $c = 7$

$\therefore$ $D = b^2 - 4ac = (20)^2 - 4\,(25)\,(7)$

$$= 400 - 700 = -300 < 0$$

Since, $D < 0$, the given quadratic has no real roots. **Ans.**

(iv) The given equation is

$$2x^2 + 5\sqrt{3}x + 6 = 0$$

Here, $a = 2$, $b = 5\sqrt{3}$ and $c = 6$

$\therefore$ $D = b^2 - 4ac$

$$= (5\sqrt{3})^2 - 4\,(2)\,(6)$$

$$= 75 - 48 = 27 > 0$$

Since, $D > 0$, the given quadratic equation has real distinct roots, given by

$$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-5\sqrt{3}+\sqrt{27}}{2(2)}$$

$$= \frac{-5\sqrt{3}+3\sqrt{3}}{4} = \frac{-\sqrt{3}}{2}$$

and $\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-5\sqrt{3}-\sqrt{27}}{2\,(2)}$

$$= \frac{-5\sqrt{3}-3\sqrt{3}}{4} = -2\sqrt{3}.$$

Hence, the two roots are

$-\frac{\sqrt{3}}{2}$ and $-2\sqrt{3}$. **Ans.**

(v) The given equation is

$$16x^2 = 24x + 1$$

or $16x^2 - 24x - 1 = 0$

Here, $a = 16$, $b = -24$ and $c = -1$

$\therefore$ $D = b^2 - 4ac$

$$= (-24)^2 - 4(16)(-1)$$

$$= 576 + 64 = 640 > 0.$$

Since, $D > 0$, the given quadratic equation has real and distinct roots, given by

$$\alpha = \frac{-b+\sqrt{D}}{2a}$$

$$= \frac{-(-24)+\sqrt{640}}{2\,(16)}$$

$$= \frac{24+8\sqrt{10}}{32} = \frac{3+\sqrt{10}}{4}$$

and $\beta = \frac{-b-\sqrt{D}}{2a}$

$$= \frac{-(-24)-\sqrt{640}}{2\,(16)}$$

$$= \frac{24-8\sqrt{10}}{32} = \frac{3-\sqrt{10}}{4}$$

Hence, the two roots are

$\frac{3+\sqrt{10}}{4}$ and $\frac{3-\sqrt{10}}{4}$. **Ans.**

(vi) The given quadratic equation is

$$x^2 - \frac{1}{3}x + \frac{3}{2} = 0$$

Here, $a = 1$, $b = -\frac{1}{3}$ and $c = \frac{3}{2}$.

$\therefore \quad D = b^2 - 4ac$

$$= \left(-\frac{1}{3}\right)^2 - 4(1)\left(\frac{3}{2}\right)$$

$$= \frac{1}{9} - 6 = \frac{-53}{9} < 0$$

Since, $D < 0$, the given quadratic equation has no real roots. **Ans.**

Q. 20. *In the following, find the value (s) of p so that the given equations has real and equal roots :*

(*i*) $3x^2 - 4x - p = 0$

(*ii*) $2px^2 - 8x + p = 0$

(*iii*) $2px^2 - 40x + 25 = 0$

(*iv*) $(p - 12)x^2 + 2(p - 12)x + 2 = 0$.

Solution : (i) $3x^2 - 4x - p = 0$

Here, $a = 3$, $b = -4$ and $c = -p$

$D = b^2 - 4ac$

$= (-4)^2 - 4(3)(-p)$

$= 16 + 12p$

We know that a quadratic equation has real and equal roots, if $D = 0$.

i.e., $\quad 16 + 12p = 0$

$\Rightarrow \quad 12p = -16$

$$p = \frac{-4}{3}$$ **Ans.**

(ii) The given equation is

$2px^2 - 8x + p = 0$

Here, $a = 2p$, $b = -8$ and $c = p$

$\therefore \quad D = b^2 - 4ac$

$= (-8)^2 - 4(2p)(p)$

$= 64 - 8p^2$

We know that a quadratic equation has real and equal roots, if $D = 0$.

$\therefore \quad 64 - 8p^2 = 0$

or $\quad p^2 = 8$

or $\quad p = \pm\sqrt{8}$

i.e., $\quad p = 2\sqrt{2}$ or $-2\sqrt{2}$. **Ans.**

(iii) The given equation is

$2px^2 - 40x + 25 = 0$

Here, $a = 2p$, $b = -40$ and $c = 25$

$\therefore \quad D = b^2 - 4ac$

$= (-40)^2 - 4(2p)(25)$

$= 1600 - 200p$

We know that a quadratic equation has real and equal roots, if $D = 0$.

$1600 - 200p = 0$

or $\quad p = 8$. **Ans.**

(iv) The given equation is

$(p - 12)x^2 + 2(p - 12)x + 2 = 0$

Here, $a = p - 12$, $b = 2(p - 12)$ and $c = 2$

$\therefore \quad D = b^2 - 4ac$

$= [2(p - 12)]^2 - 4(p - 12)(2)$

$= 4(p - 12)^2 - 8(p - 12)$

We know that a quadratic equation has real and equal roots, if $D = 0$.

$\therefore \quad 4(p - 12)^2 - 8(p - 12) = 0$

or $\quad (p - 12)^2 - 2(p - 12) = 0$

or $\quad p^2 - 24p + 144 - 2p + 24 = 0$

or $\quad p^2 - 26p + 168 = 0$

or $\quad p^2 - 14p - 12p + 168 = 0$

or $\quad p(p - 14) - 12(p - 14) = 0$

or $\quad (p - 14)(p - 12) = 0$

$\therefore \quad p = 14, 12$. **Ans.**

Q. 21. *Without finding the roots, comment on the nature of the roots of the following quadratic equations :*

(*i*) $5x^2 + 12x - 9 = 0$

(*ii*) $x^2 - 5x - 7 = 0$

(*iii*) $15x^2 - 11x + 3 = 0$

(*iv*) $9x^2 - 6x + 1 = 0$

(*v*) $k^2x^2 + kx + 1 = 0$

(*vi*) $a^2x^2 + abx - b^2 = 0, a \neq 0$.

Solution : (i) The given equation is

$5x^2 + 12x - 9 = 0$

Here, $a = 5$, $b = 12$ and $c = -9$

$\therefore \quad D = b^2 - 4ac$

$= (12)^2 - 4(5)(-9)$

$= 144 + 180$

$= 324 > 0$

So, the given equation has real and unequal roots. **Ans.**

(ii) The given equation is

$$x^2 - 5x - 7 = 0$$

Here, $a = 1, b = -5$ and $c = -7$

$$\therefore \quad D = b^2 - 4ac$$
$$= (-5)^2 - 4(1)(-7)$$
$$= 25 + 28 = 53 > 0$$

So, the given equation has real and unequal roots. **Ans.**

(iii) The given equation is

$$15x^2 - 11x + 3 = 0$$

Here, $a = 15, b = -11$ and $c = 3$

$$\therefore \quad D = b^2 - 4ac$$
$$= (-11)^2 - 4(15)(3)$$
$$= 121 - 180 = -59 < 0.$$

So, the given equation has no real roots. **Ans.**

(iv) The given equation is

$$9x^2 - 6x + 1 = 0.$$

Here, $a = 9, b = -6$ and $c = 1$

$$\therefore \quad D = b^2 - 4ac$$
$$= (-6)^2 - 4(9)(1)$$
$$= 36 - 36 = 0$$

So, the given equation has real and equal roots. **Ans.**

(v) The given equation is

$$k^2x^2 + kx + 1 = 0$$

Here, $a = k^2, b = k$ and $c = 1$

$$\therefore \quad D = b^2 - 4ac = (k^2) - 4(k^2)(1)$$
$$= k^2 - 4k^2 = -3k^2 < 0.$$

So, the given equation has no real roots.

(vi) The given equation is

$$a^2x^2 + abx - b^2 = 0, a \neq 0.$$

Here, $A = a^2, B = ab$ and $C = -b^2$

$$\therefore \quad D = B^2 - 4AC$$
$$= (ab)^2 - 4(a)^2(-b^2)$$
$$= a^2b^2 + 4a^2b^2 = 5a^2b^2 > 0$$

So, the given equation has real and unequal roots. **Ans.**

Q. 22. *For what values of k will the quadratic equation :*

$$2x^2 - kx + 1 = 0,$$

have real and equal roots ?

Solution : The given equation is

$$2x^2 - kx + 1 = 0$$

Here, $a = 2, b = -k, c = 1$

$$\therefore \quad D = b^2 - 4ac$$
$$= (-k)^2 - 4(2)(1)$$
$$= k^2 - 8$$

For the roots to be equal and real,

$$D = 0.$$

$$\therefore \quad k^2 - 8 = 0$$

or $\quad k^2 = 8$

or $\quad k = \pm\sqrt{8}$

i.e., $\quad k = 2\sqrt{2}$ or $-2\sqrt{2}$. **Ans.**

Q. 23. *Determine p, so that the following equation has coincident roots*

$$t^2 + p^2 = 2(p+1)t.$$

Solution : We have,

$$t^2 + p^2 = 2(p+1)t$$

or $t^2 - 2(p+1)t + p^2 = 0$

Here, $a = 1, b = -2(p+1)$ and $c = p^2$

$$\therefore \quad D = b^2 - 4ac$$
$$= [-2(p+1)]^2 - 4(1)(p^2)$$
$$= 4(p+1)^2 - 4p^2$$

The equation will have coincident roots, if

$$D = 0.$$

$$\therefore \quad 4(p+1)^2 - 4p^2 = 0$$

or $\quad (p+1)^2 - p^2 = 0$

or $p^2 + 2p + 1 - p^2 = 0$

or $\quad 2p = -1$

i.e., $\quad p = -\frac{1}{2}$. **Ans.**

Q. 24. *Determine the value of p such that $x^2 + 5px + 16 = 0$ has no real roots.*

Solution : The given equation is

$$x^2 + 5px + 16 = 0$$

Here, $a = 1, b = 5p$ and $c = 16$

$$\therefore \quad D = b^2 - 4ac$$
$$= (5p)^2 - 4(1)(16)$$
$$= 25p^2 - 64.$$

Since, given equation has no real roots, therefore

$$b^2 - 4ac < 0$$

$\Rightarrow \quad 25p^2 - 64 < 0$

$(5p + 8)(5p - 8) < 0$

$\therefore$ Either $5p + 8 > 0$ and $5p - 8 < 0$

$\Rightarrow \quad 5p > -8$ and $5p < 8$

$\Rightarrow \quad p > -\frac{8}{5}$ and $p < \frac{8}{5}$

$\therefore \quad -\frac{8}{5} < p < \frac{8}{5}$

or $5p + 8 < 0$ and $5p - 8 > 0$

$\Rightarrow \quad 5p < -8$ and $5p > 8$

$\Rightarrow \quad p < -\frac{8}{5}$ and $p > \frac{8}{5}$

There is no real number which is both

$< -\frac{8}{5}$ *i.e.*, $-ve$ and $> \frac{8}{5}$ *i.e.*, $+ve$.

This does not provide any value of p.

Hence, $\quad -\frac{8}{5} < p < \frac{8}{5}$. **Ans.**

Q. 25. *Determine the value of p $(p > 0)$ such that the equation $x^2 + px + 64 = 0$ and $x^2 - 8x + p = 0$ will have both real roots.*

Solution : Given, $x^2 + px + 64 = 0$

Here, $a = 1$, $b = p$ and $c = 64$

$\therefore \quad D = b^2 - 4ac = (p)^2 - 4(1)(64)$

$= p^2 - 256$

The given equation will have real roots, if $D \geq 0$.

i.e., $\quad p^2 - 256 \geq 0$

or $\quad p^2 \geq 256$

or $\quad p \geq 16, \qquad (\because p > 0)$

Also given, $x^2 - 8x + p = 0$

Here, $a = 1$, $b = -8$ and $c = p$

$\therefore \quad D = b^2 - 4ac$

$= (-8)^2 - 4(1)(p)$

$= 64 - 4p$

The given equation will have real roots, if $D \geq 0$

i.e., $\quad 64 - 4p \geq 0$

or $\quad 64 \geq 4p$

or $\quad p \leq 16$

Hence, both are satisfied for $p = 16$.

Ans.

Q. 26. *For what values of k will the equation*

$$x^2 - (3k - 1)x + 2k^2 + 2k - 11 = 0$$

have equal roots.

Solution : The given equation is

$x^2 - (3k - 1)x + (2k^2 + 2k - 11) = 0$

Here, $a = 1$, $b = -(3k - 1)$

and $c = 2k^2 + 2k - 11$

$D = b^2 - 4ac$

$= [-(3k - 1)]^2 - 4(1)(2k^2 + 2k - 11)$

$= (3k - 1)^2 - 4(2k^2 + 2k - 11)$

$= 9k^2 - 6k + 1 - 8k^2 - 8k + 44$

$= k^2 - 14k + 45.$

We know that a quadratic equation has equal roots if $D = 0$.

$\therefore \quad k^2 - 14k + 45 = 0$

Here, $a = 1$, $b = -14$, $c = 45$

$$\text{or } k = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(45)}}{2(1)}$$

$$= \frac{14 \pm \sqrt{196 - 180}}{2}$$

$$= \frac{14 \pm \sqrt{16}}{2}$$

$$= \frac{14 \pm 4}{2}$$

$$= \frac{14 \pm 4}{2}, \frac{14 - 4}{2}$$

$$= \frac{18}{2}, \frac{10}{2} = 9, 5$$

Hence, the values of k are 5 or 9.

Ans.

Q. 27. *If the equation*

$$(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$$

has equal roots, show what

$$c^2 = a^2(1 + m^2).$$

Solution : The given equation is

$(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$

Here, $A = 1 + m^2$, $B = 2mc$

and $\quad C = (c^2 - a^2)$

$\therefore D = B^2 - 4AC$

$= (2mc)^2 - 4(1 + m^2)(c^2 - a^2)$

$= 4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2)$

$= 4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2$

$= -4c^2 + 4a^2 + 4m^2a^2$

We know that a quadratic equation has equal roots, if $D = 0$.

$\therefore -4c^2 + 4a^2 + 4m^2a^2 = 0$

or $-4(c^2 - a^2 - m^2a^2) = 0$

or $c^2 - a^2 - m^2a^2 = 0$

or $c^2 = a^2(1 + m^2)$. **Ans.**

Exercise 4.4

Multiple Choice Type Questions

Q. 1. *The sum of the roots of the quadratic equation* $3x^2 + 4x = 0$ *is* :

(a) 0 (b) $-\frac{3}{4}$

(c) $-\frac{4}{3}$ (d) $\frac{4}{3}$.

Solution : The given quadratic equation is

$$3x^2 + 4x = 0$$

Here, $a = 3$, $b = 4$, $c = 0$

$\therefore$ Sum of the roots $= \frac{-b}{a} = \frac{-4}{3}$.

Hence, the sum of the roots of the quadratic equation is $\frac{-4}{3}$. **Ans.**

Q. 2. *The product of the roots of quadratic equation* $3x^2 - 4x = 0$ *is* :

(a) 0 (b) $-\frac{3}{4}$

(c) $-\frac{4}{3}$ (d) $\frac{4}{3}$.

Solution : The given quadratic equation is

$$3x^2 - 4x = 0$$

Here, $a = 3$, $b = -4$, $c = 0$

$\therefore$ The product of the roots

$$= \frac{c}{a} = \frac{0}{3} = 0$$

Hence, the product of the roots of the quadratic equation is 0.

Q. 3. *If one root of* $x^2 + kx + 3 = 0$ *is* 1, *then the value of* k *will be* :

(a) -4 (b) -3 (c) 1 (d) 5.

Solution : The given equation is

$$x^2 + kx + 3 = 0$$

Here, $a = 1$, $b = k$, $c = 3$. Given, one root = 1

$\therefore$ Product of two roots $= \frac{c}{a} = \frac{3}{1} = 3$

$\therefore$ The other roots $= \frac{\text{Product}}{\text{Given root}}$

$$= \frac{3}{1} = 3$$

Sum of the roots $= \frac{-b}{a} = -\frac{k}{1} = -k$.

Also, sum of the roots = given root + other root

$-k = 1 + 3 = 4$

$\therefore \quad k = -4$

Hence, the value of k is -4.

Q. 4. *If the sum of the roots of the equation* $3x^2 + (2k + 1)x - (k + 5) = 0$ *be equal to their product, the value of* k *is* :

(a) 5 (b) 4 (c) 3 (d) 2.

Solution : The given equation is

$$3x^2 + (2k + 1)x - (k + 5) = 0$$

Here, $a = 3$, $b = 2k + 1$, $c = -(k + 5)$

$\therefore$ Sum of roots $= \frac{-b}{a} = -\frac{(2k+1)}{3}$

and product of roots

$$= \frac{c}{a} = -\frac{(k+5)}{3}$$

According to question,

$$-\frac{(2k+1)}{3} = \frac{-(k+5)}{3}$$

$\Rightarrow \quad 2k + 1 = k + 5$

$\Rightarrow \quad 2k - k = 5 - 1$

$\therefore \quad k = 4$.

Q. 5. *The sum of the roots of quadratic equation* $5 - 7x + 3x^2 = 0$ *is* :

(a) $+\frac{7}{5}$ (b) $-\frac{7}{5}$

(c) $-\frac{7}{3}$ (c) $+\frac{7}{3}$.

Solution : The given equation is

$$5 - 7x + 3x^2 = 0$$

or $\quad 3x^2 - 7x + 5 = 0$

Here, $a = 3$, $b = -7$, $c = 5$

$\therefore$ The sum of roots

$$= \frac{-b}{a} = -\frac{(-7)}{3} = \frac{7}{3}$$

Hence, the sum of the roots is $\frac{7}{3}$.

Q. 6. *If one root of quadratic equation* $x^2 - 3x + 2 = 0$ *is* 2, *then the second root is* :

(a) 3 (b) – 1 (c) 1 (d) 2.

Solution : The given equation is

$$x^2 - 3x + 2 = 0$$

Here, $a = 1$, $b = -3$, $c = 2$.

Given one root = 2

$\therefore$ Product of two roots $= \frac{c}{a} = \frac{2}{1} = 2$

$\therefore$ Other root

$$= \frac{\text{product of two roots}}{\text{given root}}$$

$$= \frac{2}{2} = 1.$$ **Ans.**

Q. 7. *The quadratic equation whose one root is* $3 + 2\sqrt{3}$, *is* :

(a) $x^2 + 6x + 3 = 0$

(b) $x^2 - 6x + 3 = 0$

(c) $x^2 + 6x - 3 = 0$

(d) $x^2 - 6x - 3 = 0$.

Solution : Given :

One root $= 3 + 2\sqrt{3}$

$\therefore$ Other root $= 3 - 2\sqrt{3}$

Now, sum of roots

$$= 3 + 2\sqrt{3} + 3 - 2\sqrt{3} = 6$$

and product of roots

$$= (3 + 2\sqrt{3})(3 - 2\sqrt{3})$$

$$= 9 - 12 = -3$$

Hence the required equation is

x^2 – (sum of roots) x + product of roots = 0

$\Rightarrow \quad x^2 - (6)x + (-3) = 0$

$\Rightarrow \quad x^2 - 6x - 3 = 0.$ **Ans.**

Q. 8. *If* α, β *are the roots of* $x^2 - 3x + 2 = 0$, *then the equation with roots* $(\alpha + 1)$, $(\beta + 1)$ *is* :

(a) $x^2 + 5x + 6 = 0$

(b) $x^2 - 5x - 6 = 0$

(c) $x^2 + 5x - 6 = 0$

(d) $x^2 - 5x + 6 = 0$

Solution : The given equation is

$$x^2 - 3x + 2 = 0$$

Here, $a = 1$, $b = -3$, $c = 2$

$\because$ α and β are the roots of the given equation

$$\therefore \quad \alpha + \beta = \frac{-b}{a} = -\frac{(-3)}{1} = 3$$

and $\quad \alpha\beta = \frac{c}{a} = \frac{2}{1} = 2$

$\therefore$ Sum of the given roots

$$= (\alpha + 1) + (\beta + 1)$$

$$= \alpha + \beta + 2$$

$$= 3 + 2 = 5$$

and product of the given roots

$$= (\alpha + 1)(\beta + 1)$$

$$= \alpha\beta + (\alpha + \beta) + 1$$

$$= 2 + 3 + 1 = 6$$

Hence, the required equation is

$$x^2 - Sx + p = 0$$

$$x^2 - 5x + 6 = 0.$$

Q. 9. *If* α, β *are the roots of* $2x^2 - 5x + 7 = 0$, *then the equation whose roots are* $(2\alpha + 3\beta)$ *and* $(3\alpha + 2\beta)$ *is* :

(a) $2x^2 + 25x + 82 = 0$

(b) $x^2 + 25x - 82 = 0$

(c) $2x^2 - 25x + 82 = 0$

(d) $2x^2 - 25x - 82 = 0$.

Q. 10. (i) *If* α, β *are the roots of the quadratic equation* $x^2 - 6x + 6 = 0$, *then the value of* $\alpha^2 + \beta^2$ *is* :

(a) 6 (b) 12

(c) 24 (d) 36.

Solution : The given equation is

$$x^2 - 6x + 6 = 0$$

Here, $a = 1$, $b = -6$, and $c = 6$

$\because$ α and β are the roots of the given equation

$\therefore\ \alpha + \beta = -\dfrac{b}{a} = -\dfrac{(-6)}{1} = 6$

and $\alpha\beta = \dfrac{c}{a} = \dfrac{6}{1} = 6$

$\therefore\ \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (6)^2 - 2 \times 6$

$= 36 - 12 = 24.$

(ii) *Sum of the roots of the equation* $7x^2 - 3x - 4 = 0$ *will be* :

(a) $-\dfrac{3}{7}$ (b) $\dfrac{3}{7}$

(c) $-\dfrac{4}{7}$ (d) $\dfrac{4}{7}$.

Solution : Sum of roots $= -\dfrac{b}{a}$

$= \dfrac{-(-3)}{7} = \dfrac{3}{7}.$ **Ans.**

Q. 11. *If* α, β *are the roots of the quadratic equation* $x^2 + px + q = 0$, *then the value of* $\left(\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}\right)$ *is* :

(a) $\dfrac{p^2 + 2q}{q}$ (b) $\dfrac{p^2 - 2q}{q}$

(c) $\dfrac{-p^2 + 2q}{q}$ (d) $\dfrac{-p^2 - 2q}{q}$.

Solution : The given equation is

$x^2 + px + q = 0$

Here, $a = 1$, $b = p$ and $c = q$

$\because$ α and β are the roots of the given equation

$\therefore\ \alpha + \beta = \dfrac{-b}{a} = \dfrac{-p}{1} = -p$

and $\alpha\beta = \dfrac{c}{a} = \dfrac{q}{1} = q$

Then, $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2 + \beta^2}{\alpha\beta}$

$= \dfrac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \dfrac{(-p)^2 - 2 \times q}{q}$

$= \dfrac{p^2 - 2q}{q}.$

Q. 12. *If the roots of*

$3x^2 - 12x + k = 0$

are equal, then the value of k is :

(a) 3 (b) 4

(c) 6 (d) 12.

Solution : The given equation is

$3x^2 - 12x + k = 0$

Here, $a = 3$, $b = -12$, and $c = k$

$\therefore\ D = b^2 - 4ac$

$= (-12)^2 - 4 \times 3 \times k$

$= 144 - 12k$

If the roots are equal then

$D = 0$

$144 - 12k = 0$

$12k = 144$

$k = 12.$ **Ans.**

Q. 13. *If* α, β *are the roots of* $x^2 + px + 12 = 0$ *and* $\alpha - \beta = 1$, *then the value of p is* :

(a) $p = \pm 3$ (b) $p = \pm 5$

(c) $p = \pm 7$ (d) $p = \pm 8$.

Solution : The given equation is

$x^2 + px + 12 = 0$

Here, $a = 1$, $b = p$, $c = 12$,

and given $\alpha - \beta = 1$

$\because$ α and β are the roots of the given equation

$\therefore\ \alpha + \beta = \dfrac{-b}{a} = \dfrac{-p}{1} = -p$

and $\alpha\beta = \dfrac{c}{a} = \dfrac{12}{1} = 12$

We know that

$\because\ (\alpha + \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$(1)^2 = (-p)^2 - 4 \times 12$

$1 = p^2 - 48$

$p^2 = 49$

$p = \pm 7.$ **Ans.**

Q. 14. *If one root of* $2x^2 - 10x + p = 0$ *is* 3 *then other root will be* :

(a) $p = -3$ (b) $p = 6$

(c) $p = 9$ (d) $p = 12$.

Solution : The given equation is

$2x^2 - 10x + p = 0$

Here, $a = 2$, $b = -10$, $c = p$, given one root = 3

$\therefore$ Product of two roots $= \frac{c}{a} = \frac{p}{2}$

$\therefore$ The other root $= \frac{\text{product}}{\text{given root}}$

$$= \frac{p/2}{3} = \frac{p}{2} \times \frac{1}{3} = \frac{p}{6}$$

Sum of the roots

$$= -\frac{b}{a} = -\frac{(-10)}{2} = \frac{10}{2} = 5$$

Also, Sum of the roots = given root + other root

$$5 = 3 + \frac{p}{6}$$

$$\frac{p}{6} = \frac{2}{1} \Rightarrow p = 12.$$ **Ans.**

Q. 15 (i) *If one root of* $ax^2 + bx + c = 0$ *is the reciprocal of the other, then :*

(a) $a = b$ (b) $b = c$

(c) $c = a$ (d) $a + c = 0$.

(ii) *The sum of the roots of the equation* $bx^2 - cx + a = 0$ *will be :*

(a) $\frac{-b}{a}$ (b) $\frac{-c}{b}$

(c) $\frac{-c}{a}$ (d) $\frac{c}{b}$.

[Ans. : 1. (c), **2.** (a), **3.** (a), **4.** (b), **5.** (d), **6.** (c), **7.** (d), **8.** (d), **9.** (c), **10. (i)** (c), **(ii)** (b), **11.** (b), **12.** (d), **13.** (c), **14.** (d), **15. (i)** (c), **(ii)** (d).]

Very Short Answer Type Questions

Q. 16. *Find the sum and the product of the roots of the following quadratic equations :*

(*i*) $2x^2 + 3x - 5 = 0$

(*ii*) $6x^2 + x - 2 = 0$

(*iii*) $2x^2 + x - 1 = 0$

(*iv*) $3x^2 - 7x - 5 = 0$

(*v*) $\sqrt{3}\,ax^2 = -10ax - 7\sqrt{3}$, *where* $a \neq 0$

Solution : (i) The given equation is

$$2x^2 + 3x - 5 = 0$$

Here, $a = 2$, $b = 3$ and $c = -5$

$\therefore$ Sum of the roots $= \frac{-b}{a} = -\frac{3}{2}$

and product of the roots

$$= \frac{c}{a} = \frac{-5}{2} = -\frac{5}{2}.$$ **Ans.**

(ii) The given equation is

$$6x^2 + x - 2 = 0$$

Here, $a = 6$, $b = 1$ and $c = -2$

$\therefore$ Sum of the roots $= \frac{-b}{a} = -\frac{1}{6}$

and product of the roots

$$= \frac{c}{a} = \frac{-2}{6} = -\frac{1}{3}.$$ **Ans.**

(iii) The given equation is

$$2x^2 + x - 1 = 0$$

Here, $a = 2$, $b = 1$ and $c = -1$

$\therefore$ Sum of the roots $= \frac{-b}{a} = -\frac{1}{2}$

and product of the roots $= \frac{c}{a} = \frac{-1}{2}.$ **Ans.**

(iv) The given equation is

$$3x^2 - 7x - 5 = 0$$

Here, $a = 3$, $b = -7$ and $c = -5$

$\therefore$ Sum of the roots

$$= \frac{-b}{a} = \frac{-(-7)}{3} = \frac{7}{3}$$

and product of the roots

$$= \frac{c}{a} = \frac{-5}{3} = -\frac{5}{3}.$$ **Ans.**

(v) The given equation is

$$\sqrt{3}ax^2 = -10ax - 7\sqrt{3}, \; a \neq 0$$

or $\sqrt{3}ax^2 + 10ax + 7\sqrt{3} = 0$

Here,

$$A = \sqrt{3}a, \; B = 10a \text{ and } C = 7\sqrt{3}$$

$\therefore$ Sum of the roots $= \frac{-B}{A} = \frac{-10a}{\sqrt{3}a}$

$$= -\frac{10}{\sqrt{3}} = -\frac{10\sqrt{3}}{3}$$

and product of the roots

$$= \frac{C}{A} = \frac{7\sqrt{3}}{\sqrt{3}a} = \frac{7}{a}.$$ **Ans.**

Q. 17. *Construct the quadratic equation whose roots are given below :*

(*i*) $\sqrt{5}, 2\sqrt{5}$

(*ii*) $\sqrt{3}, -2\sqrt{3}$

(*iii*) $-\frac{1}{3}, \frac{2}{3}$

(*iv*) $3+\sqrt{7}, 3-\sqrt{7}$

(*v*) $1+\sqrt{2}, 1-\sqrt{2}$

(*vi*) $\frac{4+\sqrt{7}}{3}, \frac{4-\sqrt{7}}{3}$.

Solution : (i) The given roots are $\sqrt{5}$ and $2\sqrt{5}$

$\therefore$ S = Sum of the roots

$= \sqrt{5} + 2\sqrt{5} = 3\sqrt{5}$

and P = Product of the roots

$= \sqrt{5} \times 2\sqrt{5} = 10$

$\therefore$ The required quadratic equation is

$x^2 - Sx + P = 0$

i.e., $x^2 - 3\sqrt{5}\,x + 10 = 0$. **Ans.**

(ii) The given roots are

$\sqrt{3}$ and $-2\sqrt{3}$.

$\therefore$ S= sum of the roots

$= \sqrt{3} - 2\sqrt{3} = -\sqrt{3}$

and P = product of the roots

$= \sqrt{3} \times \left(-2\sqrt{3}\right) = -6$

$\therefore$ The required quadratic equation is

$x^2 - Sx + P = 0$

or $x^2 - (-\sqrt{3})\,x - 6 = 0$

$\therefore$ $x^2 + \sqrt{3}x - 6 = 0$. **Ans.**

(iii) The given roots are $-\frac{1}{3}$ and $\frac{2}{3}$.

$\therefore$ S = sum of the roots

$= -\frac{1}{3} + \frac{2}{3} = \frac{1}{3}$

and P = product of the roots

$= \left(-\frac{1}{3}\right) \times \left(\frac{2}{3}\right) = -\frac{2}{9}$.

$\therefore$ The required quadratic equation is

$x^2 - Sx + P = 0$

or $x^2 - \frac{1}{3}x - \frac{2}{9} = 0$

$\therefore$ $9x^2 - 3x - 2 = 0$. **Ans.**

(iv) The given roots are

$3+\sqrt{7}$ and $3-\sqrt{7}$.

$\therefore$ S = sum of the roots

$= (3+\sqrt{7}) + (3-\sqrt{7})$

$= 3+\sqrt{7}+3-\sqrt{7} = 6$

and P = product of the roots

$= (3+\sqrt{7}) \times (3-\sqrt{7})$

$= (3)^2 - (\sqrt{7})^2$

$= 9 - 7 = 2$

$\therefore$ The required quadratic equation is

$x^2 - Sx + P = 0$

$x^2 - 6x + 2 = 0$. **Ans.**

(v) The given roots are

$1+\sqrt{2}$ and $1-\sqrt{2}$.

$\therefore$ S = sum of the roots

$= (1+\sqrt{2}) + (1-\sqrt{2})$

$= 1+\sqrt{2}+1-\sqrt{2} = 2$

and P = product of the roots

$= (1+\sqrt{2}) \times (1-\sqrt{2})$

$= (1)^2 - (\sqrt{2})^2 = 1 - 2 = -1$

$\therefore$ The required quadratic equation is

$x^2 - Sx + P = 0$

or $x^2 - 2x + (-1) = 0$

$\therefore$ $x^2 - 2x - 1 = 0$. **Ans.**

(vi) The given roots are

$\frac{4+\sqrt{7}}{3}$ and $\frac{4-\sqrt{7}}{3}$.

$\therefore$ S = sum of the roots

$= \frac{4+\sqrt{7}}{3} + \frac{4-\sqrt{7}}{3}$

$= \frac{4+\sqrt{7}+4-\sqrt{7}}{3} = \frac{8}{3}$

and P = product of the roots

$= \left(\frac{4+\sqrt{7}}{3}\right) \times \left(\frac{4-\sqrt{7}}{3}\right)$

$$= \frac{(4)^2 - (\sqrt{7})^2}{(3)^2} = \frac{16-7}{9}$$

$= 1.$

$\therefore$ The required quadratic equation is

$$x^2 - Sx + P = 0$$

or $\quad x^2 - \frac{8}{3}x + 1 = 0$

$\therefore \quad 3x^2 - 8x + 3 = 0.$ **Ans.**

Q. 18. *Form a quadratic equation whose roots have the sum and product given below :*

(*i*) *Sum* = 5; *Product* = 6

(*ii*) *Sum* = $\sqrt{3}$; *Product* = – 6.

Solution : (i) Here, S = sum of the roots = 5 and P = product of the roots = 6

$\therefore$ The required quadratic equation is

$$x^2 - Sx + P = 0$$

i.e., $\quad x^2 - 5x + 6 = 0.$ **Ans.**

(ii) Here, S = sum of the roots = $\sqrt{3}$

and P = product of the roots = – 6.

$\therefore$ The required quadratic equation is

$$x^2 - Sx + P = 0$$

or $\quad x^2 - \sqrt{3}x + (-6) = 0$

i.e., $\quad x^2 - \sqrt{3}x - 6 = 0.$ **Ans.**

Short Answer Type Questions

Q. 19. *Find the value of p so that the sum of the roots of the equation* $3x^2 + (2p+1)x - p - 5 = 0$ *is equal to the product of the roots.*

Solution : The given equation is

$$3x^2 + (2p+1)x - p - 5 = 0$$

Here, $a = 3$, $b = (2p+1)$

and $\quad c = (-p-5)$

$\therefore$ Sum of the roots

$$= \frac{-b}{a} = \frac{-(2p+1)}{3}$$

$$= -\frac{(2p+1)}{3}$$

and product of the roots

$$\frac{c}{a} = \frac{-(p+5)}{3}.$$

Since, sum of the roots = prodcut of the roots

$\therefore \quad -\frac{(2p+1)}{3} = -\frac{(p+5)}{3}$

or $\quad 2p + 1 = p + 5$

or $\quad 2p - p = 5 - 1$

i.e., $\quad p = 4.$ **Ans.**

Q. 20. *If the roots of the quadratic equation* $ax^2 + bx + c = 0$ *are equal, then show that* $b^2 = 4ac$.

Solution : The given equation is

$$ax^2 + bx + c = 0$$

Let the roots of $ax^2 + bx + c = 0$ be α and α.

$\therefore$ Sum of the roots = $\alpha + \alpha = -\frac{b}{a}$

or $\quad 2\alpha = \frac{-b}{a}$

i.e., $\quad \alpha = -\frac{b}{2a}.$

Also, product of the roots = $\alpha.\alpha = \frac{c}{a}$

or $\quad \alpha^2 = \frac{c}{a}$

$\therefore \quad \left(-\frac{b}{2a}\right)^2 = \frac{c}{a} \quad \left[\because \alpha = -\frac{b}{2a}\right]$

or $\quad \frac{b^2}{4a^2} = \frac{c}{a}$

$\therefore \quad b^2 = 4ac.$ **Proved.**

Q. 21. *Find the value of p such that the quadratic equation*

$$x^2 - (p+6)x + 2(2p-1) = 0$$

has sum of the roots as half of their product.

Solution : The given quadratic equation is

$$x^2 - (p+6)x + 2(2p-1) = 0$$

Here, $a = 1$, $b = -(p+6)$

and $\quad c = 2(2p-1)$

$\therefore$ Sum of the roots

$$= \frac{-b}{a} = \frac{-[-(p+6)]}{1} = (p+6)$$

and product of the roots

$$= \frac{c}{a} = \frac{2(2p-1)}{1} = 2(2p-1)$$

Since, sum of the roots

$$= \frac{1}{2} \times \text{product of the roots}$$

$$\therefore \quad p + 6 = \frac{1}{2} \times 2(2p-1)$$

or $\quad p + 6 = 2p - 1$

or $\quad p - 2p = -1 - 6$

$\therefore \quad p = 7.$ **Ans.**

Q. 22. *Find the value of p such that the quadratic equation*

$$x^2 - (2p+1)x + (3p+7) = 0$$

has sum of the roots as one-third of their product.

Solution : The given quadratic equation is

$$x^2 - (2p+1)x + (3p+7) = 0$$

Here, $a = 1, b = -(2p+1)$

and $\quad c = (3p+7)$

$\therefore$ Sum of the roots

$$= \frac{-b}{a} = \frac{-[-(2p+1)]}{1}$$

$$= (2p+1)$$

and product of the roots

$$= \frac{c}{a} = \frac{3p+7}{1} = (3p+7)$$

Since, sum of the roots

$$= \frac{1}{3} \times \text{product of the roots}$$

$$\therefore \quad 2p + 1 = \frac{1}{3} \times (3p+7)$$

or $\quad 6p + 3 = 3p + 7$

or $\quad 6p - 3p = 7 - 3$

or $\quad 3p = 4$

$\therefore \quad p = \frac{4}{3}.$ **Ans.**

Q. 23. *If one root of $x^2 - 3x + p = 0$ be double the other, find the value of p.*

Solution : The given equation is

$$x^2 - 3x + p = 0$$

Here, $a = 1, b = -3, c = p$

Let the roots be α and 2α.

$\therefore$ Sum of the roots

$$= \alpha + 2\alpha = \frac{-b}{a} = \frac{-(-3)}{1}$$

or $\quad 3\alpha = 3$

or $\quad \alpha = 1$

and product of the roots

$$= (\alpha)(2\alpha) = \frac{c}{a} = \frac{p}{1}$$

or $\quad 2\alpha^2 = p$

or $\quad 2(1)^2 = p, \quad [\because a = 1]$

$\therefore \quad p = 2.$ **Ans.**

Q. 24. *If one root of the quadratic equation $3x^2 + px + 4 = 0$ is $\frac{2}{3}$, find the other root of the equation. Also find the value of p.*

Solution : The given equation is

$$3x^2 + px + 4 = 0$$

Also, given that one root of the equation $= \frac{2}{3}$.

Let the other root of the equation be α.

Here, $a = 3, b = p$ and $c = 4$

$\therefore \quad$ Sum of the roots $= \frac{-b}{a}$

i.e., $\quad \alpha + \frac{2}{3} = \frac{-p}{3} \quad$ ...(i)

and product of the roots $= \frac{c}{a}$

i.e., $\quad (\alpha) \times \left(\frac{2}{3}\right) = \frac{4}{3}$

or $\quad a = 2$

Now, putting the value of α in equation (i), we get

$$2 + \frac{2}{3} = -\frac{p}{3}$$

or $\quad \frac{8}{3} = -\frac{p}{3}$

$\therefore \quad p = -8$

Hence, other root = 2 and $p = -8$.

Ans.

Q. 25. *Find the value of k so that the equation $4x^2 - 8kx - 9 = 0$ has one root as the negative of the other.*

Solution : The given equation is

$$4x^2 - 8kx - 9 = 0$$

Here, $a = 4$, $b = -8k$ and $c = -9$

Let α and β be the roots of the given equation.

Then by the given condition,

$$\alpha = -\beta$$

or $\quad \alpha + \beta = 0$

Now, sum of the roots

$$\alpha + \beta = -\frac{b}{a} = \frac{-(-8k)}{4} = 2k$$

But $\alpha + \beta = 0$

$\therefore \quad 2k = 0$

i.e., $\quad k = 0.$ **Ans.**

Q. 26. *If one root of* $6x^2 + 11x + p = 0$ *be reciprocal of the other root, find the value of* p.

Solution : Let the roots be α and $\frac{1}{\alpha}$.

The given equation is

$$6x^2 + 11x + p = 0$$

Here, $a = 6$, $b = 11$ and $c = p$

$\therefore$ Product of the roots $= \frac{c}{a}$

i.e., $\quad \alpha \times \frac{1}{\alpha} = \frac{p}{6}$

or $\quad 1 = \frac{p}{6}$

$\therefore \quad p = 6.$ **Ans.**

Q. 27. *Form a quadratic equation with irrational coefficients, one of whose roots is* $3 + 2\sqrt{3}$.

Solution : Given, one root of the quadratic equation $= 3 + 2\sqrt{3}$.

Since, irrational roots occur in conjugate pairs, therefore the other root of the quadratic equation is $3 - 2\sqrt{3}$.

$\therefore \quad S$ = sum of the roots

$$= (3 + 2\sqrt{3}) + (3 - 2\sqrt{3})$$

$$= 3 + 2\sqrt{3} + 3 - 2\sqrt{3} = 6$$

and product of the roots,

$$p = (3 + 2\sqrt{3}) \times (3 - 2\sqrt{3})$$

$$= (3)^2 - (2\sqrt{3})^2$$

$$= 9 - 12 = -3$$

$\therefore$ The required quadratic equation is

$$x^2 - Sx + P = 0$$

or $x^2 - 6x + (-3) = 0$

$\therefore \quad x^2 - 6x - 3 = 0.$ **Ans.**

Q. 28. *Form a quadratic equation whose one root is* $2 + \sqrt{5}$ *and sum of the roots is* 4.

Solution : Given, S = sum of the roots = 4 and one root $= 2 + \sqrt{5}$

$\therefore$ Other root

$$= 4 - (2 + \sqrt{5})$$

$$= 4 - 2 - \sqrt{5}$$

$$= 2 - \sqrt{5}$$

Now, $\quad P$ = product of the roots

$$= (2 + \sqrt{5}) \times (2 - \sqrt{5})$$

$$= (2)^2 - (\sqrt{5})^2 = 4 - 5 = -1$$

Hence, the required quadratic equation is

$$x^2 - Sx + P = 0$$

or $\quad x^2 - 4x + (-1) = 0$

$\therefore \quad x^2 - 4x - 1 = 0.$ **Ans.**

Q. 29. *Form a quadratic equation whose one root is* $\sqrt{5}$ *and product of the roots is* $-2\sqrt{5}$.

Solution : Given, one root $= -\sqrt{5}$

and product of the roots $= -2\sqrt{5}$.

Let the other root be α.

$\therefore P$ = product of the roots $= -2\sqrt{5}$

i.e., $\sqrt{5} \times \alpha = -2\sqrt{5}$

$\therefore \quad \alpha = -2.$

Now, $\quad S$ = sum of the roots

$$= \sqrt{5} + (-2)$$

$$= \sqrt{5} - 2$$

$\therefore$ The required quadratic equation is

$$x^2 - Sx + P = 0$$

$\therefore \quad x^2 + (2 - \sqrt{5})x - 2\sqrt{5} = 0.$ **Ans.**

Q. 30. *If* α *and* β *are the roots of the quadratic equation*

$$ax^2 + bx + c = 0,$$

then find the values of :

(*i*) $\frac{1}{\alpha} + \frac{1}{\beta}$ (*ii*) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

(*iii*) $\alpha^3 + \beta^3$ (*iv*) $\alpha^3\beta + \beta^3\alpha$

(*v*) $\frac{\alpha + \beta}{\alpha^{-1} + \beta^{-1}}$ (*vi*) $\alpha^4 - \beta^4$.

Solution : Since, α and β are the roots of the quadratic equation

$$ax^2 + bx + c = 0$$

$$\therefore \quad \alpha + \beta = \frac{-b}{a}$$

and $$\alpha\beta = \frac{c}{a}.$$

(i) $$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{\frac{-b}{a}}{\frac{c}{a}} = \frac{-b}{c}.$$

(ii) $$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$= \frac{\left(\frac{-b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(\frac{-b}{a}\right)}{\frac{c}{a}}$$

$$= \frac{-\frac{b^3}{a^3} + \frac{3bc}{a^2}}{\frac{c}{a}}$$

i.e., $$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\frac{-b^3 + 3abc}{a^3}}{\frac{c}{a}}$$

$$= \frac{3abc - b^3}{a^2c}. \quad \textbf{Ans.}$$

(iii) $$\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$$

$$= \left(-\frac{b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(-\frac{b}{a}\right)$$

$$= \frac{-b^3}{a^3} + \frac{3bc}{a^2}$$

$$= \frac{-b^3 + 3abc}{a^3}$$

i.e., $$\alpha^3 + \beta^3 = \frac{3abc - b^3}{a^3}. \quad \textbf{Ans.}$$

(iv) $$\alpha^3\beta + \beta^3\alpha = \alpha\beta(\alpha^2 + \beta^2)$$

$$= \alpha\beta\,[(\alpha+\beta)^2 - 2\alpha\beta]$$

$$= \frac{c}{a}\left[\left(-\frac{b}{a}\right)^2 - 2 \times \frac{c}{a}\right]$$

$$= \frac{c}{a}\left[\frac{b^2}{a^2} - \frac{2c}{a}\right]$$

$$= \frac{c}{a}\left[\frac{b^2 - 2ac}{a^2}\right]$$

i.e., $$\alpha^3\beta + \beta^3\alpha = \frac{c(b^2 - 2ac)}{a^3}. \quad \textbf{Ans.}$$

(v) $$\frac{\alpha+\beta}{\alpha^{-1} + \beta^{-1}} = \frac{\alpha+\beta}{\frac{1}{\alpha} + \frac{1}{\beta}} = \frac{\alpha+\beta}{\frac{\beta+\alpha}{\alpha\beta}}$$

$$= \alpha\beta = \frac{c}{a}. \quad \textbf{Ans.}$$

(vi) $\alpha^4 - \beta^4$

$$= (\alpha+\beta)(\alpha^2+\beta^2)(\alpha-\beta)$$

$$= (\alpha+\beta)\,[(\alpha+\beta)^2 - 2\alpha\beta]\,[\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}]$$

$$= \left(\frac{-b}{a}\right)\left[\left(\frac{-b}{a}\right)^2 - 2 \times \frac{c}{a}\right]\left[\sqrt{\left(\frac{-b}{a}\right)^2 - 4 \times \frac{c}{a}}\right]$$

$$= \left(-\frac{b}{a}\right)\left[\frac{b^2}{a^2} - \frac{2c}{a}\right]\left[\sqrt{\frac{b^2}{a^2} - \frac{4c}{a}}\right]$$

$$= \left(\frac{-b}{a}\right)\left[\frac{b^2 - 2ac}{a^2}\right]\left[\sqrt{\frac{b^2 - 4ac}{a^2}}\right]$$

i.e., $\alpha^4 - \beta^4$

$$= \frac{-b}{a^4}(b^2 - 2ac)(\sqrt{b^2 - 4ac}). \quad \textbf{Ans.}$$

Q. 31. *If* α *and* β *are the roots of the quadratic equation*

$$3x^2 + 8x + 2 = 0,$$

find the value of :

(*i*) $\alpha^2 + \beta^2$ (*ii*) $\alpha^3 + \beta^3$

(*iii*) $\alpha^4\beta^3 + \alpha^3\beta^4$ (*iv*) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

(*v*) $\alpha^4 + \beta^4$.

Solution : Since, α and β are the roots of the quadratic equation

$$3x^2 + 8x + 2 = 0$$

Here, $a = 3$, $b = 8$, $c = 2$

$$\therefore \alpha + \beta = \frac{-b}{a} = -\frac{8}{3}$$

and $$\alpha\beta = \frac{c}{a} = \frac{2}{3}$$

(i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$= \left(-\frac{8}{3}\right)^2 - 2 \times \frac{2}{3} = \frac{64}{9} - \frac{4}{3}$$

$$\therefore \quad \alpha^2 + \beta^2 = \frac{64 - 12}{9} = \frac{52}{9}. \quad \textbf{Ans.}$$

(ii) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta\,(\alpha + \beta)$

$$= \left(\frac{-8}{3}\right)^3 - 3 \times \frac{2}{3}\left(\frac{-8}{3}\right)$$

$$= -\frac{512}{27} + \frac{16}{3}$$

$$= \frac{-512 + 144}{27}$$

$$\therefore \quad \alpha^3 + \beta^3 = -\frac{368}{27}. \quad \textbf{Ans.}$$

(iii) $\alpha^4\beta^3 + \alpha^3\beta^4 = \alpha^3\beta^3(\alpha + \beta)$

$$= (\alpha\beta)^3\,(\alpha + \beta)$$

$$= \left(\frac{2}{3}\right)^3\left(-\frac{8}{3}\right) = \frac{8}{27} \times \left(-\frac{8}{3}\right)$$

$$\therefore \quad \alpha^4\beta^3 + \alpha^3\beta^4 = -\frac{64}{81}. \quad \textbf{Ans.}$$

(iv) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$

$$= \frac{(\alpha + \beta)^3 - 3\alpha\beta\,(\alpha + \beta)}{\alpha\beta}$$

$$= \frac{\left(\frac{-8}{3}\right)^3 - 3\left(\frac{2}{3}\right)\left(\frac{-8}{3}\right)}{2/3}$$

$$= \frac{\frac{-512}{27} + \frac{16}{3}}{\frac{2}{3}} = \frac{\frac{-512+144}{27}}{\frac{2}{3}}$$

$$= \frac{\frac{-368}{27}}{\frac{2}{3}}$$

$$= \frac{-368 \times 3}{27 \times 2}$$

$$\therefore \quad \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{-184}{9}. \quad \textbf{Ans.}$$

(v) $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$

$$= [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$$

$$= \left[\left(-\frac{8}{3}\right)^2 - 2 \times \frac{2}{3}\right]^2 - 2\left(\frac{2}{3}\right)^2$$

$$= \left[\frac{64}{9} - \frac{4}{3}\right]^2 - 2\left(\frac{4}{9}\right)$$

$$= \left(\frac{52}{9}\right)^2 - \frac{8}{9} = \frac{2704 - 72}{81}$$

$$\therefore \quad \alpha^4 + \beta^4 = \frac{2632}{81}. \quad \textbf{Ans.}$$

Q. 32. *If* α *and* β *are the roots of the quadratic equation* $x^2 - 5x + 4 = 0$, *find the value of* $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$.

Solution : Since, α and β are the roots of the quadratic equation

$$x^2 - 5x + 4 = 0$$

Here, $a = 1$, $b = -5$, $c = 4$

$$\therefore \quad \alpha + \beta = \frac{-b}{a} = 5$$

and $$\alpha\beta = \frac{c}{a} = 4$$

$$\therefore \frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{\beta + \alpha}{\alpha\beta} - 2\alpha\beta$$

$= \frac{5}{4} - 2 \times 4 = \frac{5}{4} - 8$

$= -\frac{27}{4}$. **Ans.**

Q. 33. *If* α *and* β *are the roots of the quadratic equation*

$$x^2 + x - 2 = 0,$$

find the value of $\alpha^{-1} - \beta^{-1}$.

Solution : Since α and β are the roots of the quadratic equation

$$x^2 + x - 2 = 0$$

Here, $a = 1, b = 1, c = -2$

$\therefore \quad \alpha + \beta = \frac{-b}{a} = -1$

and $\quad \alpha\beta = \frac{c}{a} = -2$

$\therefore \quad \alpha^{-1} - \beta^{-1} = \frac{1}{\alpha} - \frac{1}{\beta}$

$= \frac{\beta - \alpha}{\alpha\beta} = \frac{-(\alpha - \beta)}{\alpha\beta}$

$= \frac{-[\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}]}{\alpha\beta}$

$= \frac{-[\sqrt{(-1)^2 - 4(-2)}]}{-2}$

$= \frac{\sqrt{1+8}}{2}$

$\therefore \quad \alpha^{-1} - \beta^{-1} = \frac{\sqrt{9}}{2} = \frac{3}{2}$. **Ans.**

Q. 34. *If* α *and* β *are the roots of the quadratic equation* $x^2 - 8x + k = 0$, *find the value of* k, *if* $\alpha^2 + \beta^2 = 20$.

Solution : Since, α and β are the roots of the quadratic equation

$$x^2 - 8x + k = 0.$$

Here, $a = 1, b = -8, c = k$

$\therefore \quad \alpha + \beta = -\frac{b}{a} = 8$

and $\quad \alpha\beta = \frac{c}{a} = k$

Also given, $\quad \alpha^2 + \beta^2 = 20$

$\therefore \quad (\alpha + \beta)^2 - 2\alpha\beta = 20$

or $\quad (8)^2 - 2 \times k = 20$,

[Putting the values of $\alpha + \beta$ and $\alpha\beta$]

or $\quad 64 - 2k = 20$

or $\quad -2k = 20 - 64 = -44$

$\therefore \quad k = 22$. **Ans.**

Long Answer Type Questions

Q. 35. *If* α *and* β *are the roots of the equation* $x^2 - 6x + 4 = 0$, *find the value of*

$$\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta.$$

Solution : Since, α and β are the roots of the equation $x^2 - 6x + 4 = 0$

Here, $a = 1, b = -6, c = 4$

$\therefore \quad \alpha + \beta = \frac{-b}{a} = 6$

and $\quad \alpha\beta = \frac{c}{a} = 4$

Therefore,

$\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$

$= \left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right) + 2\left(\frac{\beta + \alpha}{\alpha\beta}\right) + 3\alpha\beta$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} + 2\left(\frac{\alpha+\beta}{\alpha\beta}\right) + 3\alpha\beta$

$= \frac{(6)^2 - 2 \times 4}{4} + 2\left(\frac{6}{4}\right) + 3 \times 4$

$= \frac{36 - 8}{4} + 3 + 12$

$= \frac{28}{4} + 3 + 12$

$= 7 + 3 + 12 = 22$. **Ans.**

Q. 36. *If* α *and* β *are the roots of the equation* $6x^2 + x - 2 = 0$, *find the value of*:

(*i*) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ $\qquad$ (*ii*) $(\alpha - \beta)^2 + 4\alpha\beta$.

Solution : Since α and β are the roots of the equation

$$6x^2 + x - 2 = 0$$

Here, $a = 6, b = 1, c = -2$

$\therefore \quad \alpha + \beta = -\frac{b}{a} = \frac{-1}{6}$

and $\quad \alpha\beta = \frac{c}{a} = \frac{-2}{6} = \frac{-1}{3}$

(i) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{\left(-\frac{1}{6}\right)^2 - 2\left(-\frac{1}{3}\right)}{-\frac{1}{3}}$$

$$= \frac{\frac{1}{36}+\frac{2}{3}}{-\frac{1}{3}}$$

$$= \frac{\frac{25}{36}}{-\frac{1}{3}} = -\frac{25}{12}. \qquad \textbf{Ans.}$$

(ii) $(\alpha-\beta)^2 + 4\alpha\beta$

$= \alpha^2+\beta^2 - 2\alpha\beta + 4\alpha\beta$

$= \alpha^2+\beta^2+2\alpha\beta = (\alpha+\beta)^2$

$$= \left(-\frac{1}{6}\right)^2 = \frac{1}{36}. \qquad \textbf{Ans.}$$

Q. 37. *If* α *and* β *are roots of the quadratic equation* $x^2 - 1 = 0$, *form an equation whose roots are* $\frac{2\alpha}{\beta}$ *and* $\frac{2\beta}{\alpha}$.

Solution : α and β are the roots of the quadratic equation

$$x^2 - 1 = 0$$

Here, $a = 1$, $b = 0$ and $c = -1$

$$\therefore \quad \alpha+\beta = -\frac{b}{a} = 0$$

and $\alpha\beta = \frac{c}{a} = \frac{-1}{1} = -1.$

We have to form an equation whose roots are $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$.

$\therefore$ Sum of the given roots (S)

$$= \frac{2\alpha}{\beta}+\frac{2\beta}{\alpha}$$

$$= \frac{2\alpha^2+2\beta^2}{\alpha\beta} = \frac{2(\alpha^2+\beta^2)}{\alpha\beta}$$

$$= 2\,\frac{[(\alpha+\beta)^2 - 2\alpha\beta]}{\alpha\beta}$$

i.e., $S = \frac{2[(0)^2 - 2(-1)]}{-1} = -4$

and product of the given roots

$$(P) = \frac{2\alpha}{\beta}\times\frac{2\beta}{\alpha} = 4$$

$\therefore$ The required quadratic equation is

$$x^2 - Sx + P = 0$$

or $\quad x^2 - (-4)x + 4 = 0$

$\therefore \quad x^2 + 4x + 4 = 0.$ **Ans.**

Q. 38. *If* α *and* β *are the roots of the equation* $x^2 - 3x - 2 = 0$, *form an equation whose roots are*

$$\frac{1}{2\alpha+\beta} \textit{ and } \frac{1}{2\beta+\alpha}.$$

Solution : α and β are the roots of the equation

$$x^2 - 3x - 2 = 0$$

Here, $a = 1$, $b = -3$ and $c = -2$

$$\therefore \quad \alpha+\beta = -\frac{b}{a} = \frac{-(-3)}{1} = 3$$

and $\alpha\beta = \frac{c}{a} = \frac{-2}{1} = -2$

We have to form an equation whose roots are $\frac{1}{2\alpha+\beta}$ and $\frac{1}{2\beta+\alpha}$.

$\therefore$ Sum of the given roots (S)

$$= \frac{1}{2\alpha+\beta}+\frac{1}{2\beta+\alpha.}$$

$$= \frac{2\beta+\alpha+2\alpha+\beta}{(2\alpha+\beta)(2\beta+\alpha)}$$

$$= \frac{3\alpha+3\beta}{4\alpha\beta+2\beta^2+2\alpha^2+\alpha\beta}$$

$$= \frac{3(\alpha+\beta)}{5\alpha\beta+2(\alpha^2+\beta^2)}$$

$$= \frac{3(\alpha+\beta)}{5\alpha\beta+2[(\alpha+\beta)^2 - 2\alpha\beta]}$$

$$= \frac{3(3)}{5(-2)+2[(3)^2 - 2(-2)]}$$

i.e., $S = \frac{9}{-10+26} = \frac{9}{16}$

and product of the given roots (P)

$$= \frac{1}{2\alpha+\beta} \times \frac{1}{2\beta+\alpha}$$

$$= \frac{1}{4\alpha\beta + 2\beta^2 + 2\alpha^2 + \alpha\beta}$$

$$= \frac{1}{5\alpha\beta + 2[(\alpha+\beta)^2 - 2\alpha\beta]}$$

$$= \frac{1}{5(-2) + 2[(3)^2 - 2(-2)]}$$

i.e., $P = \frac{1}{-10+26} = \frac{1}{16}$

$\therefore$ The required equation is

$$x^2 - Sx + P = 0$$

or $x^2 - \frac{9}{16}x + \frac{1}{16} = 0$

$\therefore$ $16x^2 - 9x + 1 = 0$. **Ans.**

Q. 39. *The two roots α and β of a quadratic equation are related by the equations $\alpha + \beta = 9$, $\alpha - \beta = 1$. Form an equation whose roots are 3α and β.*

Solution : Given, $\alpha + \beta = 9$ ...(i)

$\alpha - \beta = 1$ ...(ii)

Adding equations (i) and (ii), we get

$$2\alpha = 10$$

i.e., $\alpha = 5$.

Substituting this value of α in equation (i), we get

$$5 + \beta = 9$$

or $\beta = 9 - 5 = 4$.

We have to form an equation whose roots are 3α and β.

$\therefore$ Sum of the given roots (S)

$= 3\alpha + \beta$

$= 3(5) + 4 = 19$,

[$\because \alpha = 5, \beta = 4$]

and product of the given roots (P)

$= 3\alpha \times \beta = 3 \times 5 \times 4 = 60$

$\therefore$ The required quadratic equation is

$$x^2 - Sx + P = 0$$

$\therefore$ $x^2 - 19x + 60 = 0$. **Ans.**

Q. 40. *If α and β are the roots of the equation $ax^2 + bx + c = 0$, show that 2α, 2β are the roots of the equation*

$$ax^2 + 2bx + 4c = 0.$$

Solution : Since α and β are the roots of the equation

$$ax^2 + bx + c = 0$$

$\therefore$ $\alpha + \beta = \frac{-b}{a}$

and $\alpha\beta = \frac{c}{a}$.

We have to form a quadratic equation whose roots are 2α and 2β.

$\therefore$ Sum of the given roots (S)

$= 2\alpha + 2\beta = 2(\alpha + \beta)$

$= 2\left(\frac{-b}{a}\right) = -\frac{2b}{a}$

and product of the given roots (P)

$= 2\alpha \times 2\beta = 4\alpha\beta = \frac{4c}{a}$.

$\therefore$ The required quadratic equation is

$$x^2 - Sx + P = 0$$

or $x^2 - \left(\frac{-2b}{a}\right)x + \frac{4c}{a} = 0$

or $ax^2 + 2bx + 4c = 0$

Hence, 2α and 2β are the roots of the quadratic equation

$$ax^2 + 2bx + 4c = 0.$$

Q. 41. *Form an equation whose roots are the cube of the roots of the equation, $x^2 + mx + n = 0$.*

Solution : Let α and β be the roots of the equation

$$x^2 + mx + n = 0$$

Here, $a = 1, b = m, c = n$

$\therefore$ $\alpha + \beta = \frac{-b}{a} = -m$

and $\alpha\beta = \frac{c}{a} = n$

We have to form an equation whose roots are the cube of the roots of the given equation, *i.e.*, the roots are α^3 and β^3.

$\therefore$ Sum of the given roots (S)

$= \alpha^3 + \beta^3$

$= (\alpha + \beta)^3 - 3\alpha\beta\,(\alpha + \beta)$

$= (-m)^3 - 3(n)(-m)$

$= -m^3 + 3mn$

and product of the given roots (P)

$= \alpha^3 \times \beta^3 = (\alpha\beta)^3$

$= (n)^3 = n^3.$

∴ The required quadratic equation is

$$x^2 - Sx + P = 0$$

or $x^2 - (-m^3 + 3mn)\,x + n^3 = 0$

or $x^2 + (m^3 - 3mn)\,x + n^3 = 0$. **Ans.**

Exercise 4.5

Multiple Choice Type Questions

Q. 1. *Which of the following can be factorized into the product of real linear factors ?*

(a) $2x^2 - 5x + 9$ (b) $3x^2 + 4x + 6$

(c) $2x^2 + 4x - 5$ (d) $5x^2 - 3x + 2$.

Q. 2. *Which of the following can be factorized into the product of real linear factors ?*

(a) $2x^2 - 4x - 1$ (b) $2x^2 - 5x + 4$

(c) $x^2 - 2x + 2$ (d) $7x^2 - 3x + 2$.

Q. 3. *Which of the following cannot be factorized into the product of real linear factors ?*

(a) $4x^2 - 12x + 9$ (b) $7x^2 - 3x + 2$

(b) $x^2 + 6x + 8$ (d) $3x^2 - 4x + 1$.

[Ans. : 1. (c), **2.** (a), **3.** (b).**]**

Very Short Answer Type Questions

Q. 4. *Determine which of the following quadratic polynomials can be factorized into a product of real linear factors :*

(*i*) $2x^2 + 4x - 5$

(*ii*) $2x^2 - 5x + 7$

(*iii*) $x^2 + 2x + 4$

(*iv*) $5x^2 - 2x - 1$

(*v*) $\sqrt{2}x^2 - 3\sqrt{3}x + 4\sqrt{2}$

(*vi*) $\sqrt{3}\,x^2 - x + 2\sqrt{3}$.

Solution : (i) $2x^2 + 4x - 5$

Here, $a = 2$, $b = 4$ and $c = -5$

∴ $D = b^2 - 4ac$

$= (4)^2 - 4\,(2)\,(-5)$

$= 16 + 40 = 56 > 0$

Since, $b^2 - 4ac > 0$

∴ The given quadratic polynomial can be factorized into a product of real linear factors. **Ans.**

(ii) $2x^2 - 5x + 7$

Here, $a = 2$, $b = -5$ and $c = 7$

∴ $D = b^2 - 4ac$

$= (-5)^2 - 4\,(2)\,(7)$

$= 25 - 56 = -31 < 0$

Since, $b^2 - 4ac < 0$

∴ The given quadratic polynomial cannot be factorized into real factors. **Ans.**

(iii) $x^2 + 2x + 4$

Here, $a = 1$, $b = 2$ and $c = 4$

∴ $D = b^2 - 4ac = (2)^2 - 4\,(1)\,(4)$

$= 4 - 16 = -12 < 0$

Since, $b^2 - 4ac < 0$

∴ The given quadratic polynomial cannot be factorized into real factors. **Ans.**

(iv) $5x^2 - 2x - 1$

Here, $a = 5$, $b = -2$ and $c = -1$

∴ $D = b^2 - 4ac$

$= (-2)^2 - 4\,(5)\,(-1)$

$= 4 + 20 = 24 > 0$

Since, $b^2 - 4ac > 0$

∴ The given quadratic polynomial can be factorized into a product of real linear factors. **Ans.**

(v) $\sqrt{2}x^2 - 3\sqrt{3}x + 4\sqrt{2}$

Here, $a = \sqrt{2}$, $b = -3\sqrt{3}$ and $c = 4\sqrt{2}$.

∴ $D = b^2 - 4ac$

$= (-3\sqrt{3})^2 - 4\,(\sqrt{2})(4\sqrt{2})$

$= 27 - 32 = -5 < 0$

Since, $b^2 - 4ac < 0$

∴ The given quadratic polynomial cannot be factorized into a product of real linear factors. **Ans.**

(vi) $\sqrt{3}x^2 - x + 2\sqrt{3}$

Here, $a = \sqrt{3}$, $b = -1$ and $c = 2\sqrt{3}$

$\therefore \quad D = b^2 - 4ac$

$= (-1)^2 - 4(\sqrt{3})(2\sqrt{3})$

$= 1 - 24 = -23 < 0$

Since, $b^2 - 4ac < 0$

$\therefore$ The given quadratic polynomial cannot be factorized into real factors.

Ans.

Q. 5. *In the following, find the set of value (s) of p for which the quadratic polynomial has real linear factors :*

(*i*) $x^2 - px + 4$

(*ii*) $u^2 - 4u + p$

(*iii*) $2x^2 + 6x + p$

(*iv*) $px^2 + 2x - 3$

(*v*) $2x^2 - 3x + p$

(*vi*) $px^2 + 8x - 2$.

Solution : (i) $x^2 - px + 4$

Here, $a = 1$, $b = -p$ and $c = 4$

$\therefore \quad D = b^2 - 4ac$

$= (-p)^2 - 4(1)(4)$

$= p^2 - 16$

The given quadratic polynomial has real linear factors, if $b^2 - 4ac \geq 0$

i.e., $\quad p^2 - 16 \geq 0$

$\Rightarrow \quad (p-4)(p+4) \geq 0$

Either, $p - 4 \geq 0$ and $p + 4 \geq 0$

$\Rightarrow \quad p \geq 4$ and $p \geq -4$

$\Rightarrow \quad p \geq 4$

or $\quad p - 4 \leq 0$ and $p + 4 \leq 0$

$\Rightarrow \quad p \leq 4$ and $p \leq -4$

$\Rightarrow \quad p \leq -4$.

Hence, the given quadratic polynomial has the real linear factors for $p \leq -4$ or $p \geq 4$. **Ans.**

(ii) $u^2 - 4u + p$

Here, $a = 1$, $b = -4$ and $c = p$

$\therefore D = b^2 - 4ac = (-4)^2 - 4(1)(p)$

$= 16 - 4p$.

The given quadratic polynomial has real linear factors, if $b^2 - 4ac \geq 0$.

i.e., $\quad 16 - 4p \geq 0$

$\Rightarrow \quad 4p \leq 16$

or $\quad p \leq 4$. **Ans.**

(iii) $2x^2 + 6x + p$

Here, $a = 2$, $b = 6$ and $c = p$

$\therefore \quad D = b^2 - 4ac = (6)^2 - 4(2)(p)$

$= 36 - 8p$

The given quadratic polynomial has real linear factors, if $b^2 - 4ac \geq 0$.

i.e., $36 - 8p \geq 0$

$\Rightarrow \quad 8p \leq 36$

or $\quad p \leq \frac{36}{8}$

or $\quad p \leq \frac{9}{2}$. **Ans.**

(iv) $px^2 + 2x - 3$

Here, $a = p$, $b = 2$ and $c = -3$

$\therefore \quad D = b^2 - 4ac$

$= (2)^2 - 4(p)(-3)$

$= 4 + 12p$.

The given quadratic polynomial has real linear factors, if

$b^2 - 4ac \geq 0$.

i.e., $\quad 4 + 12p \geq 0$

$\Rightarrow \quad 12p \geq -4$

or $\quad p \geq -\frac{4}{12}$

or $\quad p \geq -\frac{1}{3}$. **Ans.**

(v) $2x^2 - 3x + p$

Here, $a = 2$, $b = -3$ and $c = p$

$\therefore \quad D = b^2 - 4ac$

$= (-3)^2 - 4(2)(p) = 9 - 8p$

The given quadratic polynomial has real linear factors, if

$b^2 - 4ac \geq 0$

i.e., $\quad 9 - 8p \geq 0$

or $\quad 9 \geq 8p$

or $\quad 8p \leq 9$

or $\quad p \leq \frac{9}{8}$. **Ans.**

(vi) $px^2 + 8x - 2$

Here, $a = p$, $b = 8$ and $c = -2$

$\therefore \quad D = b^2 - 4ac$

$= (8)^2 - 4(p)(-2)$

$= 64 + 8p$

The given quadratic polynomial has real linear factors, if

$b^2 - 4ac \geq 0$

i.e., $64 + 8p \geq 0$

or $8p \geq -64$

$\therefore$ $p \geq -8.$ **Ans.**

Q. 6. *Factorize the following quadratic polynomial, whenever possible :*

(*i*) $x^2 + 4x + 1$

(*ii*) $x^2 + 2x - 6$

(*iii*) $3x^2 + 5x + 2$

(*iv*) $x^2 + 2x - 4$

(*v*) $x^2 + 4\sqrt{2}x + 6$

(*vi*) $8x^2 + 43x + 15$

(*vii*) $40 + 3x - x^2$

(*viii*) $10x^2 - 9x - 7$

(*ix*) $2\sqrt{2}x^2 + 4x + \sqrt{2}$

(*x*) $\sqrt{7}x^2 - 6x - 13\sqrt{7}.$

Solution : (i) $x^2 + 4x + 1$

Here, $a = 1$, $b = 4$ and $c = 1$

$\therefore$ $D = b^2 - 4ac$

$= (4)^2 - 4(1)(1) = 16 - 4$

$= 12 > 0.$

Since, $b^2 - 4ac > 0$

$\therefore$ The quadratic equation

$x^2 + 4x + 1 = 0$

has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-4 + \sqrt{12}}{2(1)}$$

$$= \frac{-4 + 2\sqrt{3}}{2} = -2 + \sqrt{3}$$

and $$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-4 - \sqrt{12}}{2(1)}$$

$$= \frac{-4 - 2\sqrt{3}}{2}$$

$$= -2 - \sqrt{3}.$$

Since,

$ax^2 + bx + c = a(x - \alpha)(x - \beta)$

$\therefore x^2 + 4x + 1$

$= [x - (-2 + \sqrt{3})][x - (-2 - \sqrt{3})]$

or $x^2 + 4x + 1$

$= (x + 2 - \sqrt{3})(x + 2 + \sqrt{3}).$ **Ans.**

(ii) $x^2 + 2x - 6$

Here, $a = 1$, $b = 2$ and $c = -6$

$\therefore$ $D = b^2 - 4ac$

$= (2)^2 - 4(1)(-6)$

$= 4 + 24 = 28 > 0$

Since, $b^2 - 4ac > 0$

$\therefore$ The quadratic equation $x^2 + 2x - 6 = 0$ has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-2 + \sqrt{28}}{2(1)}$$

$$= \frac{-2 + 2\sqrt{7}}{2} = -1 + \sqrt{7}$$

and $$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-2 - \sqrt{28}}{2(1)}$$

$$= \frac{-2 - 2\sqrt{7}}{2} = -1 - \sqrt{7}$$

Since,

$ax^2 + bx + c = a(x - \alpha)(x - \beta)$

$\therefore x^2 + 2x - 6 = 1[x - (-1 + \sqrt{7})]$

$[x - (-1 - \sqrt{7})]$

or $x^2 + 2x - 6 =$

$(x + 1 - \sqrt{7})(x + 1 + \sqrt{7}).$ **Ans.**

(iii) $3x^2 + 5x + 2$

Here, $a = 3$, $b = 5$ and $c = 2$

$\therefore$ $D = b^2 - 4ac$

$= (5)^2 - 4(3)(2)$

$= 25 - 24 = 1 > 0$

Since, $b^2 - 4ac > 0$

$\therefore$ The quadratic equation $3x^2 + 5x + 2 = 0$ has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-5 + \sqrt{1}}{2(3)}$$

$$= \frac{-5 + 1}{6} = \frac{-4}{6} = \frac{-2}{3}$$

and $$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-5 - \sqrt{1}}{2(3)}$$

$$= \frac{-5 - 1}{6} = \frac{-6}{6} = -1$$

Since,

$ax^2 + bx + c = a(x - \alpha)(x - \beta)$

$\therefore$ $3x^2 + 5x + 2$

$= 3\left[x - \left(\frac{-2}{3}\right)\right][x - (-1)]$

$= 3\left[x + \frac{2}{3}\right][x + 1]$

i.e., $3x^2 + 5x + 2 = (3x + 2)(x + 1)$. **Ans.**

(iv) $x^2 + 2x - 4$

Here, $a = 1$, $b = 2$ and $c = -4$

$\therefore \quad D = b^2 - 4ac$

$= (2)^2 - 4(1)(-4)$

$= 4 + 16 = 20 > 0$

Since, $b^2 - 4ac > 0$

$\therefore$ The quadratic equation $x^2 + 2x - 4 = 0$ has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-2 + \sqrt{20}}{2(1)}$$

$$= \frac{-2 + 2\sqrt{5}}{2} = -1 + \sqrt{5}$$

and $$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-2 - \sqrt{20}}{2(1)}$$

$$= \frac{-2 - 2\sqrt{5}}{2} = -1 - \sqrt{5}$$

Since, $ax^2 + bx + c = a(x - \alpha)(x - \beta)$

$\therefore x^2 + 2x - 4$

$= 1[x - (-1 + \sqrt{5})]$

$[x - (-1 - \sqrt{5})]$

i.e., $x^2 + 2x - 4$

$= (x + 1 - \sqrt{5})(x + 1 + \sqrt{5})$. **Ans.**

(v) $x^2 + 4\sqrt{2}x + 6$

Here, $a = 1$, $b = 4\sqrt{2}$ and $c = 6$

$\therefore \quad D = b^2 - 4ac$

$= (4\sqrt{2})^2 - 4(1)(6)$

$= 32 - 24 = 8 > 0$

Since, $b^2 - 4ac > 0$

$\therefore$ The quadratic equation $x^2 + 4\sqrt{2}x + 6 = 0$ has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-4\sqrt{2} + \sqrt{8}}{2(1)}$$

$$= \frac{-4\sqrt{2} + 2\sqrt{2}}{2} = \frac{-2\sqrt{2}}{2}$$

$$= -\sqrt{2}$$

and $$\beta = \frac{-b - \sqrt{D}}{2a}$$

$$= \frac{-4\sqrt{2} - \sqrt{8}}{2(1)}$$

$$= \frac{-4\sqrt{2} - 2\sqrt{2}}{2} = \frac{-6\sqrt{2}}{2}$$

$$= -3\sqrt{2}.$$

Since,

$ax^2 + bx + c = a(x - \alpha)(x - \beta)$

$= 1[x - (-\sqrt{2})][x - (-3\sqrt{2})]$

$= (x + \sqrt{2})(x + 3\sqrt{2})$. **Ans.**

(vi) $8x^2 + 43x + 15$

Here, $a = 8$, $b = 43$ and $c = 15$

$\therefore \quad D = b^2 - 4ac$

$= (43)^2 - 4(8)(15)$

$= 1849 - 480 = 1369 > 0$

Since, $b^2 - 4ac > 0$

$\therefore$ The quadratic equation $8x^2 + 43x + 15 = 0$ has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-43 + \sqrt{1369}}{2(8)}$$

$$= \frac{-43 + 37}{16} = \frac{-6}{16} = -\frac{3}{8}$$

and $$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-43 - \sqrt{1369}}{2(8)}$$

$$= \frac{-43 - 37}{16} = \frac{-80}{16} = -5$$

Since, $ax^2 + bx + c = a(x - \alpha)(x - \beta)$

$\therefore 8x^2 + 43x + 15$

$= 8\left[x - \left(-\frac{3}{8}\right)\right][x - (-5)]$

$= 8\left(x + \frac{3}{8}\right)(x + 5)$

$= 8\left(\frac{8x + 3}{8}\right)(x + 5)$

i.e., $8x^2 + 43x + 15$

$= (8x + 3)(x + 5)$. **Ans.**

(vii) $40 + 3x - x^2$

Here, $a = -1$, $b = 3$ and $c = 40$

$\therefore \quad D = b^2 - 4ac$

$= (3)^2 - 4(-1)(40)$

$= 9 + 160 = 169 > 0$

Since, $b^2 - 4ac > 0$

$\therefore$ The quadratic equation $40 + 3x - x^2 = 0$ has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-3 + \sqrt{169}}{2(-1)}$$

$$= \frac{-3 + 13}{-2} = -\frac{10}{2} = -5$$

and $$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-3 - \sqrt{169}}{2(-1)}$$

$$= \frac{-3 - 13}{-2} = \frac{16}{2} = 8$$

Since, $ax^2 + bx + c = a(x - \alpha)(x - \beta)$

$\therefore 40 + 3x - x^2$

$= (-1)[x - (-5)](x - 8)$

$= (-1)(x + 5)(x - 8)$

i.e., $40 + 3x - x^2 = (x + 5)(8 - x)$.

Ans.

(viii) $10x^2 - 9x - 7$

Here, $a = 10$, $b = -9$ and $c = -7$

$\therefore \quad D = b^2 - 4ac$

$= (-9)^2 - 4(10)(-7)$

$= 81 + 280 = 361 > 0$

Since, $b^2 - 4ac > 0$.

$\therefore$ The quadratic equation $10x^2 - 9x - 7 = 0$ has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a}$$

$$= \frac{-(-9) + \sqrt{361}}{2(10)}$$

$$= \frac{9 + 19}{20} = \frac{28}{20} = \frac{7}{5}$$

and $$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(-9) - \sqrt{361}}{2(10)}$$

$$= \frac{9 - 19}{20} = \frac{-10}{20} = \frac{-1}{2}.$$

Since,

$ax^2 + bx + c = a(x - \alpha)(x - \beta)$

$\therefore 10x^2 - 9x - 7$

$$= 10\left(x - \frac{7}{5}\right)\left[x - \left(-\frac{1}{2}\right)\right]$$

$$= 10\left(\frac{5x - 7}{5}\right)\left(\frac{2x + 1}{2}\right)$$

i.e., $10x^2 - 9x - 7 = (5x - 7)(2x + 1)$.

Ans.

(ix) $2\sqrt{2}\,x^2 + 4x + \sqrt{2}$

Here, $a = 2\sqrt{2}$, $b = 4$ and $c = \sqrt{2}$

$\therefore \quad D = b^2 - 4ac$

$= (4)^2 - 4(2\sqrt{2})(\sqrt{2})$

$= 16 - 16 = 0$

Since, $b^2 - 4ac = 0$

$\therefore$ The quadratic equation $2\sqrt{2}x^2 + 4x + \sqrt{2} = 0$ has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-4 + \sqrt{10}}{2(2\sqrt{2})}$$

$$= \frac{-4}{4\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

and $$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-4 - \sqrt{0}}{2(2\sqrt{2})}$$

$$= \frac{-4}{4\sqrt{2}} = -\frac{1}{\sqrt{2}}.$$

Since, $ax^2 + bx + c = a(x - \alpha)(x - \beta)$

$\therefore 2\sqrt{2}\,x^2 + 4x + \sqrt{2}$

$$= 2\sqrt{2}\left[x - \left(-\frac{1}{\sqrt{2}}\right)\right]\left[x - \left(-\frac{1}{\sqrt{2}}\right)\right]$$

$$= 2\sqrt{2}\left[\frac{\sqrt{2}\,x + 1}{\sqrt{2}}\right]\left[\frac{\sqrt{2}\,x + 1}{\sqrt{2}}\right]$$

$= \sqrt{2}(\sqrt{2}\,x + 1)^2$. **Ans.**

(x) $\sqrt{7}x^2 - 6x - 13\sqrt{7}$

Here, $a = \sqrt{7}$, $b = -6$

and $\quad c = -13\sqrt{7}$

$\therefore\ D = b^2 - 4ac$

$= (-6)^2 - 4(\sqrt{7})(-13\sqrt{7})$

$= 36 + 364 = 400 > 0$

Since, $b^2 - 4ac > 0$

$\therefore$ The quadratic equation $\sqrt{7}x^2 - 6x - 13\sqrt{7} = 0$ has real roots, given by

$$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-(-6)+\sqrt{400}}{2(\sqrt{7})}$$

$$= \frac{6+20}{2\sqrt{7}} = \frac{26}{2\sqrt{7}} = \frac{13}{\sqrt{7}}$$

and $$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(-6)-\sqrt{400}}{2(\sqrt{7})}$$

$$= \frac{6-20}{2\sqrt{7}} = \frac{-14}{2\sqrt{7}} = -\sqrt{7}$$

Since, $ax^2 + bx + c = 0$

$\therefore \sqrt{7}\,x^2 - 6x - 13\sqrt{7}$

$$= \sqrt{7}\left[x - \frac{13}{\sqrt{7}}\right][x - (-\sqrt{7})]$$

$$= \sqrt{7}\left[\frac{\sqrt{7}x - 13}{\sqrt{7}}\right][x + \sqrt{7}]$$

i.e., $\sqrt{7}x^2 - 6x - 13\sqrt{7}$

$= (\sqrt{7}x - 13)(x + \sqrt{7})$. **Ans.**

Short Answer Type Questions

Q. 7. *Find the value of a and b such that* $(x + 1)$ *and* $(x + 2)$ *are factors of the polynomial* $x^3 + ax^2 - bx + 10$.

Solution : Here,

$$p(x) = x^3 + ax^2 - bx + 10$$

Since, $(x + 1)$ as well as $(x + 2)$ is a factor of $p(x)$, so -1 as well as -2 are the zeros of $p(x)$.

$\therefore\quad p(-1) = 0$ and $p(-2) = 0$

Now, $\quad p(-1) = 0$

$\Rightarrow (-1)^3 + a(-1)^2 - b(-1) + 10 = 0$

or $\quad -1 + a + b + 10 = 0$

or $\quad a + b = -9 \quad$ ...(i)

and $\quad p(-2) = 0$

$\Rightarrow (-2)^3 + a(-2)^2 - b(-2) + 10 = 0$

or $-8 + 4a + 2b + 10 = 0$

or $\quad 2a + b = -1 \quad$ ...(ii)

On solving equations (i) and (ii), we get

$a = 8, b = -17$. **Ans.**

Exercise 4.6

Multiple Choice Type Questions

Q. 1. *The solution set of the equation* $5^{x+1} + 5^{2-x} = 126$ *is :*

(a) $\{1, 2\}$ (b) $\{1, -2\}$

(c) $\{-1, 2\}$ (d) $\{-1, -2\}$.

Solution : The given equation :

$$5^{x+1} + 5^{2-x} = 126$$

$$\Rightarrow\quad 5^x \cdot 5^1 + \frac{5^2}{5^x} = 126$$

Putting $5^x = y$, the given equation becomes

$$5y + \frac{25}{y} = 126$$

Multiplying both sides by y, we get

$$5y^2 + 25 = 126y$$

or $\quad 5y^2 - 126y + 25 = 0$

or $\quad 5y^2 - 125y - y + 25 = 0$

or $5y(y - 25) - 1(y - 25) = 0$

or $\quad (y - 25)(5y - 1) = 0$

$\therefore\quad y = 25$ or $y = \frac{1}{5}$

Now, when $y = 25$, then

$5^x = 25 \Rightarrow 5^x = 5^2$ *i.e.*, $x = 2$

and when $y = \frac{1}{5}$ then

$5^x = \frac{1}{5} \Rightarrow 5^x = 5^{-1}$ *i.e.*, $x = -1$

Hence, the solution are $x = -1, 2$.

Q. 2. *The solution set of*

$$\left[x + \frac{1}{x}\right]^2 - \frac{3}{2}\left[x - \frac{1}{x}\right] = 4 \text{ is :}$$

(a) $\{1, -1, 2, -2\}$

(b) $\{1, -1, -1/2, 1/2\}$

(c) $\{1, -1, -1/2, 2\}$

(d) $\{1, -1, -2, 1/2\}$

Solution : The given equation is

$$\left[x + \frac{1}{x}\right]^2 - \frac{3}{2}\left[x - \frac{1}{x}\right] = 4$$

$$\Rightarrow \left(x^2+\frac{1}{x^2}+2\right)-\frac{3}{2}\left(x-\frac{1}{x}\right)-4=0$$

$$\Rightarrow \left[\left(x-\frac{1}{x}\right)^2+2\right]-\frac{3}{2}\left(x-\frac{1}{x}\right)-2 = 0$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2+2-\frac{3}{2}\left(x-\frac{1}{x}\right)-2=0$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2-\frac{3}{2}\left(x-\frac{1}{x}\right)=0$$

$$\Rightarrow \left(x-\frac{1}{x}\right)\left[\left(x-\frac{1}{x}\right)-\frac{3}{2}\right]=0$$

$$\therefore \quad x-\frac{1}{x}=0$$

$$\Rightarrow \frac{x^2-1}{x}=0$$

$$\Rightarrow x^2-1=0$$

$$\Rightarrow x^2=1$$

$$\therefore \quad x=\pm 1$$

$$\text{or} \quad x-\frac{1}{x}-\frac{3}{2}=0$$

$$\Rightarrow x-\frac{1}{x}=\frac{3}{2}$$

$$\Rightarrow \frac{x^2-1}{x}=\frac{3}{2}$$

$$\Rightarrow 2x^2-2-3x=0$$

$$\Rightarrow 2x^2-3x-2=0$$

$$\Rightarrow 2x^2-4x+x-2=0$$

$$\Rightarrow 2x(x-2)+1(x-2)=0$$

$$\Rightarrow (x-2)(2x+1)=0$$

$$\Rightarrow x-2=0 \text{ or } x=-1/2$$

$$x=2$$

Hence, the solutions are

$[1, -1, -1/2, 2]$. **Ans.**

Q. 3. *The solution set of*

$$\sqrt{4x-3}+\sqrt{2x+3}=6 \textit{ is}:$$

(a) 37, 9 (b) 111, 3

(c) 3, 9 (d) None of these

Solution : The given equation is

$$\sqrt{4x-3}+\sqrt{2x+3}=6$$

$$\Rightarrow \sqrt{4x-3}=6-\sqrt{2x+3}$$

Squaring on both sides, we get

$$4x-3=36+2x+3-12\sqrt{2x+3}$$

$$\text{or } 4x-3-36-2x-3=-12\sqrt{2x+3}$$

$$\text{or } 2x-42=-12\sqrt{2x+3}$$

$$\text{or } x-21=-6\sqrt{2x+3}$$

Again squaring on both sides, we get

$$x^2-42x+441=36(2x+3)$$

$$\text{or } x^2-42x+441-72x-108=0$$

$$\text{or } x^2-114x+333=0$$

$$\text{or } x^2-3x-111x+333=0$$

$$\text{or } x(x-3)-111(x-3)=0$$

i.e., $x=3$ or $x=111$

Hence, the solutions are $x = 3, 111$.

Q. 4. *The solution set of*

$$\left(\frac{x}{x+1}\right)^2-5\left(\frac{x}{x+1}\right)+6=0 \textit{ is}:$$

(a) $\left(2, \frac{3}{2}\right)$ (b) $\left(\frac{2}{3}, \frac{1}{2}\right)$

(c) $\left(-2, \frac{3}{2}\right)$ (c) $\left(-\frac{3}{2}, -2\right)$.

Solution : The given equation is

$$\left(\frac{x}{x+1}\right)^2-5\left(\frac{x}{x+1}\right)+6=0$$

Let $\frac{x}{x+1}=y$

$$\therefore \quad y^2-5y+6=0$$

$$\Rightarrow y^2-3y-2y+6=0$$

$$\Rightarrow y(y-3)-2(y-3)=0$$

$$(y-3)(y-2)=0$$

i.e., $y=3$ or $y=2$

Now, when $y = 3$, we have

$$\frac{x}{x+1}=3$$

$$\text{or } x=3x+3 \Rightarrow 2x=-3$$

$$\Rightarrow x=-3/2$$

and when $y = 2$, we have

$$\frac{x}{x+1}=2$$

$\Rightarrow \quad 2x + 2 = x$

$\Rightarrow \quad x = -2$

Hence, the solutions are

$x = -2, -3/2.$ **Ans.**

Q. 5. *The solution set of* $(x+1)(x+2)(x+3)(x+4) - 120 = 0$ *is* :

(a) (1, 6) (b) (– 6, 1)

(c) (– 1, – 6) (d) None of these.

Solution : The given equation is

$(x+1)(x+2)(x+3)(x+4) - 120 = 0$

$\Rightarrow [(x+1)(x+4)][(x+2)(x+3)] = 120$

$\Rightarrow (x^2+5x+4)(x^2+5x+6) - 120 = 0$

Putting $x^2 + 5x = y$, we get

$(y+4)(y+6) - 120 = 0$

$\Rightarrow \quad y^2 + 10y + 24 - 120 = 0$

$\Rightarrow \quad y^2 + 10y - 96 = 0$

$\Rightarrow \quad y^2 + 16y - 6y - 96 = 0$

$\Rightarrow \quad y(y+16) - 6(y+16) = 0$

$\Rightarrow \quad (y+16)(y-6) = 0$

i.e., $y = -16$ or $y = 6$

Now, when $y = -16$

$x^2 + 5x = -16$

$\Rightarrow \quad x^2 + 5x + 16 = 0$

$$\therefore \quad x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(16)}}{2 \times 1}$$

$$= \frac{-5 \pm \sqrt{25 - 64}}{2}$$

$$= \frac{-5 \pm \sqrt{-39}}{2}$$

which is not real.

Now, when $y = 6$, then $x^2 + 5x = 6$

$\Rightarrow \quad x^2 + 5x - 6 = 0$

$\Rightarrow \quad x^2 + 6x - x - 6 = 0$

$\Rightarrow \quad x(x+6) - 1(x+6) = 0$

$\Rightarrow \quad (x+6)(x-1) = 0$

i.e., $x = -6$ or $x = 1$

Hence, the required solutions are

$x = -6, 1.$

[Ans. : 1. (c), **2.** (c), **3.** (b), **4.** (d), **5.** (b).]

Very Short Answer Type Questions

Solve the following :

Q. 6. $x^4 - 10x^2 + 9 = 0.$

Solution : Putting $x^2 = y$, the given equation reduces to

$y^2 - 10y + 9 = 0$

or $\quad y^2 - 9y - y + 9 = 0$

or $y(y-9) - 1(y-9) = 0$

or $\quad (y-9)(y-1) = 0$

$\therefore \quad y = 9$ or $y = 1$

Now, substituting $y = x^2$, we get

$x^2 = 9$ or $x^2 = 1$

$x = \pm 3$ or $x = \pm 1$

Hence the solutions are

$x = 1, -1, 3, -4.$ **Ans.**

Q. 7. $4x^4 - 45x^2 + 81 = 0.$

Solution : Suppose

$x^2 = y \quad \quad ...(i)$

$4y^2 - 45y + 81 = 0.$

It is quadratic equation in y.

Here $a = 4$, $b = -45$ and $c = 81$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-45) \pm \sqrt{(-45)^2 - 4(4)(81)}}{2 \times 4}$$

$$= \frac{45 \pm \sqrt{2025 - 1296}}{8}$$

$$= \frac{45 \pm \sqrt{729}}{8} = \frac{45 \pm 27}{8}$$

$$= \frac{45 + 27}{8} \text{ or } \frac{45 - 27}{8}$$

$$= \frac{72}{8} \text{ or } \frac{18}{8}$$

$$y = 9 \text{ or } \frac{9}{4}$$

Put in (i),

$$x^2 = 9 \quad \text{or} \quad x^2 = \frac{9}{4}$$

$$\Rightarrow \quad x = \pm 3 \quad \text{or} \quad x = \pm \frac{3}{2}.$$ **Ans.**

Q. 8. $z^4 - 2z^2 - 3 = 0$.

Solution : Putting $z^2 = x$, the given equation reduces to

$$x^2 - 2x - 3 = 0$$

or $x^2 - 3x + x - 3 = 0$

or $x(x - 3) + 1(x - 3) = 0$

or $(x - 3)(x + 1) = 0$

$\therefore$ $x = 3$ or $x = -1$

Now, substituting $z^2 = x$, we get

$$z^2 = 3 \text{ or } z^2 = -1$$

$\therefore$ $z = \pm\sqrt{3}$

$z^2 = -1$ does not give any real values of z.

Thus, the real values of z satisfying the given equation are $\pm\sqrt{3}$. **Ans.**

Q. 9. $x^4 - 13x^2 + 36 = 0$.

Solution : Putting $x^2 = y$, the given equation reduces to

$$y^2 - 13y + 36 = 0$$

or $y^2 - 9y - 4y + 36 = 0$

or $y(y - 9) - 4(y - 9) = 0$

or $(y - 9)(y - 4) = 0$

$\therefore$ $y = 9$ or $y = 4$

Now, substituting $x^2 = y$, we get

$$x^2 = 9 \text{ or } x^2 = 4$$

$\therefore$ $x = \pm 3$ or $x = \pm 2$

Hence, the solutions are

$x = 3. -3, 2, -2.$ **Ans.**

Q. 10. $9y^4 + 20 = 29y^2$.

Solution : The given equation may be written as

$$9y^4 - 29y^2 + 20 = 0$$

Putting $y^2 = x$, the given equation reduces to

$$9x^2 - 29x + 20 = 0$$

or $9x^2 - 9x - 20x + 20 = 0$

or $9x(x - 1) - 20(x - 1) = 0$

or $(x - 1)(9x - 20) = 0$

$x - 1 = 0$ or $9x - 20 = 0$

$\therefore$ $x = 1$ or $x = \dfrac{20}{9}$

Now, substituting $y^2 = x$, we get

$$y^2 = 1 \text{ or } y^2 = \frac{20}{9}$$

$\therefore$ $y = \pm 1$ or $y = \pm\dfrac{2\sqrt{5}}{3}$

Hence, the solutions are

$y = 1, -1, \dfrac{2\sqrt{5}}{3}, -\dfrac{2\sqrt{5}}{3}$. **Ans.**

Q. 11. $(x^2 + 3x)^2 - (x^2 + 3x) - 6 = 0$.

Solution : Putting $x^2 + 3x = y$, the given equation reduces to

$$y^2 - y - 6 = 0$$

or $y^2 - 3y + 2y - 6 = 0$

or $y(y - 3) + 2(y - 3) = 0$

or $(y - 3)(y + 2) = 0$

$\therefore$ $y - 3 = 0$ or $y + 2 = 0$

$\Rightarrow$ $y = 3$ or $y = -2$

Now, substituting $x^2 + 3x = y$, when $y = 3$, we have $y + 2 = 0$

$\therefore$ $x^2 + 3x = 3$

or $x^2 + 3x - 3 = 0$

Here, $a = 1$, $b = 3$, $c = -3$.

$$\therefore \quad x = \frac{-3 \pm \sqrt{9 - 4(1)(-3)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 + 12}}{2}$$

$$= \frac{-3 \pm \sqrt{21}}{2}$$

and, when $y = -2$, we have

$$x^2 + 3x = -2$$

or $x^2 + 3x + 2 = 0$

or $x^2 + 2x + x + 2 = 0$

or $x(x + 2) + 1(x + 2) = 0$

or $(x + 2)(x + 1) = 0$

$\therefore$ $x + 2 = 0$ or $x + 1 = 0$

$x = -2$ or $x = -1$

Hence, the solutions are

$$x = \frac{-3 + \sqrt{21}}{2}, \frac{-3 - \sqrt{21}}{2}, -1, -2.$$

Ans.

Q. 12. $y - \dfrac{3}{y} = \dfrac{1}{2}$.

Solution : The given equation is

$$y - \frac{3}{y} = \frac{1}{2}$$

Multiplying both sides by y, we get

$$y^2 - 3 = \frac{1}{2}y$$

or $2y^2 - 6 = y$

or $2y^2 - y - 6 = 0$

or $2y^2 - 4y + 3y - 6 = 0$

or $2y(y-2) + 3(y-2) = 0$

or $(y-2)(2y+3) = 0$

$\therefore$ $y - 2 = 0$ or $2y + 3 = 0$

$\therefore$ $y = 2$ or $2y = -3$

$\therefore$ $y = -3/2$

Hence, the solutions are $y = 2, \frac{-3}{2}$.

Ans.

Q. 13. $\frac{1}{a} + \frac{1}{b} + \frac{1}{x} = \frac{1}{a+b+x}$.

Solution :

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{x} = \frac{1}{a+b+x}$$

$$\Rightarrow \quad \frac{1}{a} + \frac{1}{b} = \frac{1}{a+b+x} - \frac{1}{x}$$

$$\Rightarrow \quad \frac{a+b}{ab} = \frac{x-(a+b+x)}{(a+b+x)\,x}$$

$$\Rightarrow \quad \frac{a+b}{ab} = \frac{-(a+b)}{(a+b+x)x}$$

$\Rightarrow$ $(a+b)(a+b+x)x = -ab(a+b)$

$\Rightarrow$ $(a+b+x)x = -ab$

$\Rightarrow$ $x^2 + (a+b)x + ab = 0$

$x^2 + ax + bx + ab = 0$

$\therefore$ $x(x+a) + b(x+a) = 0$

$\therefore$ $(x+a)(x+b) = 0$

$x + a = 0$ or $x + b = 0$

$x = -a$ or $x = -b$. **Ans.**

Q. 14. $2x + \frac{4}{x} = 9$.

Solution : The given equation is

$$2x + \frac{4}{x} = 9$$

Multiplying both sides by x, we get

$$2x^2 + 4 = 9x$$

or $2x^2 - 9x + 4 = 0$

or $2x^2 - 8x - x + 4 = 0$

or $2x(x-4) - 1(x-4) = 0$

or $(x-4)(2x-1) = 0$

$\therefore$ $x - 4 = 0$ or $2x - 1 = 0$

$\therefore$ $x = 4$ or $2x = 1$

$\therefore$ $x = 1/2$

Hence, the solutions are $x = 4, \frac{1}{2}$.

Ans.

Q. 15. $2x - \frac{3}{x} = 5$.

Solution : The given equation is

$$2x - \frac{3}{x} = 5$$

Multiplying both sides by x, we get

$$2x^2 - 3 = 5x$$

or $2x^2 - 5x - 3 = 0$

or $2x^2 - 6x + x - 3 = 0$

or $2x(x-3) + 1(x-3) = 0$

or $(x-3)(2x+1) = 0$

$\therefore$ $x - 3 = 0$ or $2x + 1 = 0$

$\therefore$ $x = 3$ or $2x = -1$

$x = -1/2$

Hence, the solutions are

$x = 3$ or $x = -\frac{1}{2}$. **Ans.**

Short Answer Type Questions

Q. 16. $x - 2\sqrt{x} - 6 = 0$.

Solution : Putting $\sqrt{x} = y$, or $x = y^2$ the given equation reduces to

$$y^2 - 2y - 6 = 0$$

Here, $a = 1, b = -2, c = -6$

$$\therefore\ y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4+24}}{2} = \frac{2 \pm \sqrt{28}}{2}$$

$$= \frac{2 \pm 2\sqrt{7}}{2} = \frac{2(1 \pm \sqrt{7})}{2}$$

$$= 1 \pm \sqrt{7}$$

$\therefore$ $y = 1 + \sqrt{7}$ or $y = 1 - \sqrt{7}$

Now, $y = 1 + \sqrt{7}$

$\Rightarrow$ $\sqrt{x} = 1 + \sqrt{7}$

On squaring both sides, we get

$$x = 1 + 7 + 2\sqrt{7} = 8 + 2\sqrt{7}$$

and when $y = 1 - \sqrt{7} \Rightarrow \sqrt{x} = 1 - \sqrt{7}$

On squaring both sides, we get

$$x = 1 + 7 - 2\sqrt{7} = 8 - 2\sqrt{7}.$$

Hence, the solutions are

$$x = 8 + 2\sqrt{7}, 8 - 2\sqrt{7}.$$ **Ans.**

Q. 17. $\left(\frac{x-1}{x+1}\right)^2 - 13\left(\frac{x-1}{x+1}\right)^2 + 36 = 0.$

Solution : Putting $\left(\frac{x-1}{x+1}\right)^2 = y$, the given equation reduces to

$$y^2 - 13y + 36 = 0$$

or $y^2 - 9y - 4y + 36 = 0$

or $y(y - 9) - 4(y - 9) = 0$

or $(y - 9)(y - 4) = 0$

$\therefore$ $y - 9 = 0$ or $y - 4 = 0$

$\therefore$ $y = 9$ or $y = 4$

Now when $y = 9$, then

$$\left(\frac{x-1}{x+1}\right)^2 = 9$$

Taking square root of both sides, we get

$$\frac{x-1}{x+1} = \pm 3$$

$\therefore$ $x - 1 = 3(x + 1)$, (Taking + ve sign)

or $x - 1 = 3x + 3$

or $-2x = 4$

i.e., $x = -2$

and $x - 1 = -3(x + 1)$, (Taking –ve sign)

or $x - 1 = -3x - 3$

or $4x = -2$

i.e., $x = -\frac{1}{2}$.

and when $y = 4$, then

$$\left(\frac{x-1}{x+1}\right)^2 = 4$$

Taking square root of both sides, we get

$$\frac{x-1}{x+1} = \pm 2$$

$\therefore$ $x - 1 = 2(x + 1)$, (Taking + ve sign)

or $x - 1 = 2x + 2$

or $-x = 3$

i.e., $x = -3$

and $x - 1 = -2(x + 1)$, (Taking – ve sign)

or $x - 1 = -2x - 2$

or $3x = -1$

i.e., $x = -\frac{1}{3}$.

Hence the solutions are

$$-2, -\frac{1}{2}, -3 -\frac{1}{3}.$$ **Ans.**

Q.18. $6\left(\frac{x-3}{2x+1}\right) - 5\sqrt{\frac{x-3}{2x+1}} + 1 = 0$, $x \neq -\frac{1}{2}$.

Solution : Putting $\sqrt{\frac{x-3}{2x+1}} = y$, $\frac{x-3}{2x+1} = y^2$ the given equation reduces to

$$6y^2 - 5y + 1 = 0$$

or $6y^2 - 3y - 2y + 1 = 0$

or $3y(2y - 1) - 1(2y - 1) = 0$

or $(2y - 1)(3y - 1) = 0$

$\therefore$ $2y - 1 = 0$ or $3y - 1 = 0$

$2y = 1$ or $3y = 1$

$\therefore$ $y = \frac{1}{2}$ or $y = \frac{1}{3}$

Now when $y = \frac{1}{2}$, then

$$\sqrt{\frac{x-3}{2x+1}} = \frac{1}{2}$$

Squaring both sides, we get

$$\frac{x-3}{2x+1} = \frac{1}{4}$$

On cross multiplication, we get

$$4x - 12 = 2x + 1$$

or $4x - 2x = 1 + 12$

$$2x = 13$$

i.e., $x = \frac{13}{2}$

and, when $y = \frac{1}{3}$, then

$$\sqrt{\frac{x-3}{2x+1}} = \frac{1}{3}$$

Squaring both sides, we get

$$\frac{x-3}{2x+1} = \frac{1}{9}$$

On cross multiplication, we get

$$9x - 27 = 2x + 1$$

or $9x - 2x = 1 + 27$

or $7x = 28$

i.e., $x = 4$

Hence, the solutions are $x = 4, \frac{13}{2}$. **Ans.**

Q. 19. $\left(\frac{2x+3}{x+1}\right) + 6\left(\frac{x+1}{2x+3}\right) = 7$, $x \neq -1$ *and* $x \neq -\frac{3}{2}$.

Solution : The given equation is

$$\left(\frac{2x+3}{x+1}\right) + 6\left(\frac{x+1}{2x+3}\right) = 7$$

Putting $\left(\frac{2x+3}{x+1}\right) = y$ and

so $\left(\frac{x+1}{2x+3}\right) = \frac{1}{y}$,

the given equation becomes

$$y + \frac{6}{y} = 7$$

Multiplying both sides by y, we get

$$y^2 + 6 = 7y$$

or $y^2 - 7y + 6 = 0$

or $y^2 - 6y - y + 6 = 0$

or $y(y-6) - 1(y-6) = 0$

or $(y-6)(y-1) = 0$

$\therefore$ $y - 6 = 0$ or $y - 1 = 0$

$\therefore$ $y = 6$ or $y = 1$

Now, when $y = 6$, then

$$\frac{2x+3}{x+1} = 6$$

or $2x + 3 = 6x + 6$

$\Rightarrow$ $2x - 6x = 6 - 3$

or $-4x = 3$ *i.e.*, $x = \frac{-3}{4}$

and, when $y = 1$, then

$$\frac{2x+3}{x+1} = 1$$

or $2x + 3 = x + 1$

or $2x - x = 1 - 3$

or $x = -2$

Hence, the solutions are

$x = -2, \frac{-3}{4}$. **Ans.**

Q. 20. $\left(\frac{2x-3}{x-1}\right) - 4\left(\frac{x-1}{2x-3}\right) = 3$, *where* $x \neq 1$ *and* $x \neq \frac{3}{2}$.

Solution : The given equation is

$$\left(\frac{2x-3}{x-1}\right) - 4\left(\frac{x-1}{2x-3}\right) = 3$$

Putting $\left(\frac{2x-3}{x-1}\right) = y$ and then $\left(\frac{x-1}{2x-3}\right) = \frac{1}{y}$, the given equation becomes $y - \frac{4}{y} = 3$

Multiplying both sides by y, we get

$$y^2 - 4 = 3y$$

or $y^2 - 3y - 4 = 0$

or $y^2 - 4y + y - 4 = 0$

or $y(y-4) + 1(y-4) = 0$

or $(y-4)(y+1) = 0$

$\therefore$ $y - 4 = 0$ or $y + 1 = 0$

$\therefore$ $y = 4$ or $y = -1$

Now, when $y = 4$, then

$$\frac{2x-3}{x-1} = 4$$

or $2x - 3 = 4x - 4$

$\Rightarrow$ $2x - 4x = -4 + 3$

or $-2x = -1$ *i.e.*, $x = \frac{1}{2}$

and, when $y = -1$, then

$$\frac{2x-3}{x-1} = -1$$

or $2x - 3 = -x + 1$

$\Rightarrow$ $2x + x = 1 + 3$

or $3x = 4$ *i.e.*, $x = \frac{4}{3}$

Hence, the solution are $x = \frac{1}{2}, \frac{4}{3}$.

Ans.

Q. 21. $3\sqrt{\frac{x}{5}} + 3\sqrt{\frac{5}{x}} = 10, x \neq 0.$

Solution : The given equation is

$$3\sqrt{\frac{x}{5}} + 3\sqrt{\frac{5}{x}} = 10$$

On putting $\sqrt{\frac{x}{5}} = y$ and $\sqrt{\frac{5}{x}} = \frac{1}{y}$, the given equation becomes

$$3y + \frac{3}{y} = 10$$

Multiplying both sides by y, we get

$$3y^2 + 3 = 10y$$

or $3y^2 - 10y + 3 = 0$

or $3y^2 - 9y - y + 3 = 0$

or $3y(y-3) - 1(y-3) = 0$

or $(y-3)(3y-1) = 0$

$\therefore$ $y - 3 = 0$ or $y = 3$

or $3y - 1 = 0$

$3y = 1$

$\therefore$ $y = \frac{1}{3}$

Now, when $y = 3$, then

$$\sqrt{\frac{x}{5}} = 3$$

Squaring both sides, we get

$$\frac{x}{5} = 9 \text{ i.e., } x = 45$$

and, when $y = \frac{1}{3}$, then

$$\sqrt{\frac{x}{5}} = \frac{1}{3}$$

Squaring both sides, we get

$$\frac{x}{5} = \frac{1}{9} \text{ i.e., } x = \frac{5}{9}$$

Hence, the solutions are $x = 45, \frac{5}{9}$.

Ans.

Q. 22. $3^{(x+2)} + 3^{-x} = 10.$

Solution : The given equation is

$$3^{(x+2)} + 3^{-x} = 10$$

or $3^x . 3^2 + 3^{-x} = 10$

or $9 . 3^x + \frac{1}{3^x} = 10$

Putting $3^x = y$, the given equation becomes

$$9y + \frac{1}{y} = 10$$

Multiplying both sides by y, we get

$$9y^2 + 1 = 10y$$

or $9y^2 - 10y + 1 = 0$

or $9y^2 - 9y - y + 1 = 0$

or $9y(y-1) - 1(y-1) = 0$

or $(y-1)(9y-1) = 0$

$\therefore$ $y - 1 = 0$ or $9y - 1 = 0$

$y = 1$ or $9y = 1$

$y = \frac{1}{9}$

Now, when $y = 1$, then $3^x = 1$

$\Rightarrow$ $3^x = (3)^0$ *i.e.*, $x = 0$

and, when $y = \frac{1}{9}$, then

$$3^x = \frac{1}{9}$$

$\Rightarrow 3^x = \frac{1}{3^2} = 3^{-2}$ *i.e.*, $x = -2$

Hence, the solutions are $x = 0, -2$.

Ans.

Q. 23. $5^{(x+1)} + 5^{(2-x)} = 5^3 + 1$.

Solution : The given equation is

$$5^{(x+1)} + 5^{(2-x)} = 5^3 + 1$$

or $\quad 5^x 5^1 + 5^2.5^{-x} = 125 + 1$

or $\quad 5^1.5^x + \frac{5^2}{5^x} = 126$

Putting $5^x = y$, the given equation becomes

$$5y + \frac{25}{y} = 126$$

Multiplying both sides by y, we get

$$5y^2 + 25 = 126y$$

or $\quad 5y^2 - 126y + 25 = 0$

or $\quad 5y^2 - 125y - y + 25 = 0$

or $\quad 5y(y - 25) - 1(y - 25) = 0$

$$(y - 25)(5y - 1) = 0$$

$\therefore \quad y - 25 = 0$ or $5y - 1 = 0$

$\therefore \quad y = 25$ or $5y = 1$

$\therefore \quad y = 1/5$

Now, when $y = 25$, then

$$5^x = 25 \Rightarrow 5^x = 5^2 \text{ i.e., } x = 2$$

and, when $y = \frac{1}{5}$, then

$$5^x = \frac{1}{5} \Rightarrow 5^x = 5^{-1} \text{ i.e., } x = -1$$

Hence, the solutions are $x = -1, 2$.

Ans.

Q. 24. $2^{2x+3} = 65(2^x - 1) + 57$.

Solution : The given equation is

$$2^{2x+3} = 65(2^x - 1) + 57$$

or $\quad 2^{2x} . 2^3 = 65.2^x - 65 + 57$

or $\quad 8(2^x)^2 - 65.2^x + 8 = 0$

Putting $2^x = y$, the above equation becomes

$$8y^2 - 65y + 8 = 0$$

or $\quad 8y^2 - 64y - y + 8 = 0$

or $\quad 8y(y - 8) - 1(y - 8) = 0$

or $\quad (y - 8)(8y - 1) = 0$

$\therefore \quad y - 8 = 0$

$\therefore \quad y = 8$

or $\quad 8y - 1 = 0$

or $\quad 8y = 1$

$\therefore \quad y = \frac{1}{8}$

Now, when $y = 8$, then

$$2^x = 8 \Rightarrow 2^x = 2^3 \text{ i.e., } x = 3$$

and, when $y = \frac{1}{8}$, then

$$2^x = \frac{1}{8}$$

$\Rightarrow \quad 2^x = \frac{1}{2^3} = 2^{-3}$ *i.e.*, $x = -3$

Hence, the solutions are $x = 3, -3$.

Ans.

Q. 25. $\sqrt{217 - x} = x - 7$.

Solution : The given equation is

$$\sqrt{217 - x} = x - 7$$

Solution of this equation must satisfy,

$$217 - x \geq 0 \text{ and } x - 7 \geq 0$$

i.e., $\quad 217 \geq x$ and $x \geq 7$

i.e., $\quad 7 \leq x \leq 217$

Now squaring both sides of given equation, we get

$$217 - x = x^2 - 14x + 49$$

or $\quad x^2 - 13x - 168 = 0$

or $\quad x^2 - 21x + 8x - 168 = 0$

or $\quad x(x - 21) + 8(x - 21) = 0$

or $\quad (x - 21)(x + 8) = 0$

i.e., $\quad x - 21 = 0$

$\therefore \quad x = 21$

or $\quad x + 8 = 0$

$\therefore \quad x = -8$

But $x = -8$ does not satisfy

$$7 < x \leq 217$$

Hence, the solution is $x = 21$. **Ans.**

Q. 26. $\sqrt{25 - x^2} = x - 1$.

Solution : The given equation is

$$\sqrt{25 - x^2} = x - 1$$

Solution of this equation must satisfy

$$25 - x^2 \geq 0 \text{ and } x - 1 \geq 0$$

i.e., $\quad 25 \geq x^2$ and $x \geq 1$

i.e., $\quad 1 \leq x \leq 5$

Now, squaring both sides of given equation, we get

$$25 - x^2 = x^2 - 2x + 1$$

or $\quad 2x^2 - 2x - 24 = 0$

or $\quad x^2 - x - 12 = 0$

or $\quad x^2 - 4x + 3x - 12 = 0$

or $\quad x(x - 4) + 3(x - 4) = 0$

or $\quad (x - 4)(x + 3) = 0$

i.e., $\quad x - 4 = 0$

$\therefore \quad x = 4$

or $\quad x + 3 = 0$

$\therefore \quad x = -3$

But $x = -3$ does not satisfy

$$1 \leq x \leq 5$$

Hence, the solution is $x = 4$. **Ans.**

Q. 27. $\sqrt{2x+9} + x = 13$.

Solution : The given equation may be written as $\sqrt{2x+9} = 13 - x$

Solution of this equation must satisfy

$2x + 9 \geq 0$ and $13 - x \geq 0$

i.e., $\quad 2x \geq -9$ and $13 \geq x$

i.e., $-\frac{9}{2} \leq x \leq 13$

Now, squaring both sides of given equation, we get

$2x + 9 = 169 - 26x + x^2$

or $\quad x^2 - 28x + 160 = 0$

or $\quad x^2 - 20x - 8x + 160 = 0$

or $\quad x(x - 20) - 8(x - 20) = 0$

or $\quad (x - 20)(x - 8) = 0$

i.e., $\quad x - 20 = 0 \Rightarrow x = 20$

or $\quad x - 8 = 0 \Rightarrow x = 8$

But $x = 20$ does not satisfy

$$-\frac{9}{2} \leq x \leq 13.$$

Hence, the solution is $x = 8$. **Ans.**

Long Answer Type Questions

Q. 28. $\sqrt{4-x} + \sqrt{x+9} = 5$.

Solution : Given equation is

$$\sqrt{4-x} + \sqrt{x+9} = 5$$

Solution of this equation must satisfy

$4 - x \geq 0$ and $x + 9 \geq 0$

i.e., $\quad x \leq 4$ and $x \geq -9$

i.e., $\quad -9 \leq x \leq 4$

Now, we first transform one of the radicals to the R.H.S. Then

$$\sqrt{4-x} = 5 - \sqrt{x+9}$$

Squaring both sides, we get

$$4 - x = 25 - 10\sqrt{x+9} + (x + 9)$$

or $-2x - 30 = -10\sqrt{x+9}$

or $\quad x + 15 = 5\sqrt{x+9}$

Again squaring both sides, we get

$x^2 + 30x + 225 = 25(x + 9)$

or $x^2 + 30x + 225 - 25x - 225 = 0$

or $\quad x^2 + 5x = 0$

or $\quad x(x + 5) = 0$

i.e., $\quad x = 0$ or $x = -5$

Here $x = 0, -5$ satisfy $-9 \leq x \leq 4$

Hence, the solutions are $x = 0, -5$.

Ans.

Q. 29. $\sqrt{4x+1} - \sqrt{x+2} = 1$.

Solution : Given equation is

$$\sqrt{4x+1} - \sqrt{x+2} = 1$$

Solution of this equation must satisfy

$4x + 1 \geq 0$ and $x + 2 \geq 0$

i.e., $\quad 4x \geq -1$ and $x \geq -2$

i.e., $\quad x \geq -\frac{1}{4}$ and $x \geq -2$

i.e., $\quad x \geq -2$

Now, we transform one of the radicals to the R.H.S. Then we have

$$\sqrt{4x+1} = 1 + \sqrt{x+2}$$

Squaring on both sides, we get

$$4x + 1 = 1 + x + 2\sqrt{x+2}$$

or $\quad 4x - x + 1 - 1 - 2 = 2\sqrt{x+2}$

or $\quad 3x - 2 = 2\sqrt{x+2}$

Again, squaring both sides, we get

$9x^2 - 12x + 4 = 4(x + 2)$

$\therefore 9x^2 - 12x - 4x + 4 - 8 = 0$

$\therefore \quad 9x^2 - 16x - 4 = 0$

$\therefore \quad 9x^2 - 18x + 2x - 4 = 0$

$\therefore \quad 9x(x - 2) + 2(x - 2) = 0$

$\therefore \quad (x - 2)(9x + 2) = 0$

$\therefore \quad x - 2 = 0 \quad \text{or} \quad 9x + 2 = 0$

$\therefore \quad x = 2 \quad \text{or} \quad 9x = -2$

$\therefore \quad x = -2/9$

Hence, the solution are $x = 2$ or $x = -2/9$. **Ans.**

Q. 30. $\sqrt{2x+3} - \sqrt{x+1} = 1$.

Solution : Given equation is

$$\sqrt{2x+3} - \sqrt{x+1} = 1$$

Solution of this equation must satisfy

$$2x + 3 \geq 0 \text{ and } x + 1 \geq 0$$

i.e., $2x \geq -3$ and $x \geq -1$.

i.e., $x \geq -\frac{3}{2}$ and $x \geq -1$.

i.e., $-\frac{3}{2} \leq x \leq -1$.

Now, we first transform one of the radicals to the R.H.S. Then we have

$$\sqrt{2x+3} = 1 + \sqrt{x+1}$$

Squaring both sides, we get

$$2x + 3 = 1 + 2\sqrt{x+1} + (x+1)$$

or $2x + 3 - 1 - x - 1 = 2\sqrt{x+1}$

or $x + 1 = 2\sqrt{x+1}$

Again, squaring both sides, we get

$$x^2 + 1 + 2x = 4(x+1)$$

or $x^2 + 1 + 2x - 4x - 4 = 0$

or $x^2 - 2x - 3 = 0$

or $x^2 - 3x + x - 3 = 0$

or $x(x-3) + 1(x-3) = 0$

or $(x-3)(x+1) = 0$

i.e., $x - 3 = 0$

$\therefore \quad x = 3$

or $x + 1 = 0$

$\therefore \quad x = -1$

Hence, the solution are $x = 3, -1$. **Ans.**

Q. 31. $\sqrt{3x+10} + \sqrt{6-x} = 6$.

Solution : Given equation is

$$\sqrt{3x+10} + \sqrt{6-x} = 6$$

Solution of this equation must satisfy

$$3x + 10 \geq 0 \text{ and } 6 - x \geq 0$$

i.e., $3x \geq -10$ and $6 \geq x$

i.e., $x \geq -\frac{10}{3}$ and $x < 6$.

i.e., $-\frac{10}{3} \leq x \leq 6$.

Now, we first transform one of the radicals to the R.H.S. Then we have

$$\sqrt{3x+10} = 6 - \sqrt{6-x}$$

Squaring on both sides, we get

$$3x + 10 = 36 - 12\sqrt{6-x} + 6 - x$$

or $4x + 10 - 42 = -12\sqrt{6-x}$

or $4x - 32 = -12\sqrt{6-x}$

or $x - 8 = -3\sqrt{6-x}$

Again, squaring on both sides, we get

$$x^2 - 16x + 64 = 9(6-x)$$

or $x^2 - 16x + 9x + 64 - 54 = 0$

or $x^2 - 7x + 10 = 0$

or $x^2 - 5x - 2x + 10 = 0$

or $x(x-5) - 2(x-5) = 0$

or $(x-5)(x-2) = 0$

i.e., $x - 5 = 0$ or $x - 2 = 0$

$\therefore \quad x = 5 \quad \text{or} \quad x = 2$

Hence, the solution are $x = 5$ or 2. **Ans.**

Q. 32. $\sqrt{x+5} + \sqrt{x+21} = \sqrt{6x+40}$.

Solution : Given equation is

$$\sqrt{x+5} + \sqrt{x+21} = \sqrt{6x+40}$$

Solution of this equation must satisfy

$$x + 5 \geq 0,\ x + 21 \geq 0$$

and $6x + 40 \geq 0$

i.e., $x \geq -5$, $x \geq -21$ and $x \geq -\frac{20}{3}$

In order to satisfy all these conditions, we must have $x \geq -5$.

Now, squaring both sides of given equation, we get

$$x + 5 + 2\sqrt{x+5}\,\sqrt{x+21} + x + 21 = 6x + 40$$

or $2\sqrt{(x+5)(x+21)} = 6x + 40 - x - 5 - x - 21$

or $2\sqrt{(x+5)(x+21)} = 4x + 14$

or $\sqrt{(x+5)(x+21)} = 2x + 7.$

Again, squaring both sides, we get

$(x + 5)(x + 21) = 4x^2 + 28x + 49$

or $x^2 + 21x + 5x + 105 = 4x^2 + 28x + 49$

or $x^2 + 26x + 105 - 4x^2 - 28x - 49 = 0$

or $-3x^2 - 2x + 56 = 0$

or $3x^2 + 2x - 56 = 0$

or $3x^2 + 14x - 12x - 56 = 0$

or $x(3x + 14) - 4(3x + 14) = 0$

or $(3x + 14)(x - 4) = 0$

$\therefore$ $3x + 14 = 0$

$3x = -14$

$x = \frac{-14}{3}$

or $x - 4 = 0$

$\therefore$ $x = 4$

Clearly each one of these values satisfies the condition $x \geq -5$. Putting $x = \frac{-14}{3}$ in the given equation, we get

$$\sqrt{\frac{-14}{3}+5} + \sqrt{\frac{-14}{3}+21} = \sqrt{6\left(\frac{-14}{3}\right)+40}$$

i.e., $\sqrt{\frac{1}{3}} + \sqrt{\frac{49}{3}} = \sqrt{12}$, which is not true.

Putting $x = 4$ in the given equation, we get

$$\sqrt{4+5} + \sqrt{4+21} = \sqrt{6(4)+40}$$

or $\sqrt{9} + \sqrt{25} = \sqrt{64}$

i.e., $3 + 5 = 8$ *i.e.*, $8 = 8$, which is true.

Hence, the solution is $x = 4$. **Ans.**

Q. 33. $3\left(x^2 + \frac{1}{x^2}\right) - 16\left(x + \frac{1}{x}\right) + 26 = 0.$

Solution : We use,

$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$$

Therefore, the given equation can be written as

$$3\left(x + \frac{1}{x}\right)^2 - 6 - 16\left(x + \frac{1}{x}\right) + 26 = 0$$

or $3\left(x + \frac{1}{x}\right)^2 - 16\left(x + \frac{1}{x}\right) + 20 = 0$

Now, putting $y = x + \frac{1}{x}$, we get

$3y^2 - 16y + 20 = 0$

or $3y^2 - 10y - 6y + 20 = 0$

or $y(3y - 10) - 2(3y - 10) = 0$

or $(3y - 10)(y - 2) = 0$

i.e., $3y - 10 = 0$

$3y = 10$

$\therefore$ $y = \frac{10}{3}$

or $y - 2 = 0$

$\therefore$ $y = 2$

Therefore, $x + \frac{1}{x} = \frac{10}{3}$ ...(i)

and $x + \frac{1}{x} = 2$ ...(ii)

Now, from (i), we get

$\frac{x^2 + 1}{x} = \frac{10}{3}$

$3(x^2 + 1) = 10x$

or $3x^2 - 10x + 3 = 0$

or $3x^2 - 9x - x + 3 = 0$

or $3x(x - 3) - 1(x - 3) = 0$

or $(x - 3)(3x - 1) = 0$

i.e., $x - 3 = 0$

$\therefore$ $x = 3$

or $3x - 1 = 0$

$3x = 1$

$\therefore$ $x = \frac{1}{3}$

and, from (ii), we get

$x^2 + 1 = 2x$

or $x^2 - 2x + 1 = 0$

or $(x - 1)^2 = 0$

or $(x - 1)(x - 1) = 0$

i.e., $x = 1, 1$

Hence, the solutions are

$x = 1, 1, 3, \frac{1}{3}$. **Ans.**

Q. 34. $6\left(x^2+\frac{1}{x^2}\right)-35\left(x+\frac{1}{x}\right)+62=0.$

Solution : We use,

$$x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2\text{, we get}$$

$$\therefore 6\left[\left(x+\frac{1}{x}\right)^2-2\right]-35\left(x+\frac{1}{x}\right)+62=0$$

Now, put $x+\frac{1}{x}=y$, we get

$$6(y^2-2)-35y+62=0$$
$$\therefore \quad 6y^2-35y+50=0$$
$$\therefore \quad 6y^2-15y-20y+50=0$$
$$\text{or } 3y(2y-5)-10(2y-5)=0$$
$$\text{or } (2y-5)(3y-10)=0$$
$$2y-5=0 \text{ or } 3y-10=0$$
$$\therefore \quad y=\frac{5}{2} \text{ or } y=\frac{10}{3}$$

Therefore,

$$x+\frac{1}{x}=\frac{5}{2} \text{ and } x+\frac{1}{x}=\frac{10}{3}$$
$$\therefore \frac{x^2+1}{x}=\frac{5}{2} \text{ and } \frac{x^2+1}{x}=\frac{10}{3}$$
$$\therefore \quad 2x^2+2-5x=0$$
$$\text{and} \quad 3x^2+3=10x$$
$$\therefore \quad 2x^2-5x+2=0$$
$$\text{and} \quad 3x^2-10x+3=0$$
$$\therefore \quad 2x^2-4x-x+2=0$$
$$\text{and} \quad 3x^2-9x-x+3=0$$
$$\therefore \quad 2x(x-2)-1(x-2)=0$$
$$\text{and} \quad 3x(x-3)-1(x-3)=0$$
$$\therefore \quad (x-2)(2x-1)=0$$
$$\text{and} \quad (x-3)(3x-1)=0$$
$$\therefore \quad x=2 \text{ or } 2x-1=0$$
$$\text{and } (x-3)=0 \text{ or } 3x-1=0$$
$$\therefore \quad x=3 \text{ or } 3x=1$$
$$x=\frac{1}{3}$$

Hence the solutions are $2, \frac{1}{2}, 3$ and $\frac{1}{3}$.

Q. 35. $4\left(x^2+\frac{1}{x^2}\right)-8\left(x+\frac{1}{x}\right)+3=0.$

Solution : We use,

$$x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2.$$

Therefore, the given equation can be written as

$$4\left(x+\frac{1}{x}\right)^2-8-8\left(x+\frac{1}{x}\right)+3=0$$
$$\text{or } 4\left(x+\frac{1}{x}\right)^2-8\left(x+\frac{1}{x}\right)-5=0$$

Now, putting $x+\frac{1}{x}=y$, we get

$$4y^2-8y-5=0$$
$$\text{or} \quad 4y^2-10y+2y-5=0$$
$$\text{or} \quad 2y(2y-5)+1(2y-5)=0$$
$$\text{or} \quad (2y-5)(2y+1)=0$$
$$\textit{i.e.,} \quad 2y-5=0$$
$$\therefore \quad 2y=5$$
$$y=\frac{5}{2}$$
$$\text{or} \quad 2y+1=0$$
$$2y=-1$$
$$y=-\frac{1}{2}$$

Therefore, $x+\frac{1}{x}=\frac{5}{2}$ ...(i)

and $x+\frac{1}{x}=-\frac{1}{2}$ ...(ii)

Now, from (i), we get

$$\frac{x^2+1}{x}=\frac{5}{2}$$
$$2(x^2+1)=5x$$
$$\text{or} \quad 2x^2-5x+2=0$$
$$\text{or} \quad 2x^2-4x-x+2=0$$
$$\text{or} \quad 2x(x-2)-1(x-2)=0$$
$$\text{or} \quad (x-2)(2x-1)=0$$
$$\textit{i.e.,} \quad x-2=0$$
$$\therefore \quad x=2$$
$$\text{or} \quad 2x-1=0$$
$$\therefore \quad x=\frac{1}{2}$$

and from (ii), we get

$$\frac{x^2+1}{x} = \frac{-1}{2}$$

$$2(x^2+1) = -x$$

$$2x^2 + x + 2 = 0$$

Here, $a = 2, b = 1, c = 2$

$$x = \frac{-1 \pm \sqrt{1-4(2)(2)}}{2(2)}$$

$$= \frac{-1 \pm \sqrt{1-16}}{4}$$

$$= \frac{-1 \pm \sqrt{-15}}{4}$$

which are not real.

Hence, the solution are $x = 2, \frac{1}{2}$.

Ans.

Q. 36. $6\left(x^2+\frac{1}{x^2}\right) - 25\left(x+\frac{1}{x}\right) + 12 = 0.$

Solution : We use,

$$x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$$

Therefore, the given equation becomes

$$6\left(x-\frac{1}{x}\right)^2 + 12 - 25\left(x-\frac{1}{x}\right) + 12 = 0$$

$$\text{or } 6\left(x-\frac{1}{x}\right)^2 - 25\left(x-\frac{1}{x}\right) + 24 = 0$$

Now putting $x - \frac{1}{x} = y$, we get

$$6y^2 - 25y + 24 = 0$$

$$\therefore \quad 6y^2 - 16y - 9y + 24 = 0$$

$$\therefore \quad (3y-8)(2y-3) = 0$$

$$\therefore \quad 3y - 8 = 0 \text{ or } 2y - 3 = 0$$

$$y = \frac{8}{3} \text{ or } y = \frac{3}{2}$$

Therefore,

$$\text{or} \quad x - \frac{1}{x} = \frac{8}{3} \text{ or } x - \frac{1}{x} = \frac{3}{2}$$

$$\therefore \quad \frac{x^2-1}{x} = \frac{8}{3} \text{ or } \frac{x^2-1}{x} = \frac{3}{2}$$

$$\therefore \quad 3x^2 - 3 = 8x \text{ or } 2x^2 - 2 = 3x$$

$$\therefore \quad 3x^2 - 8x - 3 = 0$$

$$\text{or } 2x^2 - 3x - 2 = 0$$

$$\therefore \quad 3x^2 - 9x + x - 3 = 0$$

$$\text{or } 2x^2 - 4x + x - 2 = 0$$

$$\therefore \quad 3x(x-3) + 1(x-3) = 0$$

$$\text{or } 2x(x-2) + 1(x-2) = 0$$

$$\therefore \quad (x-3)(3x+1) = 0$$

$$\text{or } (x-2)(2x+1) = 0$$

$$\therefore \quad x - 3 = 0 \text{ or } 2x + 1 = 0$$

$$\text{or } x - 2 = 0 \text{ or } 2x + 1 = 0$$

$$x = 3, x = -1/3, x = 2, x = -1/2$$

Hence the solutions are $2, \frac{-1}{2}, 3$ and $\frac{-1}{3}$.

Ans.

Q. 37. $\left(x^2+\frac{1}{x^2}\right) - 3\left(x-\frac{1}{x}\right) - 2 = 0.$

Solution : We use,

$$x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$$

Therefore, the given equation can be written as

$$\left(x-\frac{1}{x}\right)^2 + 2 - 3\left(x-\frac{1}{x}\right) - 2 = 0$$

$$\text{or} \quad \left(x-\frac{1}{x}\right)^2 - 3\left(x-\frac{1}{x}\right) = 0$$

$$\text{or} \quad \left(x-\frac{1}{x}\right)\left[\left(x-\frac{1}{x}\right) - 3\right] = 0$$

$$\text{i.e.,} \quad x - \frac{1}{x} = 0$$

$$\Rightarrow \quad x^2 = 1 \;\therefore x = \pm 1$$

$$\text{and} \quad x - \frac{1}{x} - 3 = 0$$

$$\text{or} \quad \frac{x^2-1}{x} - 3 = 0$$

$$x^2 - 1 - 3x = 0$$

$$\text{or} \quad x^2 - 3x - 1 = 0$$

Here, $a = 1$

$$b = -3$$
$$c = -1$$

or $x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$

$$= \frac{3 \pm \sqrt{9+4}}{2} = \frac{3 \pm \sqrt{13}}{2}$$

Hence, the solutions are

$x = 1, -1, \dfrac{3+\sqrt{13}}{2}, \dfrac{3-\sqrt{13}}{2}$. **Ans.**

Q. 38. $4\left(x^2 + \dfrac{1}{x^2}\right) + 8\left(x - \dfrac{1}{x}\right) - 29 = 0.$

Solution : We use

$$x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$$

Therefore, the given equation can be written as

$$4\left(x - \frac{1}{x}\right)^2 + 8 + 8\left(x - \frac{1}{x}\right) - 29 = 0$$

or $4\left(x - \dfrac{1}{x}\right)^2 + 8\left(x - \dfrac{1}{x}\right) - 21 = 0.$

Now, putting $x - \dfrac{1}{x} = y$, we get

$$4y^2 + 8y - 21 = 0$$

or $4y^2 + 14y - 6y - 21 = 0$

or $2y(2y + 7) - 3(2y + 7) = 0$

or $(2y + 7)(2y - 3) = 0$

i.e., $(2y + 7) = 0$

$\therefore$ $2y = -7$

$y = -\dfrac{7}{2}$

or $2y - 3 = 0$

$\therefore$ $y = \dfrac{3}{2}$.

Therefore, $x - \dfrac{1}{x} = -\dfrac{7}{2}$ ...(i)

and $x - \dfrac{1}{x} = \dfrac{3}{2}$ ...(ii)

From (i), we get

$$\frac{x^2 - 1}{x} = \frac{-7}{2}$$

$$2(x^2 - 1) = -7x$$

or $2x^2 + 7x - 2 = 0$

Here, $a = 2$, $b = 7$, $c = -2$

or $x = \dfrac{-7 \pm \sqrt{(7)^2 - 4(2)(-2)}}{2(2)}$

$$= \frac{-7 \pm \sqrt{49 + 16}}{4}$$

$$= \frac{-7 \pm \sqrt{65}}{4}$$

and from (ii), we get

$$\frac{x^2 - 1}{x} = \frac{3}{2}$$

$$2(x^2 - 1) = 3x$$

or $2x^2 - 3x - 2 = 0$

or $2x^2 - 4x + x - 2 = 0$

or $2x(x - 2) + 1(x - 2) = 0$

or $(x - 2)(2x + 1) = 0$

i.e., $x - 2 = 0$

$\therefore$ $x = 2$

or $2x + 1 = 0$

$\therefore$ $x = -\dfrac{1}{2}$

Hence, the solutions are

$x = 2, -\dfrac{1}{2}, \dfrac{-7 \pm \sqrt{65}}{4}$. **Ans.**

Q. 39. $(x + 1)(x + 2)(x + 3)(x + 4) = 120.$

Solution : Given,

$(x + 1)(x + 2)(x + 3)(x + 4) = 120$

or $[(x + 1)(x + 4)]$

$[(x + 2)(x + 3)] = 120,$

[Note 1 + 4 = 2 + 3]

or $(x^2 + 5x + 4)$

$(x^2 + 5x + 6) - 120 = 0$

Putting $x^2 + 5x = y$, we get

$$(y + 4)(y + 6) - 120 = 0$$

or $y^2 + 6y + 4y + 24 - 120 = 0$

or $y^2 + 10y + 24 - 120 = 0$

or $y^2 + 10y - 96 = 0$

or $y^2 + 16y - 6y - 96 = 0$

or $y(y + 16) - 6(y + 16) = 0$

or $(y + 16)(y - 6) = 0$

i.e., $y + 16 = 0$

$\therefore \quad y = -16$

or $\quad y - 6 = 0$

$\therefore \quad y = 6$

Now, when $y = -16$, then

$x^2 + 5x = -16$

or $\quad x^2 + 5x + 16 = 0$

Here, $a = 1$, $b = 5$, $c = 16$

$$\therefore \; x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(16)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{25-64}}{2} = \frac{-5 \pm \sqrt{-39}}{2}$$

which is not real.

Now, when $y = 6$, then $x^2 + 5x = 6$

or $\quad x^2 + 5x - 6 = 0$

or $\quad x^2 + 6x - x - 6 = 0$

or $\quad x(x + 6) - 1(x + 6) = 0$

or $\quad (x + 6)(x - 1) = 0$

i.e., $\quad x + 6 = 0$

$\therefore \quad x = -6$

or $\quad x - 1 = 0$

$\therefore \quad x = 1$

Hence, the required solutions are

$x = -6, 1.$ **Ans.**

Q. 40. $(x + 2)(x - 5)(x - 6)(x + 1) = 144.$

Solution : Given,

$(x + 2)(x - 5)(x - 6)(x + 1) = 144$

or $[(x + 2)(x - 6)]\,[(x - 5)(x + 1)] = 144.$

[Note $2 - 6 = -5 + 1$]

or $(x^2 - 4x - 12)(x^2 - 4x - 5) - 144 = 0$

Putting $x^2 - 4x = y$, we get

$(y - 12)(y - 5) - 144 = 0$

or $y^2 - 5y - 12y + 60 - 144 = 0$

or $\quad y^2 - 17y + 60 - 144 = 0$

or $\quad y^2 - 17y - 84 = 0$

or $\quad y^2 - 21y + 4y - 84 = 0$

or $\quad y(y - 21) + 4(y - 21) = 0$

or $\quad (y - 21)(y + 4) = 0$

i.e., $\quad y - 21 = 0$

$\therefore \quad y = 21$

or $\quad y + 4 = 0$

$\therefore \quad y = -4$

Now, when $y = 21$, then

$x^2 - 4x = 21$

or $\quad x^2 - 4x - 21 = 0$

or $\quad x^2 - 7x + 3x - 21 = 0$

or $\quad x(x - 7) + 3(x - 7) = 0$

or $\quad (x - 7)(x + 3) = 0$

i.e., $\quad x - 7 = 0$

$\therefore \quad x = 7$

or $\quad x + 3 = 0$

$\therefore \quad x = -3$

and, when $y = -4$, then

$x^2 - 4x = -4$

or $\quad x^2 - 4x + 4 = 0$

or $\quad x^2 - 2x - 2x + 4 = 0$

or $\quad x(x - 2) - 2(x - 2) = 0$

or $\quad (x - 2)(x - 2) = 0$

i.e., $\quad x = 2, 2$

Hence, the required solutions are

$x = 7, -3, 2, 2.$ **Ans.**

Q. 41. $(x - 2)(x + 3)(x - 3)(x + 4) = 40.$

Solution : Here

$[(x - 2)(x + 3)]\,[(x - 3)(x + 4)] = 40$

$\Rightarrow [x^2 + 3x - 2x - 6]\,[x^2 + 4x - 3x - 12] = 40$

$\Rightarrow [x^2 + x - 6]\,[x^2 + x - 12] = 40$

Suppose $x^2 + x = y$ ...(i)

$\Rightarrow \quad (y - 6)(y - 12) = 40$

$\Rightarrow \quad y^2 - 12y - 6y + 72 - 40 = 0$

$\Rightarrow \quad y^2 - 18y + 32 = 0$

$\Rightarrow \quad y^2 - 16y - 2y + 32 = 0$

$\Rightarrow \quad y(y - 16) - 2(y - 16) = 0$

$\Rightarrow \quad (y - 16)(y - 2) = 0$

$\Rightarrow \quad y - 16 = 0$ or $y - 2 = 0$

$\Rightarrow \quad y = 16$ or $\quad y = 2$

Put the value of y in (i),

$x^2 + x = 16$

$\Rightarrow \quad x^2 + x - 16 = 0$

Here $a = 1$, $b = 1$ and $c = -16$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4 \times 1(-16)}}{2 \times 1}$$

$$= \frac{-1 \pm \sqrt{1+64}}{2}$$

$$= \frac{-1 \pm \sqrt{65}}{2}$$

or $x^2 + x = 2$

or $x^2 + x - 2 = 0$

$x^2 + 2x - x - 2 = 0$

$x(x + 2) - 1(x + 2) = 0$

$(x + 2)(x - 1) = 0$

$x + 2 = 0$ or $x - 1 = 0$

$x = -2$ or $x = 1$

Hence, the required solutions are

$$-2, 1 \text{ and } \frac{-1 \pm \sqrt{65}}{2}. \quad \textbf{Ans.}$$

Q. 42. $x(x + 5)(x + 7)(x + 12) = -150.$

Solution : The given equation is

$[x(x + 12)]\,[(x + 5)(x + 7)] = -150,$

[Note, 0 + 12 = 5 + 7]

or $(x^2 + 12x)(x^2 + 12x + 35) + 150 = 0$

Putting $x^2 + 12x = y$, we get

$y(y + 35) + 150 = 0$

or $y^2 + 35y + 150 = 0$

$\therefore$ Here, $a = 1$, $b = 35$, $c = 150$

$$y = \frac{-35 \pm \sqrt{(35)^2 - 4(1)(150)}}{2(1)}$$

$$= \frac{-35 \pm \sqrt{1225 - 600}}{2}$$

$$= \frac{-35 \pm \sqrt{625}}{2} = \frac{-35 \pm 25}{2}$$

$$= \frac{-35 + 25}{2}, \frac{-35 - 25}{2}$$

$$= \frac{-10}{2}, \frac{-60}{2} = -5, -30.$$

Now, when $y = -5$, then

$x^2 + 12x = -5$

or $x^2 + 12x + 5 = 0$

Here, $a = 1$, $b = 12$, $c = 5$

$$\therefore \quad x = \frac{-12 \pm \sqrt{(12)^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{-12 \pm \sqrt{144 - 20}}{2}$$

$$= \frac{-12 \pm \sqrt{124}}{2}$$

i.e., $x = -6 \pm \sqrt{31}$

and when $y = -30$, then

$x^2 + 12x = -30$

or $x^2 + 12x + 30 = 0$

Here, $a = 1$, $b = 12$, $c = 30$

$$\therefore \quad x = \frac{-12 \pm \sqrt{(12)^2 - 4(1)(30)}}{2(1)}$$

$$= \frac{-12 \pm \sqrt{144 - 120}}{2}$$

$$= \frac{-12 \pm \sqrt{24}}{2}$$

i.e., $x = -6 \pm \sqrt{6}.$

Hence, the required solutions are

$$x = -6 \pm \sqrt{31}, -6 \pm \sqrt{6}. \quad \textbf{Ans.}$$

Exercies 4.7

Multiple Choice Type Questions

Q. 1. *The sum of a number and its reciprocal is* $\frac{10}{3}$, *then number is* :

(a) 3 (b) – 3

(c) 10 (d) None of these.

Solution : Let the number be x, then according to the given information we get,

$$x + \frac{1}{x} = \frac{10}{3}$$

$$\Rightarrow \quad \frac{x^2 + 1}{x} = \frac{10}{3}$$

$$\Rightarrow \quad 3(x^2 + 1) = 10x$$

$$\Rightarrow \quad 3x^2 - 10x + 3 = 0$$

$$\Rightarrow \quad 3x^2 - 9x - x + 3 = 0$$

$$\Rightarrow \quad 3x(x - 3) - 1(x - 3) = 0$$

$$\Rightarrow \quad (x - 3)(3x - 1) = 0$$

$$\Rightarrow \quad x - 3 = 0 \text{ or } 3x - 1 = 0$$

$$\therefore \quad x = 3 \text{ or } x = \frac{1}{3}$$

Hence, the numbers are $3, \frac{1}{3}$. **Ans.**

Q. 2. *The sum of two numbers is* 17 *and their product is* 72, *then the numbers are* :

(a) 9, 7 (b) 8, 9

(c) 12, 6 (d) 10, 7.

Solution : Let the two numbers be x and y.

According to question, we get

$$x + y = 17 \quad ...(i)$$

and $$xy = 72 \quad ...(ii)$$

From equation (i),

$$x = 17 - y \quad ...(iii)$$

Putting the value of x in equation (ii), we get

$$(17 - y)\,y = 72$$

$\Rightarrow$ $17y - y^2 = 72$

$\Rightarrow$ $y^2 - 17y + 72 = 0$

$\Rightarrow$ $y^2 - 9y - 8y + 72 = 0$

$\Rightarrow$ $y(y - 9) - 8(y - 9) = 0$

$\Rightarrow$ $(y - 9)(y - 8) = 0$

$\therefore$ $y - 8 = 0$ or $y - 9 = 0$

$\therefore$ $y = 8$ or $y = 9$

Now, from equation (iii),

when $y = 8, x = 17 - 8 = 9$

and when $y = 9, x = 17 - 9 = 8$

Hence, the numbers are 9, 8.

Q. 3. *The difference of two numbers is* 5 *and their product is* 84, *then the numbers are* :

(a) 9, 4 (b) 13, 8

(c) 12, 7 (d) None of these.

Solution : Let the two numbers be x and y.

According to question, we get

$$x - y = 5 \quad ...(i)$$

and $$xy = 84 \quad ...(ii)$$

From equation (i),

$$x = y + 5 \quad ...(iii)$$

Putting the value of x in equation (ii), we get

$$(y + 5)y = 84$$

$\Rightarrow$ $y^2 + 5y - 84 = 0$

$\Rightarrow$ $y^2 + 12y - 7y - 84 = 0$

$\Rightarrow$ $y(y + 12) - 7(y + 12) = 0$

$\Rightarrow$ $(y + 12)(y - 7) = 0$

$\Rightarrow$ $y + 12 = 0$ or $y - 7 = 0$

$\therefore$ $y = -12$ $\therefore$ $y = 7$

Now, from equation (iii),

when $y = 7, x = y + 5$

$$x = 7 + 5 = 12$$

Hence, the numbers are 7, 12.

Q. 4. *The sum of a number and its square is* 240. *The number is* :

(a) 12 (b) 14

(c) 15 (d) 16.

Solution : Let the number be x.

Accoding to question, we get

$$x + x^2 = 240$$

$\Rightarrow$ $x^2 + x - 240 = 0$

$\Rightarrow$ $x^2 + 16x - 15x - 240 = 0$

$\Rightarrow$ $x(x + 16) - 15(x + 16) = 0$

$\Rightarrow$ $(x + 16)(x - 15) = 0$

$\Rightarrow$ $x + 16 = 0$ or $x - 15 = 0$

$\therefore$ $x = -16$ or $x = 15$

Hence, the number is 15.

Q. 5. *If a number is subtracted from its square, the remainder is* 20. *The number is* :

(a) 4 (b) 5

(c) 6 (d) 8.

Solution : Let the number be x.

According to question, we get

$$x^2 - x = 20$$

$\Rightarrow$ $x^2 - x - 20 = 0$

$\Rightarrow$ $x^2 - 5x + 4x - 20 = 0$

$\Rightarrow$ $x(x - 5) + 4(x - 5) = 0$

$\Rightarrow$ $(x - 5)(x + 4) = 0$

$\Rightarrow$ $x - 5 = 0$ or $x + 4 = 0$

$$x = 5 \text{ or } x = -4$$

Hence, the number is 5. **Ans.**

[Ans. : 1. (a), **2.** (b), **3.** (c), **4.** (c), **5.** (b).]

Short Answer Type Questions

Q. 6. *The sum of a number and its reciprocal is* $\frac{17}{4}\cdot$ *Find the number.*

Solution : Let the number be x, then according to the given information, we get

$$x + \frac{1}{x} = \frac{17}{4}$$

or $4(x^2 + 1) = 17x$

or $4x^2 - 17x + 4 = 0$

or $4x^2 - 16x - x + 4 = 0$

or $4x(x - 4) - 1(x - 4) = 0$

or $(x - 4)(4x - 1) = 0$

i.e., $x = 4$ or $x = \frac{1}{4}$

Hence, the number is 4 or $\frac{1}{4}$. **Ans.**

Q. 7. *The sum of two numbers is* 8 *and the difference of their squares is* 16. *Find the numbers.*

Solution : Let the two numbers be x and y.

According to question, we get

$x + y = 8$ ...(i)

and $x^2 - y^2 = 16$ ...(ii)

$\Rightarrow (x - y)(x + y) = 16$

$\Rightarrow (x - y) \times 8 = 16$

$\therefore x - y = 2$ ...(iii)

Solve the equation (i) and (iii), we get

$x + y = 8$ ...(i)

$x - y = 2$ ...(iii)

$2x = 10$

$x = 5$

Putting the value x in equation (i)

$5 + y = 8$

$y = 8 - 5 = 3$

Hence, the numbers are $x = 5, y = 3$. **Ans.**

Q. 8. *Find two consecutive natural numbers whose squares have the sum* 221.

Solution : Let the two consecutive natural numbers be x and $x + 1$.

Then according to the given informations, we have

$x^2 + (x + 1)^2 = 221$

or $x^2 + x^2 + 2x + 1 - 221 = 0$

or $2x^2 + 2x - 220 = 0$

or $x^2 + x - 110 = 0$

or $x^2 + 11x - 10x - 110 = 0$

or $x(x + 11) - 10(x + 11) = 0$

or $(x + 11)(x - 10) = 0$

$\Rightarrow x + 11 = 0$

$\therefore x = -11$

or $x - 10 = 0$

$\therefore x = 10$

Since, x must be a natural number, it cannot be negative, so we reject the solution $x = -11$.

Here, $x = 10$

$\therefore$ Other number $= x + 1 = 10 + 1 = 11$

Hence, the required numbers are 10 and 11. **Ans.**

Q. 9. *There are three consecutive positive integers such that the sum of squares of first integer and product of second and third is* 191. *Find those integers.* **(U. P. 2014)**

Solution : Suppose three positive consecutive positive integers are x, $(x + 1)$ and $(x + 2)$ respectively.

According to question,

$x^2 + (x + 1)(x + 2) = 191$

$\Rightarrow x^2 + x^2 + 3x + 2 = 191$

$\Rightarrow 2x^2 + 3x - 189 = 0$

$\Rightarrow 2x^2 + 21x - 18x - 189 = 0$

$\Rightarrow x(2x + 21) - 9(2x + 21) = 0$

$\Rightarrow (2x + 21)(x - 9) = 0$

$\Rightarrow 2x + 21 = 0$ or $x - 9 = 0$

$\Rightarrow 2x = -21$ or $x = 9$.

$x = \frac{-21}{2}$

$x = \frac{-21}{2}$ is not possible.

Hence, the three positive consecutive integers are $9, 9 + 1, 9 + 2$

$= 9, 10, 11.$ **Ans.**

Q. 10. *Find two such consecutive positive even numbers, the sum of whose squares is* 452.

Solution : Suppose two consecutive positive even numbers are x and $(x + 2)$.

According to question,

$x^2 + (x + 2)^2 = 452$

$\Rightarrow x^2 + x^2 + 4x + 4 = 452$

$\Rightarrow 2x^2 + 4x - 448 = 0$

$\Rightarrow x^2 + 2x - 224 = 0$

$\Rightarrow x^2 + 16x - 14x - 224 = 0$

$\Rightarrow x(x + 16) - 14(x + 16) = 0$

$\Rightarrow (x + 16)(x - 14) = 0$

$\Rightarrow x + 16 = 0$ or $x - 14 = 0$

$\Rightarrow x = -16$ or $x = 14$

$x = -16$ is not positive

i.e., $x = 14$ is valid.

Hence, the two positive consecutive even integers are 14 and 14 + 2.

$\Rightarrow$ 14 and 16. **Ans.**

Q. 11. *Divide* 16 *into two parts such that twice the square of the larger part exceeds the square of the smaller part by* 164.

Solution : Let the smaller part be x

Then, the larger part = $16 - x$

According to the given informations, we have

$$2(16 - x)^2 - x^2 = 164$$

or $2(256 - 32x + x^2) - x^2 - 164 = 0$

or $512 - 64x + 2x^2 - x^2 - 164 = 0$

or $x^2 - 64x + 348 = 0$

or $x^2 - 6x - 58x + 348 = 0$

or $x(x - 6) - 58(x - 6) = 0$

or $(x - 6)(x - 58) = 0$

$\Rightarrow x - 6 = 0$

$\therefore x = 6$

or $x - 58 = 0$

$\therefore x = 58$

But $x = 58$ is not acceptable, because the sum of the parts is 16.

Therefore, $x = 6$

i.e., saller part = $x = 6$

and larger part = $16 - x = 16 - 6 = 10$

Hence, the smaller part = 6 and larger part = 10. **Ans.**

Q. 12. *A two digit number is such that the product of the digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the nummber.*

Solution : Given, product of digits = 18

Let ten's digit be x. Then,

unit's digit = $\dfrac{18}{x}$

$\therefore$ Number = $10x + \dfrac{18}{x}$

When, we interchange the digits, the number becomes

$$= 10 \times \frac{18}{x} + x$$

$$= \frac{180}{x} + x$$

Now, according to the given information, we have

$$\left(10x + \frac{18}{x}\right) - 63 = \frac{180}{x} + x$$

or $10x^2 + 18 - 63x = 180 + x^2$

or $10x^2 + 18 - 63x - 180 - x^2 = 0$

or $9x^2 - 63x - 162 = 0$

or $x^2 - 7x - 18 = 0$

or $x^2 - 9x + 2x - 18 = 0$

or $x(x - 9) + 2(x - 9) = 0$

or $(x - 9)(x + 2) = 0$

i.e., $x - 9 = 0$

$\therefore x = 9$

or $x + 2 = 0$

$\therefore x = -2$

Here, $x = -2$ is not acceptable, since -2 is not a digit.

Therefore, $x = 9$

$\therefore$ Number = $10(9) + \dfrac{18}{9}$

$= 90 + 2 = 92$

Hence, the number is 92. **Ans.**

Q. 13. *The sum of the squares of two numbers is* 130. *The sum of the smaller number and twice the larger number is* 25. *Determine the number.*

Solution : Let the smaller of two numbers be x.

According to the question, we have

$x^2 + (\text{larger number})^2 = 130$

i.e., larger number = $\sqrt{130 - x^2}$

Again, according to the question, we have

$$x + 2\left(\sqrt{130 - x^2}\right) = 25$$

or $2\sqrt{130 - x^2} = 25 - x$

On squaring both sides, we get

$$4(130 - x^2) = 625 - 50x + x^2$$

or $520 - 4x^2 - 625 + 50x - x^2 = 0$

or $-5x^2 + 50x - 105 = 0$

or $x^2 - 10x + 21 = 0$

or $x^2 - 7x - 3x + 21 = 0$

or $x(x - 7) - 3(x - 7) = 0$

or $(x - 7)(x - 3) = 0$

i.e., $x = 7$ or $x = 3$

When $x = 7$, *i.e.*, smaller number = 7.

and larger number = $\sqrt{130 - (7)^2}$

$= \sqrt{130 - 49}$

$= \sqrt{81} = 9$

Check : $7 + 2(9) = 7 + 18 = 25$, which is true.

When $x = 3$, *i.e.*, smaller number = 3.

and larger number

$= \sqrt{130 - (3)^2} = \sqrt{130 - 9}$

$= \sqrt{121} = 11$

Check :

$3 + 2(11) = 3 + 22$

$= 25$, which is also true.

Hence, the numbers are 7, 9 or 3, 11.

Ans.

Q. 14. *The sum of two numbers is* 8, *and* 15 *times the sum of their reciprocals is also* 8. *Find the numbers.*

Solution : Let the one number be x.

Given, sum of two numbers = 8

i.e., x + other number = 8

i.e., other number = $8 - x$.

Now, according to the given information, we have

$$15\left(\frac{1}{x} + \frac{1}{8 - x}\right) = 8$$

or $$15\left(\frac{8 - x + x}{x(8 - x)}\right) = 8$$

or $15(8) = 8x(8 - x)$

or $120 - 64x + 8x^2 = 0$

or $x^2 - 8x + 15 = 0$

or $x^2 - 5x - 3x + 15 = 0$

or $x(x - 5) - 3(x - 5) = 0$

or $(x - 5)(x - 3) = 0$

$\Rightarrow$ $x - 5 = 0$

$\therefore$ $x = 5$

or $x - 3 = 0$

$\therefore$ $x = 3$

When $x = 5$,

the other number = $8 - x = 8 - 5 = 3$

and when $x = 3$,

the other number = $8 - x = 8 - 3 = 5$.

Hence, the numbers are 5 and 3.

Ans.

Q. 15. *Find two natural numbers, the sum of whose squares is* 25 *times their sum and also equal to* 50 *times their difference.*

Solution : Let the two natural numbers be x and y.

The according to the given informations, we have

$x^2 + y^2 = 25(x + y)$ ...(i)

and $x^2 + y^2 = 50(x - y)$ ...(ii)

From (i) and (ii), we have

$$25(x + y) = 50(x - y)$$

or $x + y = 2(x - y)$

or $x - 2x = -2y - y$

or $-x = -3y$

i.e., $x = 3y$

Now, putting $x = 3y$ in equation (i), we get

$$(3y)^2 + y^2 = 25(3y + y)$$

or $9y^2 + y^2 = 25(4y)$

or $9y^2 + y^2 - 100y = 0$

or $10y^2 - 100y = 0$

or $10y(y - 10) = 0$

i.e., $y = 0$

or $y = 10$.

Here, $y = 0$ is not possible therefore $y = 10$.

$\therefore$ $x = 3y = 3(10) = 30$

Hence, the numbers are 10 and 30.

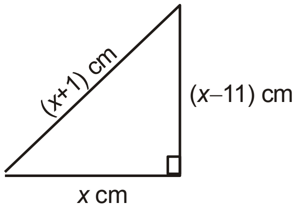
Ans.

Q. 16. *The sides (in cm) of a right angled triangle are* $x - 1$, x *and* $x + 1$. *Find the sides of the triangle.*

Solution : Given, three sides of a right angled triangle are

$$x - 1, x \text{ and } x + 1.$$

The longest side (hypotenuse) is $(x + 1)$ cm.



Now, by Pythagoras theorem, we have

$$(x + 1)^2 = (x - 1)^2 + x^2$$

or $x^2 + 2x + 1 = x^2 - 2x + 1 + x^2$

or $x^2 + 2x + 1 - 2x^2 + 2x - 1 = 0$

or $-x^2 + 4x = 0$

or $-x(x - 4) = 0$

i.e., $x = 0$

or $x = 4$

But $x \neq 0$, since length of a side cannot be zero.

$\therefore$ $x = 4$

Now, $x - 1 = 4 - 1 = 3$

and $x + 1 = 4 + 1 = 5$

Hence, the three sides of the triangle are 3 cm, 4 cm and 5 cm. **Ans.**

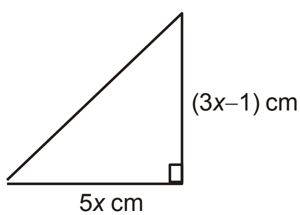
Q. 17. *The sides (in cm) of a right angled triangle containing the right angle are* $5x$ *and* $3x - 1$*. If the area of the triangle is* 60 *cm*^{2}*, find the sides of the triangle.*

Solution : Given that the sides containing right angled triangle are $5x$ and $3x - 1$ cms.

$\therefore$ Area of right angle triangle

$= \frac{1}{2} \times$ product of sides containing the right angled triangle

$= \frac{1}{2} \times 5x \times (3x - 1)$



Also, given that the area of right angled triangle is 60 cm^2.

$\therefore$ $\frac{1}{2} \times 5x \times (3x - 1) = 60$

or $5x(3x - 1) = 60 \times 2$

or $15x^2 - 5x - 120 = 0$

or $3x^2 - x - 24 = 0$

or $3x^2 - 9x + 8x - 24 = 0$

or $3x(x - 3) + 8(x - 3) = 0$

or $(x - 3)(3x + 8) = 0$

$\Rightarrow$ $x - 3 = 0$

$\therefore$ $x = 3$

or $3x + 8 = 0$

$3x = -8$

$\therefore$ $x = \frac{-8}{3}$

But $x \neq \frac{-8}{3}$, since length of side cannot be negative.

$\therefore$ $x = 3$

When $x = 3$, then $5x = 5 \times 3 = 15$

and when $x = 3$, then

$$3x - 1 = 3 \times 3 - 1 = 9 - 1 = 8$$

$\therefore$ Hypotenuse $= \sqrt{(8)^2 + (15)^2}$

$= \sqrt{64 + 225}$

$= \sqrt{289} = 17.$

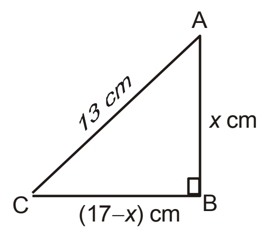
Hence, the three sides of a right angled triangle are 8 cm, 15 cm and 17 cm. **Ans.**

Q. 18. *The perimeter of a right angled triangle is* 30 *cm and its hypotenuse is* 13 *cm. Find the other two sides of the triangle.*

Solution : Given, perimeter of right angled triangle = 30 cm

and hypotenuse = 13 cm

$\therefore$ The sum of the two sides of right angled triangle = 30 – 13 = 17 cm



Let one of the sides of right angled triangle be x cm.

Then, the other side of right angled triangle

$$= (17 - x) \text{ cm}$$

By Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$(13)^2 = x^2 + (17 - x)^2$$

or $169 = x^2 + 289 - 34x + x^2$

or $2x^2 - 34x + 120 = 0$

or $x^2 - 17x + 60 = 0$

or $x^2 - 12x - 5x + 60 = 0$

or $x(x - 12) - 5(x - 12) = 0$

or $(x - 12)(x - 5) = 0$

i.e., $x = 12$ or $x = 5$

When $x = 12$, the other side

$$= 17 - 12 = 5$$

and when $x = 5$, the other side

$$= 17 - 5 = 12$$

Hence, the two sides of the right angled triangle are 5 cm and 12 cm.

Ans.

Q. 19. *The area of a right angled triangle is 63 sq cm. If the base of the triangle exceeds that of its altitude by 5 cm, find the altitude of the triangle.*

Solution : Let the altitude of the triangle be x cm.

Then, the base of the triangle

$$= (x + 5) \text{ cm}$$

New, area of the triangle

$$= \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$63 = \frac{1}{2} \times x \times (x + 5)$$

[$\because$ given, area = 63 sq cm]

or $126 = x^2 + 5x$

or $x^2 + 5x - 126 = 0$

or $x^2 + 14x - 9x - 126 = 0$

or $x(x + 14) - 9(x + 14) = 0$

or $(x + 14)(x - 9) = 0$

$\Rightarrow$ $x + 14 = 0$

$\therefore$ $x = -14$

or $x - 9 = 0$

$\therefore$ $x = 9$

But $x \neq -14$, since altitude of a triangle cannot be negative.

Therefore, $x = 9$

Hence, the altitude of the triangle

$= 9$ cm. **Ans.**

Q. 20. *The length of rectangular field is three times its breadth. If the area of the field be 147 sq. metres, find the length of the field.*

Solution : Let the breadth of rectangular field be x metre.

Then the length of rectangular field

$= 3x$ metre.

$\therefore$ Area of rectangular field

= Length × Breadth

i.e., $147 = 3x \times x$

or $3x^2 = 147$

or $x^2 = 49$

i.e., $x = 7$ metre.

Hence, the length of rectangular field

$= 3 \times 7 = 21$ metre. **Ans.**

Q. 21. *The length of a hall is 3 metre more than its breadth. If the area of the hall is 238 square metres, calculate its length and breadth.*

Solution : Let the breadth of the hall be x metre.

Then according to the question, its length

$= (x + 3)$ metre

Given, area of the hall

= 238 square metre

Now, area of the hall

= length × breadth

$\therefore$ $238 = x(x + 3)$

or $x^2 + 3x - 238 = 0$

or $x^2 + 17x - 14x - 238 = 0$

or $x(x + 17) - 14(x + 17) = 0$

or $(x + 17)(x - 14) = 0$

$\Rightarrow$ $x - 14 = 0$

$\therefore$ $x = 14$

or $x + 17 = 0$

$\therefore$ $x = -17$

But $x \neq -17$, since the breadth of the hall cannot be negative.

Therefore, $x = 14$ metre, *i.e.*,

breadth = 14 metre

$\therefore$ Length $= x + 3 = 14 + 3$

$= 17$ metre

Hence, the length of a hall is 17 metre and its breadth is 14 metre. **Ans.**

Q. 22. *The length of the hypotenuse of a right angled triangle exceeds the length of the base by* 2 *cm and exceeds twice the length of the altitude by* 1 *cm. Find the length of each side of the triangle.*

Solution : Let the base of the right angled triangle be x cm.

Then, hypotenuse of the right angled triangle $= (x + 2)$ cm.

Also, according to question, we have

$(x + 2) - (2 \times \text{altitude}) = 1$

or $\quad 2 \times \text{altitude} = x + 2 - 1$

or $\quad 2\ \text{altitude} = x + 1$

or $\quad \text{altitude} = \frac{1}{2}(x+1)$ cm

Now, by Pythagoras theorem, we get

$$(x + 2)^2 = x^2 + \frac{1}{4}(x + 1)^2$$

or $x^2 + 4 + 4x = \dfrac{4x^2 + x^2 + 1 + 2x}{4}$

or $4(x^2 + 4x + 4) = 4x^2 + x^2 + 1 + 2x$

or $4x^2 + 16x + 16 - 4x^2 - x^2 - 2x - 1 = 0$

or $\quad -x^2 + 14x + 15 = 0$

or $\quad x^2 - 14x - 15 = 0$

or $\quad x^2 - 15x + x - 15 = 0$

or $\quad x(x - 15) + 1(x - 15) = 0$

or $\quad (x - 15)(x + 1) = 0$

i.e., $\quad x = 15$ or $x = -1$

But $x = -1$ is not possible.

Therefore, $x = 15$ cm, *i.e.*, base $= 15$ cm

Now, hypotenuse $= (15 + 2) = 17$ cm

and altitude $= \frac{1}{2}(15 + 1) = 8$ cm.

Hence, the sides of the triangle are 8 cm, 15 cm and 17 cm. **Ans.**

Q. 23. *A farmer prepares a rectangular garden of area* 180 *sq. metres. With* 39 *metres of barbed wire, he can fence the three sides of the garden, leaving one of the longer sides unfenced. Find the dimensions of the garden.*

Solution : Let the length of rectangular garden be x metres.

Given, area of the rectangular garden

$= 180$ sq. metres

i.e., length × breadth = 180

or $\quad x \times \text{breadth} = 180$

i.e., $\quad \text{Breadth} = \dfrac{180}{x}$ metre.

According to the given condition, we have

$$x + \frac{180}{x} + \frac{180}{x} = 39$$

or $\quad x^2 + 180 + 180 = 39x$

or $\quad x^2 - 39x + 360 = 0$

or $\quad x^2 - 24x - 15x + 360 = 0$

or $\quad x(x - 24) - 15(x - 24) = 0$

or $\quad (x - 24)(x - 15) = 0$

$\Rightarrow \quad x - 24 = 0 \therefore x = 24$

or $\quad x - 15 = 0 \therefore x = 15.$

When $x = 24$, then breadth

$= \dfrac{180}{24} = 7.5$ metre

and when $x = 15$, then breadth

$= \dfrac{180}{15} = 12$ metre.

Hence, the dimensions of the garden are length = 24 metre, breadth = 7.5 metre or length = 15 metre, breadth = 12 metre. **Ans.**

Q. 24. *The product of Manoj's age (in years) five years ago with his age (in years)* 9 *years later is* 15. *Find Manoj's present age.*

Solution : Let Manoj's present age

$= x$ years

His age 5 years ago $= (x - 5)$ years

His age 9 years later $= (x + 9)$ years

Now, from the given condition, we have

$(x - 5)(x + 9) = 15$

or $\quad x^2 + 4x - 45 - 15 = 0$

or $\quad x^2 + 4x - 60 = 0$

or $\quad x^2 + 10x - 6x - 60 = 0$

or $x(x + 10) - 6(x + 10) = 0$

or $(x + 10)(x - 6) = 0$

$\Rightarrow$ $x + 10 = 0$

$\therefore$ $x = -10$

or $x - 6 = 0$

$\therefore$ $x = 6$

Since, x is the present age of Manoj, it cannot be negative.

Therefore, $x \neq -10$

Thus, Manoj's present age is 6 years.

Ans.

Q. 25. *If a boy's age and his father's age together is 24 years, and one-fourth part of the product of their ages, exceeds the boy's age by 9 years, find how old they are ?*

Solution : Let the boy's age be x years and father's age be y years.

According to the given condition, we have

$$x + y = 24 \quad \text{...(i)}$$

and $$\frac{xy}{4} - 9 = x \quad \text{...(ii)}$$

From equation (i), we get

$$x = 24 - y$$

Putting this value of x in equation (ii), we get

$$\frac{(24 - y)\,y}{4} - 9 = 24 - y$$

or $24y - y^2 - 36 = 96 - 4y$

or $24y + 4y - y^2 - 36 - 96 = 0$

or $-y^2 + 28y - 132 = 0$

or $y^2 - 28y + 132 = 0$

or $y^2 - 22y - 6y + 132 = 0$

or $y(y - 22) - 6(y - 22) = 0$

or $(y - 22)(y - 6) = 0$

i.e., $y = 22$ or $y = 6$.

But $y = 6$ is not acceptable, because father's age is more than boy's age.

Therefore, $y = 22$ years *i.e.*, father's age = 22 years.

Now, from (i), we get

$x + 22 = 24$ *i.e.*, $x = 2$ years.

Hence, boy's age is 2 years and father's age is 22 years. **Ans.**

Q. 26. *Sunny is m years old while his mother Mrs. Madhu is m^2 years old. 5 years hence Mrs. Madhu will be three times as old as Sunny. Find their present ages.*

Solution : The present age of Sunny

$= m$ years

and the present age of Mrs. Madhu

$= m^2$ years

After 5 years, the age of Sunny

$= (m + 5)$ years

After 5 years, the age of Mrs. Madhu

$= (m^2 + 5)$ years

Using the given informations, we get

$m^2 + 5 = 3(m + 5)$

$m^2 + 5 = 3m + 15$

or $m^2 - 3m - 10 = 0$

or $m^2 - 5m + 2m - 10 = 0$

or $m(m - 5) + 2(m - 5) = 0$

or $(m - 5)(m + 2) = 0$

$\Rightarrow$ $m - 5 = 0 \therefore m = 5$

or $m + 2 = 0 \therefore m = -2$

Neglecting the negative value of m, we get $m = 5$

Hence, the present age of Sunny

$= 5$ years

and the present age of Mrs. Madhu

$= (5)^2$

$= 25$ years. **Ans.**

Long Answer Type Questions

Q. 27. *A passenger train takes 2 hours less for a journey of 300 km, if its speed is increased by 5 km/hr from its usual speed. What is its usual speed ?*

Solution : Let, the usual speed of the train

$= x$ km/hr.

Given, distance = 300 km

$$\therefore \quad \text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{300}{x} \text{ hrs}$$

According to the question,

increased speed = $(x + 5)$ km/hr

and reduced time = $\left(\frac{300}{x} - 2\right)$ hr

Since, distance = speed × time

$$\therefore \quad 300 = (x + 5) \times \left(\frac{300}{x} - 2\right)$$

or $300x = (x + 5)(300 - 2x)$

or $300x = 300x - 2x^2 + 1500 - 10x$

or $2x^2 + 10x - 1500 = 0$

or $x^2 + 5x - 750 = 0$

or $x^2 + 30x - 25x - 750 = 0$

or $x(x + 30) - 25(x + 30) = 0$

or $(x + 30)(x - 25) = 0$

i.e., $x = -30$ or $x = 25$

Neglect $x = -30$ (– ve quantity)

Hence, the usual speed of the train $= 25$ km/hr. **Ans.**

Q. 28. *If Neeraj had walked* 1 *km / hr faster, he would have taken* 10 *minutes less to walk* 2 *km. Find the rate of his walking.*

Solution : Let the rate walking $= x$ km/hr, $[\because x > 0]$

Given, distance = 2 km

$$\therefore \quad \text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{2}{x}\text{hrs}$$

Again, speed $= (x + 1)$ km/hr

and time = 10 minutes less than previous time

$$= \frac{2}{x} - \frac{1}{6},$$

$$\left[\because 10 \text{ minutes} = \frac{1}{6}\text{ hrs.}\right]$$

$$\therefore \quad \text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{2}{(x+1)}$$

$$\text{or} \quad \frac{2}{x} - \frac{1}{6} = \frac{2}{(x+1)}$$

$$\text{or} \quad \frac{2}{x} - \frac{2}{(x+1)} = \frac{1}{6}$$

$$\text{or} \quad \frac{2(x+1) - 2x}{x(x+1)} = \frac{1}{6}$$

or $6(2x + 2 - 2x) = x(x + 1)$

or $12 - x^2 - x = 0$

or $x^2 + x - 12 = 0$

or $x^2 + 4x - 3x - 12 = 0$

or $x(x + 4) - 3(x + 4) = 0$

or $(x + 4)(x - 3) = 0$

i.e., $x = -4$ or $x = 3$.

But $x = -4$ is not possible because $x > 0$.

Therefore, $x = 3$ km/hr

Hence, the rate of walking $= 3$ km/hr. **Ans.**

Q. 29. *A plane left* 30 *minutes later than the scheduled time and in order to reach its destination* 1500 *km away in time, it has to increase its speed by* 250 *km / hr from its usual speed. Find its usual speed.*

Solution : Let the usual speed of the plane be x km/hr.

∴ Usual time to cover 1500 km distance

$$\text{time} = \frac{1500}{x}\text{ hrs.}$$

Given, that the speed is increased by 250 km/hr.

Therefore, the increased speed $= (x + 250)$ km/hr

Now, time to cover the journey with speed $(x + 250)$ km/hr.

$$= \frac{1500}{x + 250}\text{ hrs.}$$

Also, given that the time with increased speed is 30 minutes $\left(= \frac{1}{2}\text{ hrs}\right)$, less than the usual time.

$$\frac{1500}{x+250} = \frac{1500}{x} - \frac{1}{2}.$$

On multiplying both sides by $2x(x + 250)$, we have

$2x(1500) = 1500(2x + 500) - x(x + 250)$

or $3000x = 3000x + 750000 - x^2 - 250x$

or $x^2 + 250x - 750000 = 0$

or $x^2 + 1000x - 750x - 750000 = 0$

or $x(x + 1000) - 750(x + 1000) = 0$

or $(x + 1000)(x - 750) = 0$

i.e., $x = -1000$ or $x = 750$

Neglect $x = -1000$, because speed cannot be negative.

Therefore, $x = 750$ km/hr.

Hence, the usual speed of the plane $= 750$ km/hr. **Ans.**

Q. 30. *A man can row downstream* 3 *km / hr faster than he can row upstream. In one hour, he rows* 1 *km upstream and downstream back to the starting point. Find his speed while going downstream.*

Solution : Let the speed of downstream

$= x$ km/hr

The, the speed of upstream

$= (x - 3)$ km/hr

According to the given condition, we have

$$\frac{1}{x-3} + \frac{1}{x} = 1$$

Multiplying both sides by $x(x - 3)$, we get

$$x + x - 3 = x(x - 3)$$

or $\quad 2x - 3 - x^2 + 3x = 0$

or $\quad x^2 - 5x + 3 = 0$

Here, $a = 1$, $b = -5$, $c = 3$

$$\therefore \quad x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(3)}}{2(1)}$$

or $\quad x = \dfrac{5 \pm \sqrt{25 - 12}}{2} = \dfrac{5 \pm \sqrt{13}}{2}$

i.e., $\quad x = 4.3$ or 0.7

Neglect $x = 0.7$ km/hr because downstream speed cannot be less then 3 km/hr in this case.

Therefore $x = 4.3$ km/hr.

Hence, the downstream speed = 4.3 km/hr. **Ans.**

Q. 31. *One-fourth of a herd of camels was seen in forest. Twice the suqare root of the herd had gone to mountain and remaining* 15 *camels were on the bank of a river. Find the total number of camels.*

Solution : Let the number of camels be x.

Number of camels seen in the forest

$= \dfrac{x}{4}$.

Number of camels gone to mountain slopes $= 2\sqrt{x}$

Number of camels on the bank of the river $= 15$.

Therefore, according to the given information, we have

$$\frac{x}{4} + 2\sqrt{x} + 15 = x$$

or $\quad x + 8\sqrt{x} + 60 = 4x$

or $\quad -3x + 8\sqrt{x} + 60 = 0$

or $\quad 3x - 8\sqrt{x} - 60 = 0$

Now, putting $\sqrt{x} = y$, we get

$$3y^2 - 8y - 60 = 0$$

or $\quad 3y^2 - 18y + 10y - 60 = 0$

or $\quad 3y(y - 6) + 10(y - 6) = 0$

or $\quad (y - 6)(3y + 10) = 0$

$\Rightarrow \quad y - 6 = 0 \therefore y = 6$

or $\quad 3y + 10 = 0$

or $\quad 3y = -10$

or $\quad y = \dfrac{-10}{3}$

But $y = \dfrac{-10}{3}$ is not possible.

Therefore, $y = 6$

$\therefore \quad \sqrt{x} = 6$ or $x = 36$

Hence, the number of camels = 36. **Ans.**

Q. 32. *In a flight of* 600 *km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by* 200 *km / hr and the time of flight increased by* 30 *minutes. Find the duration of flight.*

Solution : Let the speed of aircraft

$= x$ km/hr

Given, distance = 600 km

$$\therefore \text{ Time} = \frac{\text{distance}}{\text{speed}} = \frac{600}{x} \text{ hrs.}$$

Reduced speed $= (x - 200)$ km/hr

$$\therefore \text{ Increased time} = \frac{600}{x} + \frac{30}{60}$$

$$= \left(\frac{600}{x} + \frac{1}{2}\right) \text{hrs}$$

$\therefore$ Distance = speed × time

$$600 = (x - 200) \times \left(\frac{600}{x} + \frac{1}{2}\right)$$

or $\quad 1200x = (x - 200)(1200 + x)$

or $1200x = 1200x + x^2 - 240000 - 200x$

or $x^2 - 200x - 240000 = 0$

or $x^2 - 600x + 400x - 240000 = 0$

or $x(x - 600) + 400(x - 600) = 0$

or $(x - 600)(x + 400) = 0$

i.e., $x = 600$ or $x = -400$.

Neglect $x = -400$ (negative quantity)

$\therefore$ Speed of the aircraft = 600 km/hr

Hence, required time $= \frac{600}{600}$

$= 1$ hour. **Ans.**

Q. 33. *A lady purchased one piece of cloth for Rs.* 1600. *Had the price rate of cloth been Rs.* 40 *per metre less, then she would have purchase* 2 *metre cloth more in the same amount. Find the measure of cloth and price per metre.*

Solution : Let the price per metre of cloth be ₹ x.

Amount spend = ₹ 1600

$\therefore$ Length of cloth purchase

$= \frac{1600}{x}$ metre

Now new price of cloth per metre become

$= ₹\,(x - 40)$

$\therefore$ Length of cloth purchased

$= \frac{1600}{x-40}$ metre

According to the question,

$$\frac{1600}{x} - \frac{1600}{x-40} = 2$$

$$\Rightarrow 1600\left[\frac{1}{x} - \frac{1}{x-40}\right] = 2$$

$$\Rightarrow 800\left[\frac{x - 40 - x}{x(x-40)}\right] = 1$$

$$\Rightarrow 800\left[\frac{-40}{x^2 - 40x}\right] = 1$$

$$\Rightarrow \frac{-32000}{x^2 - 40x} = 1$$

$\Rightarrow x^2 - 40x = -32000$

$\Rightarrow x^2 - 40x + 32000 = 0$

$\Rightarrow x^2 - 200x - 160x + 32000 = 0$

$\Rightarrow x(x - 200) - 160(x - 200) = 0$

$\Rightarrow (x - 200)(x - 160) = 0$

$\Rightarrow x - 200 = 0 \therefore x = 200$

or $x - 160 = 0 \therefore x = 160$

Hence, cost of cloth per metre

$= ₹\,200$

and length of cloth $= \frac{1600}{200} = 8$ metre.

Ans.

Q. 34. ₹ 250 *were divided equally among a certain number of students. If there were* 25 *students more, each would have received* ₹ $\frac{1}{2}$ *less. Find the number of students.*

Solution : Let the required number of students be x.

Then, share of each

$= \left(\frac{25000}{x}\right)$ paise

If there were $(x + 25)$ students, share of each $= \left(\frac{25000}{x+25}\right)$ paise

$$\therefore \frac{25000}{x} - \frac{25000}{x+25} = 50$$

$$\Rightarrow \frac{1}{x} - \frac{1}{x+25} = \frac{1}{500}$$

$$\Rightarrow \frac{(x+25) - x}{x(x+25)} = \frac{1}{500}$$

$\Rightarrow x^2 + 25x - 12500 = 0$

$\Rightarrow x^2 + 125x - 100x - 12500 = 0$

$\Rightarrow x(x + 125) - 100(x + 125) = 0$

$\Rightarrow (x + 125)(x - 100) = 0$

$\Rightarrow x + 125 = 0 \therefore x = -125$

or $x - 100 = 0 \therefore x = 100$

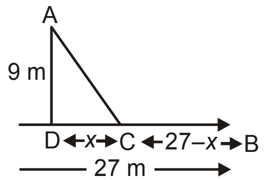
$\Rightarrow x \neq 125$ [Since number of student cannot be negative]

$\therefore$ Required number of students = 100.

Ans.

Q. 35. *A peacock is sitting on a pillar 9 m high. A snake at a distance of 27 m from the pillar is comming to a hole at the base of the pillar. Seeing the snake, the peacock pounces upon it. If their speeds are equal, then find at what distance from the hole is snake caught ?*

Solution : Suppose the snake is caught at a distance of x metre from the hole.



Since the speed of the peacock is the same as that of the snake, the distances covered by them are equal

$$AC = CB$$

$$\Rightarrow \sqrt{(9)^2 + x^2} = 27 - x$$

Squaring both sides,

$$81 + x^2 = 729 - 54x + x^2$$

$$\Rightarrow \quad 54x = 729 - 81 = 648$$

$$\Rightarrow \quad x = \frac{648}{54} = 12 \text{ metre.}$$

Hence, the distance from the hole is 12 metre. **Ans.**

Q. 36. *A man sells a watch for ₹ 75 and gains as much per cent as the watch cost him. Find the cost price of watch.*

Solution : Let cost price of watch be ₹ x then, gain = $x\%$

$$\therefore \text{ Selling price} = \text{Rs.}\left[\frac{(10 + x)\,x}{100}\right]$$

According to question,

$$\frac{(100 + x)\,x}{100} = 75$$

$$\Rightarrow \quad 100x + x^2 = 7500$$

$$\Rightarrow \quad x^2 + 100x - 7500 = 0$$

$$\Rightarrow \quad x^2 + 150x - 50x - 7500 = 0$$

$$\Rightarrow x(x + 150) - 50(x + 150) = 0$$

$$\Rightarrow \quad (x + 150)(x - 50) = 0$$

$$\Rightarrow \quad x + 150 = 0 \therefore x = -150$$

$$\text{or} \quad x - 50 = 0 \therefore x = 50$$

Price can't be negative.

$\therefore$ $x = -150$ is not acceptable.

Hence, the cost price of the watch is Rs. 50. **Ans.**

Q. 37. *A swimming pool is fitted with three pipes with uniform flow. The first two pipes operating simultaneously, fill the pool in the same time during which the pool is filled by the third pipe alone. The second pipe fills the pool 5 hours faster than the first pipe and 4 hours slower than the third pipe. Find the time required by each pipe to fill the pool individually.*

Solution : Let the time taken by the second pipe to fill the pool be x hours.

Then, the first pipe takes $(x + 5)$ hours,

While the third one takes $(x - 4)$ hours to fill it.

Then, the work done by first two pipes in 1 hours

= the work done by the third pipe in 1 hour

According to question,

$$\frac{1}{x + 5} + \frac{1}{x} = \frac{1}{x - 4}$$

$$\Rightarrow \quad \frac{x + x + 5}{x(x + 5)} = \frac{1}{x - 4}$$

$$\Rightarrow \quad \frac{2x + 5}{x^2 + 5x} = \frac{1}{x - 4}$$

$$\Rightarrow (x - 4)(2x + 5) = x^2 + 5x$$

$$\Rightarrow 2x^2 + 5x - 8x - 20 - x^2 - 5x = 0$$

$$\Rightarrow \quad x^2 - 8x - 20 = 0$$

$$\Rightarrow \quad x^2 - 10x + 2x - 20 = 0$$

$$\Rightarrow \quad x(x - 10) + 2(x - 10) = 0$$

$$\Rightarrow \quad (x - 10)(x + 2) = 0$$

$$\Rightarrow \quad x - 10 = 0 \therefore x = 10$$

$$\text{or} \quad x + 2 = 0 \therefore x = -2$$

$\therefore$ $x = -2$ is not acceptable.

Hence, the first pipe takes

$$10 + 5 = 15 \text{ hours}$$

The second pipe takes 10 hours and the third pipe takes $10 - 4 = 6$ hours. **Ans.**

❑

5 Arithmetic Progression

EXERCISE 5.1

Multiple Choice Type Questions

1. Common difference of A.P. 8, 11, 14, 17, 20, ... is :

(a) 7 (b) 3

(c) 2 (d) 5.

Sol. Common difference

$\Rightarrow \quad 11 - 8 = 3$

$\Rightarrow \quad 14 - 11 = 3$

$\Rightarrow \quad 17 - 14 = 3$

$\Rightarrow \quad 20 - 17 = 3$

Hence, common difference = 3.

2. Common difference of 2, 0.5, – 1, – 2.5, – 4, ... is :

(a) – 1.5 (b) – 1.3

(c) 2.4 (d) 2.6.

Sol. Common difference :

$\Rightarrow \quad 0.5 - 2 = -1.5$

$\Rightarrow \quad -1 - 0.5 = -1.5$

$\Rightarrow \quad -2.5 - (-1) = -1.5$

$\Rightarrow \quad -4 - (-2.5) = -1.5$

Hence, the common difference $= -1.5$.

Short Answer Type Questions

3. The n^{th} term of a progression is $3n - 8$. Is the pattern of numbers so formed, in A. P. ? If so, find its 16th term.

Sol. $a_n = 3n - 8$

Putting $n = 1$,

$a_1 = 3 \times 1 - 8 = -5$

Putting $n = 2$

$a_2 = 3 \times 2 - 8 = -2$

Putting $n = 3$

$a_3 = 3 \times 3 - 8 = 1.$

$\because$ Difference is common = 3

So, Yes, this is an A.P.

Now, 16^{th} term

$a_{16} = a + 15d \quad [\because a_n = a + (n-1)d]$

$= -5 + 15 \times 3$

$= -5 + 45$

$a_{16} = 40.$

4. The n^{th} term of a progression is $3n^2 + 4$. Is the pattern of numbers so formed, in A.P. ?

Sol. $a_n = 3n^2 + 4$

Putting $n = 1$,

$a_1 = 3 \times (1)^2 + 4 = 7.$

Putting $n = 2$

$a_2 = 3 \times (2)^2 + 4 = 16.$

Putting $n = 3$

$a_3 = 3 \times (3)^2 + 4 = 31.$

Hence, Difference is not common, so, This is not is an A.P.

5. The n^{th} term of a progression is $3n + 4$. Is the pattern of numbers, so, formed in A.P. ? If so, find its 16^{th} term.

Sol. $a_n = 3n + 4$

Putting $n = 1$,

$a_1 = 3 \times 1 + 4 = 7$

Putting $n = 2$,

$a_2 = 3 \times 2 + 4 = 10$

Putting $n = 3$,

$a_3 = 3 \times 3 + 4 = 13$

$\because$ Difference is common = 3

$\therefore$ Yes, this is an A.P.

Now, 6^{th} term

$a_6 = a + 5d \quad [\because a_n = a + (n-1)d]$

$a_6 = 7 + 5 \times 3$

$a_6 = 7 + 15$

$a_6 = 22$

6. Find p and q, such that $2p$, $2p + q$, $p + 4q$, 35 are in A.P.

Sol. Given, $t_1 = 2p$

$t_2 = 2p + q$

$t_3 = p + 4q$

$t_4 = 35$

For the value of p and q

$t_2 - t_1 = t_3 - t_2 = t_4 - t_3$

$2p + q - 2p = p + 4q - (2p + q)$

$= 35 - (p + 4q)$

$2p + q - 2p = p + 4q - 2p - q$

$= 35 - p - 4q$

$2p + q - 2p = 35 - p - 4q$

$2p + q - 2p + p + 4q = 35$

$p + 5q = 35$ ...(i)

Now, $p + 4q - 2p - q = 35 - p - 4q$

$p + 4q - 2p - q + p + 4q = 35$

$7q = 35$

$q = 5.$

Put the value of q in equation (i)

$p + 5 \times 5 = 35$

$p + 25 = 35$

$p = 35 - 25$

$p = 10.$

7. Find *a* and *b*, such that 12, *a* + *b*, 2*a*, *b* are in A.P.

Sol. Given, $t_1 = 12$

$t_2 = a + b$

$t_3 = 2a$

$t_4 = b$

Since it is in A.P. therefore the common difference between two consecutive terms will be same so,

$t_2 - t_1 = t_4 - t_3$

$a + b - 12 = b - 2a$

$3a = 12$

$a = 4.$

$t_2 - t_1 = t_3 - t_2$

also, $a + b - 12 = 2a - (a + b)$

$b - 8 = 4 - b$

$2b = 12$

$b = 6.$

8. Show that P–7, P–10, P–13, P–16, P–19...are in A.P. Find its 8th term and the common difference.

Sol. $T_1 = P - 7$

$T_2 = P - 10$

$T_3 = P - 13$

$T_4 = P - 16$

$T_5 = P - 19$

A is the variable which demonstrates the 1st term (+ T_1) and D is the variable that gives us the uniform difference between two terms.

So, $D = T_2 - T_1$

therefore, $D = (P - 10) - (P - 7)$

$D = -3.$

In each case, the value of D is uniform so, we can say that the terms are in uniform intervals, so they are in AP.

Now, 8^{th} term $= A + (7 \times D)$

$= (P - 7) + [7 \times (-3)]$

$= P - 7 - 21$

$= P - 28$

8^{th} term $= -28.$

9. Find *x*, so that $2x + 1$, $x^2 + x + 1$ and $3x^2 - 3x + 3$ are consecutive terms of an A.P.

Sol. Given : $2x + 1$, $x^2 + x + 1$ and $3x^2 - 3x + 3$ are the consecutive terms of an A.P.

As the terms are in A.P. Therefore, common difference between the given consecutive terms will be equal.

So, $t_2 - t_1 = t_3 - t_2$

$x^2 + x + 1 - (2x + 1) = 3x^2 - 3x + 3 - (x^2 + x + 1)$

$x^2 + x + 1 - 2x - 1 = 3x^2 - 3x + 3 - x^2 - x - 1$

$x^2 - 3x^2 + x^2 + x - 2x + 3x + x + 1 - 1 - 3 + 1 = 0$

$-x^2 + 3x - 2 = 0$

$-(x^2 - 3x + 2) = 0$

$x^2 - 3x + 2 = 0$

$x^2 - 2x - x + 2 = 0$

$x(x - 2) - 1(x - 2)$

$(x - 1)(x - 2)$

Hence, $x = 2$

$x = 1.$

EXERCISE 5.2

Multiple Choice Type Questions

1. 100th term of $\frac{1}{2}, 1, \frac{3}{2}, 2$ ...... is :

(a) 50 **(b) 60**

(c) 70 **(d) 40.**

Sol. 100th term of an A.P. $= a + (100 - 1)d$

$= \frac{1}{2} + (100 - 1)\frac{1}{2}$

$= \frac{1}{2} + \frac{99}{2}$

$= \frac{100}{2}$

$= 50.$ **Ans.**

2. r^{th} term of $a + 2b$, $a - b$, $a - 4b$ ...... is :

(a) $a + (5 - 3r)b$

(b) $a + (4 - 3r)b$

(c) $\boldsymbol{a + (6 - r)b}$
(d) $\boldsymbol{a + (2 - r)b}$**.**

Sol. It's A.P. with first term $a + 2b$ and Common difference $= -3b$

Now, r^{th} term $= a + 2b + (r-1) \times -3b$
$= a + 2b - 3br + 3b$
$= a + 5b - 3br$
$= a + (5 - 3r)\, b.$

Very Short Answer Type Questions

3. Find : (i) 10^th^ term of 9, 5, 1, – 3 .....

Sol. 10^{th} term $= a + (10 - 1)d$
$= 9 + (9) - 4$
$= -27.$

(ii) 16^th^ term of $\boldsymbol{\frac{1}{9}, \frac{4}{9}, \frac{7}{9}, \ldots}$ **.**

Sol. 16^{th} term $= a + (16 - 1)d$

$$= \frac{1}{9} + (15) \times \frac{3}{9}$$

$$= \frac{46}{9}$$

$$= 5\frac{1}{9}.$$

(iii) 20^th^ term of $\boldsymbol{\frac{1}{\sqrt{2}}, \sqrt{2}, \frac{3}{\sqrt{2}} \ldots}$

Sol. $$20^{th} \text{ term} = \frac{1}{\sqrt{2}} + (20 - 1)\frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} + \frac{19}{\sqrt{2}}$$

$$= \frac{20}{\sqrt{2}}$$

$$= 10\sqrt{2}.$$

(iv) $\boldsymbol{(n+1)^{th}}$ **term of** $\boldsymbol{n, n - \frac{1}{n}, n - \frac{2}{n} \ldots}$

Sol. $$(n + 1) \text{ term} = n + (n + 1 - 1) - \frac{1}{n}$$

$$= n - 1.$$

4. (i) The last term of the sequence 27, 24, 21 ..... is 0. Find the number of terms.

Sol. n^{th} term $= a + (n - 1)d$
$0 = 27 + (n - 1) - 3$
$0 = 27 - 3n + 3$
$3n = 30$

$$n = \frac{30}{3}$$

$n = 10^{th}$ term.

(ii) Find which term in progress 76, 72, 68, 64, ... is 0.

Sol. n^{th} term $= a + (n - 1)d$
$0 = 76 + (n - 1) - 4$
$0 = 76 - 4n + 4$
$4n = 80$

$$n = \frac{80}{4}$$

$n = 20^{th}$ term.

5. (i) Find the number of terms in the sequence 8, 11, 14 .... 86.

Sol. n^{th} term $= a + (n - 1)d$
$86 = 8 + (n - 1)3$
$86 = 8 + 3n - 3$
$86 - 5 = 3n$
$81 = 3n$

$$\frac{81}{3} = n$$

$n = 27^{th}$ term.

(ii) Find the number of terms in the sequence – 10, – 7, – 4 .... 29.

Sol. n^{th} term $= a + (n - 1)d$
$29 = -10 + (n - 1)3$
$29 = -10 + 3n - 3$
$29 = 3n - 13$
$29 + 13 = 3n$
$42 = 3n$

$$\frac{42}{3} = n$$

$n = 14^{th}$ term.

6. (i) Which term of the sequence 4, 9, 14, 19 ...... is 104 ?

Sol. n^{th} term $= a + (n - 1)d$
$104 = 4 + (n - 1)5$
$104 = 4 + 5n - 5$
$104 = 5n - 1$
$104 + 1 = 5n$

$$\frac{105}{5} = n$$

$n = 21^{st}$ term.

(ii) Which term of the sequence 21, 18, 15 ...... is 81 ?

Sol. n^{th} term $= a + (n - 1)d$

$$-81 = 21 + (n-1) - 3$$
$$-81 = 21 - 3n + 3$$
$$-81 - 24 = -3n$$
$$-105 = -3n$$
$$\frac{105}{3} = n$$
$$n = 35^{th} \text{ term.}$$

7. (i) If the third term of an A.P. is 5 and seventh term is 9. Find the 17th term.

Sol. 3^{rd} term $= a + (3 - 1)d$

$$5 = a + (3-1)d$$
$$5 = a + 3d - d$$
$$5 = a + 2d \quad ...(1)$$

7^{th} term $= a + (7 - 1)d$

$$9 = a + (7-1)d$$
$$9 = a + 7d - d$$
$$9 = a + 6d \quad ...(2)$$

On subtracting equation (1) from equation (2)

$$4d = 4,\ d = 1.$$

On substituting the value of d in equation (1) we get

$$5 = a + 2 \times 1;\ a = 3$$

17th term $= a + (17 - 1)d$
$= 3 + 16 \times 1$
$= 19.$

(ii) The 5th term of an A.P. is 1 and 31st term is – 77. Find the 11th term.

Sol. 5^{th} term $= a + (5 - 1)d$

$$1 = a + 4d \quad ...(1)$$

31^{st} term $= a + (31 - 1)d$

$$-77 = a + 30d \quad ...(2)$$

On subtracting equation (2) from equation (1) we get

$$-26d = 78;\ d = -3$$

On substituting the value of d in equation (1) we get,

$$1 = a + 4 \times (-3)$$
$$a = 13.$$

11^{th} term $= a + (11 - 1)d$
$= 13 + 10 \times - 3$
$= - 17.$

Short Answer Type Questions

8. (i) The 7th term and 13th term of an A.P. are 34 and 64 respectively. Find the first three terms.

Sol. 7^{th} term $= a + (7 - 1)d$

$$34 = a + 6d \quad ...(1)$$

13^{th} term $= a + (13 - 1)d$

$$64 = a + 12d \quad ...(2)$$

On subtracting equation (1) from (2) we get,

$$6d = 30 \Rightarrow d = 5.$$

On substituting the value of d in equation (1) we get :

$$34 = a + 6 \times 5;\ a = 4$$

1^{st} term $= a = 4$

2^{nd} term $= a + (n - 1)d$
$= 4 + (2 - 1)5$
$= 9$

3^{rd} term $= 4 + (3 - 1)5$
$= 14.$ **Ans.**

(ii) In an A.P. the third term is 4 times the first term. The 6th term is 17. Find the series.

Sol. Let a be the first term and D be common difference

Given, $a_3 = 4a$

$$a + 2d = 4a$$
$$2d = 4a - a$$
$$2d = 3a \quad ...(i)$$
$$a_6 = 17$$
$$a + 5d = 17$$
$$\frac{2d}{3} + \frac{5d}{1} = 17$$
$$2d + 15d = 51$$
$$17d = 51$$
$$d = 3.$$

Put the value of d in (i)

$$2 \times 3 = 3a$$
$$6 = 3a$$
$$a = 2.$$

Therefore, The A.P. is $a, a + d, a + 2d$, 2, 5, 8, 11....

9. (i) For what value of a the following terms are in arithmetics progression ? a + 1, 3a, 4a + 2

Sol. Arithmetic Progression :

$$t_2 - t_1 = t_3 - t_2$$
$$(3a) - (a + 1) = (4a + 2) - (3a)$$
$$3a - a - 1 = 4a + 2 - 3a$$
$$2a - 1 = a + 2$$
$$2a - a = 2 + 1$$
$$a = 3.$$

$\therefore$ A. P. is $3 + 1, 3 \times 3, 4 \times 3 + 2$
$= 4, 9, 14.$

(ii) If 3*x*, *x* + 2 and 8 are three consecutive terms of an A.P. Find its fourth term.

Sol. $3x$, $x+2$, 8 = are in A.P.

Given $a_1 = 3x$

$a_2 = x + 2$

$a_3 = 8$

$\therefore \quad a_2 - a_1 = a_3 - a_2$

$x + 2 - 3x = 8 - (x + 2)$

$2 - 2x = 8 - x - 2$

$2 - 2x = 6 - x$

$-2x + x = 6 - 2$

$-x = 4; x = -4$

1^{st} term $= 3x = 3(-4) = -12$

2^{nd} term $= x + 2 = -4 + 2 = -2$

$d = a_2 - a_1$

$= -2 - (-12) = 10$

$a_4 = a + (4 - 1)d$

$a_4 = -12 + 3 \times 10$

$a_4 = 18.$

10. The m^{th} term of an A.P. is $\frac{1}{n}$ and the n^{th} term is $\frac{1}{m}$. Prove that its mn^{th} term is 1.

Sol. Given that m^{th} term = $1/n$ and n^{th} term = $1/m$

Let a and d the first term and common difference.

So, $a + (m - 1)d = \frac{1}{n}$ ...(1)

$a + (n - 1)d = \frac{1}{m}$ ...(2)

subtracting equation (1) by (2) we get,

$md - d - nd + d = \frac{1}{n} - \frac{1}{m}$

$d(m - n) = \frac{m-n}{mn}$

$d = \frac{1}{mn}$

again if we put this value in equation (1) and (2) we get

$a = \frac{1}{mn}$

then, let A be the mnth term of the A.P.

$= a + (mn - 1)d$

$= \frac{1}{mn} + (mn - 1)\frac{1}{mn}$

$= \frac{1}{mn} + \frac{mn}{mn} - \frac{1}{mn}$

$= \frac{1}{mn} + 1 - \frac{1}{mn}$

$= 1$

Hence, it is proved that mn^{th} term is 1.

11. The p^{th} term of an A.P. is q and the q^{th} term is p. Find its m^{th} term.

Sol. Let the first term be a and common difference is d.

p^{th} term $= a + (p - 1)d = q$...(1)

q^{th} term $= a + (q - 1)d = p$...(2)

on subtracting equation (2) from (1) we get,

$(q - p)d = p - q$

$d = -1$

on substituting the value of d in equation (1) we get

$a = q - (p - 1)d$

$= q - (p - 1)(-1)$

$= p + q - 1$

Hence, m^{th} term $= a + (m - 1)$

$= p + q - 1 + (m - 1)(-1)$

$p + q - 1 - m + 1$

$= p + q - m.$

12. If m times the m^{th} term of an A. P. is equal to n times the n^{th} term, show that $(m + n)^{th}$ term of this A. P. is zero.

Sol. Let the first term is a and common difference is d.

Given : m times of m^{th} term = n times of n^{th} term

To Find : mn^{th} term of A.P.

Sol. $t_n = a + (n - 1)d$

$t_m = a + (m - 1)d$

$mtm = mtn$

$m[a + (m - 1)d] = n[a + (n - 1)d]$

$m[a + (m - 1)d] - n[a + (n - 1)d] = 0$

$am + m(m - 1)d - an - n(n - 1)d = 0$

$a(m - n) + (m^2 - m)d - (n^2 - n)d = 0$

$a(m - n) + [d(m^2 - n^2) - d(m - n)] = 0$

$a(m - n) + d[(m + n)(m - n) - (m - n)] = 0$

$(m - n)[a + d(m + n) - 1] = 0$

$a + [(m + n) - 1]d = 0$

$t_{m+n} = a + [(m+n) - 1]d$

$\therefore \quad t_{m+n} = 0$

Hence $(m + n)^{th}$ term of A.P. is 0.

13. Find the 15th term from the end of an A. P. 6, 10, 14, 18 .... 102.

Sol. 15th term from the end then let the end term is first term

So, $\quad 15^{th}$ term $= a + (15 - 1)d$

$= 102 + 14 \times -4$

$= 102 - 56$

$= 46.$

14. Find the number of terms in the A.P. 10, 4, – 2, – 8 ..... – 290. Also, find its 30th term from the end.

Sol. Given : $a = 10 \; d = -6$

$-290 = a + (n - 1)(-6)$

$-290 = 10 + (n - 1)(-6)$

$\frac{-300}{-6} = n - 1 \Rightarrow n - 1 = 50$

$\Rightarrow \quad n = 51^{th}$

so, there are 51 terms in A.P.

30th from the end is $(51 - 30 + 1)^{th}$ term

$\therefore$ from begining $= 22^{nd}$ term

22^{th} term $= a + (22 - 1)d$

$= 10 + 21 \times -6$

$= 10 - 126$

$= -116.$ **Ans.**

15. Sita was appointed as a lecturer. She was offered monthly salary of ₹ 15,000 with annual increment of ₹ 500. In the tenth year what would be her monthly salary ?

Sol. Current salary = ₹ 15,000

Annual salary in 1st year = 12 × 15,000 = 1,80,000.

Increment every year = ₹ 500

Annual salary in 2nd year = 15,500 × 12 = 1,86,000.

Annual salary in 3rd year = 16,000 × 12 = 1,92,000.

This form is A.P. with $a = 1{,}80{,}000$

$d = 6{,}000.$

$n = 10$ years

Thus,

10^{th} term $= a + (10 - 1)d$

$= 1{,}80{,}000 + 9 \times 6{,}000$

$= 1{,}80{,}000 + 54{,}000$

$= 2{,}34{,}000$

10th year salary = 2,34,000.

monthly salary in 10th year =

$\frac{2{,}34{,}000}{12} = 19{,}500.$ **Ans.**

16. Ritu joined a bank on the initial salary ₹ 5,000 per month with annual increment ₹ 400. In 20th year what will be her monthly salary.

Sol. Initial salary = 5,000

Annual salary in 1st year = 5,000 × 12 = 60,000

Annual increment per year = 400

Annual salary in 2nd year = 5,400 × 12 = 64,800.

Annual salary in 3rd year = 5,800 × 12 = 69,600.

This form is A.P. with $a = 60{,}000$

$d = 4{,}800$

$n = 20$ years

Thus,

20^{th} term $= a + (20 - 1)d$

$= 60{,}000 + 19 \times 4{,}800$

$= 1{,}51{,}200.$

Monthly salary in 20th year =

$\frac{1{,}51{,}200}{20} = 12{,}600.$ **Ans.**

17. Determine the number of term in A.P. 3, 7, 11, ... 407. Also, find its 20th term form the end.

Sol. n^{th} term $= a + (n - 1)d$

$407 = 3 + (n - 1)4$

$407 - 3 = (n - 1)4$

$\frac{404}{4} = n - 1$

$n - 1 = 101$

$n = 101 + 1$

$n = 102^{th}$

Hence this is 102 terms in this A.P.

20th term from the end = 102 – 20 + 1 = 83

$\therefore$ From the begining = 83th term

83^{th} term $= a + (n - 1)d$

$= 3 + (83 - 1)4$

$= 3 + 82 \times 4 = 331.$

18. Find four numbers in A.P. whose sum is 20 and the sum of whose squares is 120.

Sol. Let the terms be $a - 3d, a - d, a + d, a + 3d$

sum of the terms $= (a - 3d) + (a - d) + (a + d) + (a + 3d) = 20$

$4a = 20$

$a = 5$

sum of the squares of the terms :

$= (a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 20$

$a^2 - 6ad + 9d^2 + a^2 - 2ad + d^2 + a^2 + 2ad + d^2 + a^2 + 6ad + 9d^2 = 120$

$4a^2 + 20d^2 = 120$ ...(1)

substituting $a = 5$ in equation (1)

$4(5)^2 + 20d^2 = 120$

$100 + 20d^2 = 120$

$20d^2 = 120 - 100$

$20d^2 = 20$

$d^2 = \frac{20}{20}$

$d^2 = 1$

$a = \pm 1.$

Thus, the four number are

Taking $d = 1$

$(a - 3d)(a - d)(a + d)(a + 3d)$

$= (5 - 3),(5 - 1),(5 + 1),(5 + 3)$

$= 2, 4, 6, 8.$

19. Which term of the A.P. 3, 15, 27, 39, ... will be 120 more than its 64th term ?

Sol. Let the required term be n^{th} term

64^{th} term $= a + (n - 1)d$

$= 3 + (64 - 1)12$

$= 3 + 63 \times 12$

$= 759$

Therefore, $759 + 120 = 879$ will be n^{th} term

$879 = 3 + (n - 1)12$

$\frac{879 - 3}{12} = n - 1$

$73 = n - 1$

$n = 74.$

20. Find the number of terms in A.P. 8.5, 6.9, 5.3 ..... – 151.5. Also, find the 50th term from the end.

Sol. Let – 151.5 is the n^{th} term of A.P.

$\therefore \quad -151.5 = a + (n - 1)d$

$-151.5 = 8.5 + (n - 1) - 1.6$

$\frac{-151.5 - 8.5}{-1.6} = n - 1$

$100 = n - 1$

$n = 101$

Hence, – 151.5 is 101^{th} term.

50^{th} term from the end $= 101 - (50 - 1) = 52^{th}$ term from the starting

52^{th} term $= a + (52 - 1)d$

$= 8.5 + 51 \times - 1.6$

$= - 73.1.$

21. Which term of A.P. 163, 160, 167 .... is the first negative term.

Sol. $D = 160 - 163 = - 3$

First negative term will be smaller than 0.

So, $0 > a + (n - 1)d$

$0 > 163 + (n - 1) - 3$

$-163 > - 3n + 3$

$-166 > - 3n$

$166 > 3n$

$\frac{166}{3} > n$

$55 \times \frac{1}{3} > n$

hence, $n = 56$

therefore, the 56^{th} term is first negative.

22. Check whether (137) is a term of A.P. 3, 11, 19, 22, ?

Sol. $t_n = a + (n - 1)d$

$t_n = 137$

$a = 3$

$d = 11 - 3 = 8$

now, by putting the values we get,

$137 = 3 + (n - 1)8$

$\frac{137 - 3}{8} = n - 1$

$16.75 = n - 1$

$n = 17.75$

Hence, $n = 17.75$ and term can not be in decimal.

Therefore, (137) is not a term of AP. 3, 11, 19, 22.

23. How many numbers of two digits are divisible by 9 ?

Sol. We know that the two digits numbers divisible by 9 starts from 18.

Therefore A.P. formed is

18, 27, 36, 45 99

$a = 18,$

$d = 9$ last term (a_n)

$= (99), n = ?$

using the formula

$a_n = a + (n - 1)d$

$99 = 18 + (n - 1)9$

$\frac{99 - 18}{9} = n - 1$

$9 = n - 1$

$n = 10.$

Therefore 10, 2 digits numbers are divisible by 9.

24. Divide 32 into four parts which are in A.P. such that the ratio of product of extremes to the product of means is 7 : 15.

Sol. Let the four parts are $a - 3d$, $a - d$, $a + d$ and $a + 3d$ or $4a = 32$

$$a = 8.$$

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$$

$$15a^2 - 135d^2 = 7a^2 - 7d^2$$

$$8a^2 = 128d^2$$

$$8 \times 8^2 = 128d^2 \; d = \pm 2$$

The four parts are :

$$a - 3d = 8 - 3 \times 2 = 2$$

$$a - d = 8 - 2 = 6$$

$$a + 8 = 8 + 2 = 10$$

$$a + 3d = 8 + 3 \times 2 = 14$$

Thus, A. P. is 2, 6, 10, 14....

EXERCISE 5.3

Multiple Choice Type Questions

1. The sum of first 16 terms of the A. P. 10, 6, 2 .... is :

(a) 320 **(b) – 320**

(c) – 352 **(d) – 400.**

Sol. Sum of first 16 terms of AP :

$$S = \frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow \frac{16}{2}[2\times 10+(16-1)-4]$$

$$\frac{16}{2}[20-64+4]$$

$$\frac{16}{2}[-40]$$

$$\frac{-640}{2}$$

$$\Rightarrow \quad -320.$$

2. The sum of first 20 terms of the A.P. 1, 3, 5, 7, 9 .... is.

(a) 264 **(b) 400**

(c) 472 **(d) 563.**

Sol.
$$S = \frac{n}{2}[2a+(n-1)d]$$

$$= \frac{20}{2}[2\times 1+(20-1)2]$$

$$= \frac{20}{2}[2+40-2]$$

$$= \frac{20}{2}[40]$$

$$= \frac{800}{2}$$

$$= 400.$$

3. (5 + 13 + 21 + ... + 181) = ?

(a) 2476 **(b) 2337**

(c) 2219 **(d) 2139.**

Sol. Sum of A.P. $= \frac{n}{2}(a+l)$

Where, $a = 5$, $l = 181$, $d = 8$

$n \Rightarrow a + (n - 1)\,d$ = last term

$5 + (n - 1)8 = 181$

$$n - 1 = \frac{181-5}{8}$$

$$n - 1 = 22$$

$$n = 23$$

$$\text{Sum} = \frac{23}{2}(5+181)$$

$$\text{Sum} = 2139.$$

4. The sum of n terms of the A.P. $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$ .... is

(a) 1 **(b) $2n(n + 1)$**

(c) $\frac{1}{2}n(n+1)$ **(d) $\frac{1}{\sqrt{2}}n(n+1)$.**

Sol. The series can be written as $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}$ ....

$$\text{Sum of } n \text{ terms} = \frac{n}{2}[2a+(n-1)d]$$

$$= \frac{n}{2}[2\times\sqrt{2}+(n-1)\sqrt{2}]$$

$$= \frac{n}{2}\left[2\sqrt{2}+\sqrt{2}n-\sqrt{2}\right]$$

$$= \frac{n}{2}\left[\sqrt{2}n+\sqrt{2}\right]$$

$$= \frac{1}{2}n\left[\sqrt{2}\,(n+1)\right]$$

$$= \frac{1}{\sqrt{2}}n(n+1)$$

5. How many terms of the A.P. 3, 7, 11, 15, ... will make the sum 406 ?
(a) 10 (b) 12
(c) 14 (d) 20.

Sol.
$$\text{Sum} = \frac{n}{2}[2a+(n-1)d]$$
$$406 = \frac{n}{2}[2\times3+(n-1)4]$$
$$406 \times 2 = n(6 + 4n - 4)$$
$$812 = n(2 + 4n)$$
$$812 = 2n + 4n^2$$
$$2n + 4n^2 - 812 = 0$$
$$2n^2 + n - 406 = 0$$
(dividing whole equation by 2)
$$2n^2 - 28n + 29n - 406 = 0$$
$$2n(n - 14) + 29(n - 14) = 0$$
$$2n + 29 = 0$$
$$n = \frac{-29}{2}.$$
$$n - 14 = 0;\ n = 14.$$
Since value of n cannot be infraction therefore, $n = 14$.

6. The sum of first 100 natural numbers is :
(a) 4,950 (b) 5,050
(c) 5,000 (d) 5,150.

Sol. The sum of first n terms of an A.P. is
$$S = \frac{n}{2}\ [2a + (n - 1)\ d]$$
$$S = \frac{100}{2}\ [2 \times 1 + (100 - 1)\ 1]$$
$$S = 50\ [2 + 99]$$
$$S = 50 \times 101$$
$$S = 5{,}050.$$

7. The sum of all odd numbers between 100 and 200 is :
(a) 3,750 (b) 6,200
(c) 6,500 (d) 7,500.

Sol. The first term is (a) 101
Difference (b) = 2
Last term (c) = 199
Then A.P. = 101, 103....197, 199
$$l = a + (n - 1)d$$
$$199 = 101 + (n - 1)\ 2$$
$$\frac{199-101}{2} = n - 1$$
$$n - 1 = 49$$
$$n = 49 + 1$$
$$n = 50.$$
$$S = \frac{n}{2}(a + l)$$
$$= \frac{50}{2}\ (101 + 199)$$
$$= 25 \times 300$$
$$= 7{,}500.$$

8. The sum of all even natural number less than 100 is :
(a) 2,272 (b) 2,352
(c) 2,450 (d) 2468.

Sol. For all even natural number less than 100 A.P. is :
2, 4, 6, 8, 10 .....96, 98.
$\therefore\ a = 2\ d = 2\ l = 98.$
$$l = a + (n - 1)d$$
$$98 = 2 + (n - 1)2$$
$$\frac{98-2}{2} = n - 1$$
$$n - 1 = 48$$
$$n = 48 + 1$$
$$n = 49.$$
$$S = \frac{n}{2}(a + l)$$
$$S = \frac{49}{2}(2 + 98)$$
$$= \frac{49}{2} \times 100$$
$$= 2450.$$

9. The sum of first fifteen multiples of 8 is :
(a) 840 (b) 960
(c) 872 (d) 1,080.

Sol. For multiples of 8 A.P. is :
8, 16, 24
$\therefore\quad a = 8\ d = 8\ n = 15$
$$\text{Sum (S)} = \frac{n}{2}\ [2a + (n - 1)d]$$
$$= \frac{15}{2}(2 \times 8 + (15 - 1)8]$$
$$= \frac{15}{2}(16 + 112)$$
$$= \frac{15}{2} \times 128$$

$= 15 \times 64$
$= 960.$

10. In an AP, the first term is 22, n^{th} term is –11 and the sum of first n terms is 66. The value of n is :
(a) 10 **(b) 12**
(c) 14 **(d) 16.**

Sol. Given : $a = 22$ $l = -11$ S = 66

$$S = \frac{n}{2}[a + l]$$

$$66 = \frac{n}{2}[22 + (-11)]$$

$$66 \times 2 = n\ (22 - 11)$$
$$132 = n \times 11$$
$$n = \frac{132}{11}$$
$$n = 12.$$

Hence, value of n is 12.

11. In an AP, the first term is 8, n^{th} term is 33 and the sum of first n terms is 123. Then, d = ?

Sol. In A.P. $a = 8$, $l = 33$ Sn = 123

$$Sn = \frac{n}{2}(a+l)$$

$$123 = \frac{n}{2}(8+33)$$

$$\frac{123\times 2}{41} = n$$
$$n = 6.$$

$$Sn = \frac{n}{2}[2a+(n-1)d]$$

$$123 = \frac{6}{2}[2\times 8+(6-1)d]$$

$$\frac{123\times 2}{6} = 16 + (6 - 1)\ d$$

$$\frac{41-16}{5} = d$$
$$d = 5.$$

12. The sum of n terms of an A.P. is given by $S_n = (2n^2 + 3n)$. What is the common difference of the AP ?
(a) 3 **(b) 4**
(c) 5 **(d) 9.**

Sol. Let $n = 1$

$$S_1 = [2(1)^2 + 3(1)]$$
$$= 5$$
$$= t_1$$
$$S_2 = [2(2)^2 + 3(2)]$$
$$= 14$$
$$= t_1 + t_2$$
$$\therefore\ S_2 - S_1 = 9 = t_2$$
$$d = t_2 - t_1$$
$$= 9 - 5$$
$$= 4.$$

13. Find the sum of :
(i) 1 + 7 + 13 + 19 + ... to 40 terms.

Sol. Given : $a = 1$ and $d = 7–1 = 6$, $n = 40$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{40}{2}[2 \times 1 + (40-1)\ 6]$$

$$S_n = \frac{40}{2}[2 + (39)\ 6]$$

$$S_n = \frac{40}{2}[2 + 234]$$

$$S_n = \frac{40}{2}[236]$$

$$S_n = \frac{9440}{2}$$
$$= 4720.$$

(ii) – 12 – 8 – 4 + 0 + 4 ... to 20 terms.

Sol. Given : $a = -12$, $n = 20$, $d = 4$

$$S_n = \frac{n}{2}[2a + (n-1)\ d]$$

$$S_n = \frac{20}{2}[2 \times -12 + (20-1)\ 4]$$

$$S_n = \frac{20}{2}[-24 + (19)\ 4]$$

$$S_n = \frac{20}{2}[-24 + 76]$$

$$S_n = 10\ [52]$$
$$S_n = 520.$$

(iii) $2 + \frac{7}{2} + 5 + \frac{13}{2}$ + ... to 10 terms.

Sol. Given : $a = 2$, $d = \frac{3}{2}$, $n = 10$

$$S_n = \frac{n}{2}\ [2a + (n-1)\,d]$$

$$= \frac{10}{2}\ [2 \times 2 + (10-1)\ \frac{3}{2}]$$

$$= \frac{10}{2} \times 4 + \frac{27}{2}$$

$$= \frac{10}{2} \times \frac{35}{2} = \frac{175}{2}.$$

14. Find the sum of all natural numbers between 100 and 1000, which are multiples of 5.

Sol. Multiples of 5 are 5, 10, 15, 20, 25 ... Multiples of 5 between 100 and 1000 are 105, 110, 115 ... 990, 995. This sequence form an A.P. as difference between the consective terms is constant.

Here, First term $= a = 105$

Common difference $= d = 110 - 105 = 5$

and last term $= l = 995$

First we need to find number of terms, *i.e.*, n we know that

$$a_n = a + (n-1)\,d$$

where $a_n = n^{\text{th}}$ term

n = number of terms

a = first term

d = common difference

Here a_n = last term $= l = 995$, $a = 105$, $d = 5$ on putting values, we get–

$$995 = 105 + (n-1)\,5$$
$$995 = 105 + 5n - 5$$
$$995 = 100 + 5n$$
$$995 - 100 = 5n$$
$$895 = 5n$$
$$\frac{895}{5} = n$$
$$179 = n$$

For finding sum we use formula

$$S_n = \frac{n}{2}\ [a + l]$$

$$S_n = \frac{179}{2}\ [105 + 995]$$

$$= \frac{179}{2}\ (1100)$$

$$= 179\ (550)$$
$$= 98450.$$

15. Find the sum of all odd integers between 300 and 498.

Sol. odd integers are 1, 3, 5, 7, 9 ... odd integers between 300 and 498 are 301, 303, 305 ... 495, 497

This form is in A.P. as difference between the consecutive terms are constant

Here, $a = 301$, $d = 2$, $l = 497$

$$a_n = a + (n-1)\,d$$
$$497 = 301 + (n-1)\,2$$
$$\frac{497 - 301}{2} = n - 1$$
$$98 = n - 1$$
$$n = 99.$$

$$\text{Sum of terms} = \frac{n}{2}\ (a + l)$$

$$= \frac{99}{2}\ (301 + 497)$$

$$= \frac{99}{2}\ (798)$$

$$= 99\ (399)$$
$$= 39501.$$

16. Find the sum of all integers between 400 and 579, which are divisible by 10.

Sol. Multiple by 10 are 10, 20, 30, 40, 50 ... Multiple by 10 between 400 and 579 are 410, 420, 430 ... 560, 570. This sequence form in A.P. as difference between the consecutive terms are constant.

Here, $a = 410$, $d = 10$, $l = 570$

$$a_n = a + (n-1)\,d$$
$$570 = 410 + (n-1)\,10$$
$$\frac{570 - 410}{10} = n - 1$$
$$\frac{160}{10} = n - 1$$
$$16 = n - 1,\ n = 17.$$

$$S_n = \frac{n}{2}\ [2a + (n-1)\,d]$$

$$S_n = \frac{17}{2}\ [2 \times 410 +$$

(17 – 1) 10]

$S_n = \frac{17}{2}$ [820 + 160]

$S_n = \frac{17}{2}$ [980]

S_n = 17 [490]

S_n = 8330.

17. Show that the sum of first *n* even natural numbers is equal to $\left(1 + \frac{1}{n}\right)$ times the sum of first *n* odd natural numbers.

Sol. Even number's = 2, 4, 6, 8 ... *n*

Odd numbers = 1, 3, 5, 7 ... *n*

Sum of first '*n*' even numbers

$$\Rightarrow \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2 \times 2 + (n-1)2]$$

$$= \frac{n}{2}[4 + 2(n-1)]$$

$$= \frac{n}{\not{2}} \times \not{2}\,[2 + (n-1)]$$

$$= 2n + n^2 - n$$

$$= n^2 + n$$

Sum of first odd '*n*' natural numbers

$$\Rightarrow \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2 \times 1 + (n-1)2]$$

$$= \frac{n}{2}[2 + 2(n-1)]$$

$$= \frac{n}{2} \times 2\,[1 + (n-1)]$$

$$= n \times [1 + (n-1)]$$

$$= n + n^2 - n$$

$$S_n = n^2$$

Sum of Ist '*n*' even natural numbers

$= \left(\frac{n+1}{n}\right)$ times the sum of Ist '*n*' odd natural numbers

$$n^2 + n = \left(\frac{n+1}{\not{n}}\right) \times n^{\not{2}}$$

$n^2 + n = n^2 + n$ **Hence Proved.**

18. Find the sum of all four digit numbers, which when divided by 25 leaves 5 as a remainder.

Sol. We will get an A.P.–

1005, 1030, 1055 ... 9980

$$a_n = a + (n-1)d$$

$$9980 = 1005 + (n-1)25$$

$$\frac{9980 - 1005}{25} = n - 1$$

$$359 = n - 1$$

$$359 + 1 = n$$

$$n = 360.$$

as formula for sum of A. P. is

$$S_n = \frac{n}{2}(a + l)$$

$$= \frac{360}{2}(1005 + 9980)$$

$$= \frac{360}{2}(10985)$$

$$= 180\,(10985)$$

$$= 19{,}77{,}300.$$

19. In an A. P., if the first term is 22, the common difference is – 4 and the sum to *n* terms is 64, find *n*. Explain double answer.

Sol. Given : $a = 22$, $d = -4$, $S_n = 64$

Hence, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$64 = \frac{n}{2}[2 \times 22 + (n-1) - 4]$$

$$64 \times 2 = 44n - 4n^2 + 4n$$

$$-4n^2 + 48n - 128 = 0$$

$$-4n^2 + 32n + 16n - 128 = 0$$

$$-4n(n-8) + 16(n-8) = 0$$

$$(-4n + 16)(n-8) = 0$$

$-4n + 16 = 0 \qquad n - 8 = 0$

$n = 4 \qquad n = 8$

n = 4 or 8.

∴ If *n* is 4 :

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$= \frac{4}{2}$ [2 × 22 + (4 – 1) – 4]

$= \frac{4}{2}$ [44 – 16 + 4]

= 2 [32]

= 64

If n is 8 :

$S_n = \frac{8}{2}$ [2 × 22 + (8 – 1) – 4] = 64.

20. How many terms of the sequence 18, 16, 14 ... should be taken, so that their sum is 78 ?
Explain double answer.

Sol. Given : $a = 18, d = -2, S_n = 78$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$78 = \frac{n}{2}[2 \times 18 + (n-1) - 2]$$

$$78 \times 2 = 36n - 2n^2 + 2n$$

$$-2n^2 + 38n - 156 = 0$$

$$-2n^2 + 26n + 12n - 156 = 0$$

$$-2n(n-13) + 12(n-13) = 0$$

$$(-2n + 12)(n-13) = 0$$

$-2n + 12 = 0$ $\qquad$ $n - 13 = 0$

$-2n = -12$

$$n = \frac{12}{2}$$

$n = 6$ $\qquad$ $n = 13$

$\therefore$ $\quad n = 6$ or 13.

21. Find the sum of all numbers, which are divisible by 7 and lying between 50 and 500.

Sol. The first number divisible by 7 is 56 lying between 50 and 500 is 56, 63, 70 ..., 490, 497

Hence, $\quad a = 56, d = 7, a_n = 497$

$$a_n = a + (n-1)d$$

$$497 = 56 + (n-1)7$$

$$\frac{497 - 56}{7} = n - 1$$

$$63 = n - 1$$

$$n = 64$$

$$S_n = \frac{n}{2}(a + l)$$

$$S_n = \frac{64}{2}(56 + 497)$$

$$= \frac{64}{2}(553)$$

$$= 32(553)$$

$$S_n = 17696.$$

22. Find the sum of all integers between 92 and 786, which are multiples of 9.

Sol. Multiples of 9 between 92 and 786 are 99, 108, 117 ... 774, 783

Hence, $\quad a = 99, d = 9, a_n = 783$

$$a_n = a + (n-1)d$$

$$783 = 99 + (n-1)9$$

$$\frac{783 - 99}{9} = n - 1$$

$$76 = n - 1$$

$$n = 77.$$

$$S_n = \frac{n}{2}[a + l]$$

$$S_n = \frac{77}{2}[99 + 783]$$

$$S_n = \frac{77}{2}(882)$$

$$= 77(441)$$

$$S_n = 33957.$$

23. The sum of first n terms of an A.P., is zero, show that the sum of next m terms is $\frac{-am(m+n)}{n-1}$, a being the first term.

Sol. A is the first term then, sum of n terms in A.P.
When common difference is d.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = 0 = \frac{n}{2}[2a + (n-1)d]$$

$$0 = 2a + (n-1)d$$

$$d = \frac{-2a}{(n-1)} \qquad ...(1)$$

now, sum of next m terms.

$$S_m = S_{m+n} - S_n$$

$= \frac{m+n}{2}[2a+(m+n-1)d]$

$- \frac{n}{2}[2a+(n-1)d]$

$\frac{m+n}{2}(2a+md+nd-d) - \frac{n}{2}(2a+nd-d)$

$\frac{2am+2an+m^2d+mnd+mnd+n^2d-md-nd}{2} - \frac{(2an+n^2d-nd)}{2}$

$\frac{2am+2an+m^2d+mnd+mnd+n^2d-md-nd-2an-n^2d+nd}{2}$

$\frac{2am+d(m^2+mn+mn+n^2-m-n-n^2+n)}{2}$

$= am + \frac{d}{2}[m^2+n^2+2mn-m-n-n^2+n]$

$= am + \frac{d}{2}[m^2+2mn-m]$

now, put $d = \frac{-2a}{(n-1)}$

then $S_m = am + \frac{(-2a)}{2(n-1)}[m^2+2mn-m]$

$= am - \frac{am}{(n-1)}[m+2n-1]$

$= \frac{(n-1)am - am}{n-1}[n+2n-1]$

$= \frac{amn - am - am^2 - 2amn + am}{n-1}$

$= \frac{-am^2 - amn}{n-1}$

$= \frac{-am(m+n)}{n-1}$

Hence, it is proved that

$S_m = \frac{-am(m+n)}{(n-1)}$.

24. If first term of an A.P. be 100 and the sum of first six terms is five times the sum of next six terms. Find the sum of first eleven terms.

Sol. Given : $a = 100$

let the common difference = d

So, the A. P. is

$100, 100+d, 100+2d \ldots$

$\therefore 100+(100+d)+(100+2d)+(100+3d)+(100+4d)+(100+5d) = 5[(100+6d)+(100+7d)+(100+8d)+(100+9d)+(100+10d)+(100+11d)]$

$600+15d = 5(600+51d)$

$-240d = 2400$

$d = -10$

$S_{11} = 1100+55d$

$= 1100-550 = 550.$

25. How many terms of the A. P. – 6, $\frac{-11}{2}$, – 5 ... are needed to give the sum – 25 ? Explain double answer.

Sol. Given : $a = -6, d = \frac{-11}{2} + 6 = \frac{1}{2}$

$S_n = -25$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$-25 = \frac{n}{2}[2 \times -6 + (n-1)\frac{1}{2}]$

$-25 \times 2 = -12n + \frac{n^2}{2} - \frac{n}{2}$

$\frac{n^2}{2} - \frac{25n}{2} + 50 = 0$

$$\frac{n^2}{2} - \frac{20n}{2} - \frac{5n}{2} + 50 = 0$$

$$\frac{n}{2}[n-20] - \frac{5}{2}[n-20] = 0$$

$$\left[\frac{n}{2} - \frac{5}{2}\right](n-20) = 0$$

$$\frac{n}{2} - \frac{5}{2} = 0 \qquad n - 20 = 0$$

$$n = 5 \qquad n = 20$$

If $n = 5 \Rightarrow$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{5}{2}[2 \times -6 + (5-1)\frac{1}{2}]$$

$$= \frac{5}{2}\left[-12 + \frac{5}{2} - \frac{1}{2}\right]$$

$$= -\frac{60}{2} + \frac{25}{4} - \frac{5}{4}$$

$$= -\frac{120 + 25 - 5}{4}$$

$$= -\frac{125 + 25}{4}$$

$$= -\frac{100}{4}$$

$$= -25.$$

If $n = 20 \Rightarrow$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{20}{2}[2 \times -6 + (20-1)\frac{1}{2}]$$

$$= \frac{20}{2}\left[-12 + \frac{19}{2}\right]$$

$$= 10\left[\frac{-24+19}{2}\right]$$

$$= 10\left[\frac{-5}{2}\right]$$

$$= -\frac{50}{2}$$

$$= -25$$

26. Find the number of terms in each of the following :

(i) 2 + 4 + 6 + 8 + = –25 = 10100

Sol. Given : $a = 2, d = 2, S_n = 10100$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$10100 = \frac{n}{2}[2 \times 2 + (n-1)2]$$

$$10100 \times 2 = 4n + 2n^2 - 2n$$

$$2n^2 + 2n - 20200 = 0$$

$$n^2 + n - 10100 = 0$$

$$n^2 + 101n - 100n - 10100$$

$$n(n+101) - 100(n+101)$$

$$(n+101)(n-100)$$

$$n = -101 \text{ or } 100$$

n can not be negative

So, $n = 100.$

(ii) – 1.0 – 1.5 – 2.0 – 2.5 ... = – 27

Sol. Given : $a = -1, d = -0.5,$

$S_n = -27$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$-27 = \frac{n}{2}[2 \times -1 + (n-1) - 0.5]$$

$$-27 \times 2 = -2n - 0.5n^2 + 0.5n$$

$$0.5n^2 + 1.5n - 54 = 0$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-1.5 \pm \sqrt{(1.5)^2 - 4 \times 0.5 \times -54}}{2 \times 0.5}$$

Here $a = 0.5$

$b = 1.5$

$c = -54$

$$n = \frac{1.5 \pm \sqrt{2.25 + 108}}{1}$$

$$n = 1.5 \pm \sqrt{110.25}$$

$$n = 1.5 \pm 10.5$$

$n = 9$ or -12; n can not be negative

$\therefore \quad n = 9.$

(iii) $-1 + \frac{1}{4} + \frac{3}{2} + ... = 3969$

Sol. Given : $a = -1, d = \frac{5}{4}, S_n = 3969$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$3969 = \frac{n}{2}[2 \times -1 + (n-1)\frac{5}{4}]$$

$$3969 \times 2 = -2n + \frac{5}{4}n^2 - \frac{5}{4}n$$

$$3969 \times 2 = -\frac{8n + 5n^2 - 5n}{4}$$

$$31752 = 5n^2 - 13n$$

$$-5n^2 + 13n + 31752 = 0$$

Here $a = -5$

$b = 13$

$c = 31752$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-13 \pm \sqrt{(13)^2 - 4 \times -5 \times 31752}}{2 \times -5}$$

$$= \frac{-13 \pm \sqrt{169 + 635040}}{-10}$$

$$= \frac{-13 \pm \sqrt{635209}}{-10}$$

$$= \frac{-13 \pm 797}{-10}$$

$$n = \frac{-13 + 797}{-10}$$

$$n = \frac{784}{-10}$$

$$n = n = -78{\cdot}4.$$

$$n = \frac{-13 - 797}{-10}$$

$$= \frac{-810}{-10}$$

$$n = 81.$$

The value of n can not be negative or in decimal

$\therefore \quad n = 81.$

(iv) $(x + y) + (x - y) + (x - 3y) + ... = 22(x - 20y)$

Sol. $\underbrace{(x + y) + (x - y) + (x - 3y) + ...}_{\text{LHS}} = \underbrace{22(x - 20y)}_{\text{RHS}}$

Here total value of RHS = 22

$\therefore$ There must be 22 bracket in LHS.

Hence from the both side we get

$n = 22.$

27. The fourth term of an A.P. is 11 and the eighth term exceeds twice the fourth term by 5. Find the A.P. and sum of first 20 terms.

Sol.

$a_4 = 11$

$a + 3d = 11 \quad ...(1)$

$a_8 = 2a_4 + 5$

$a + 7d = 2(a + 3d) + 5$

$a + 7d = 2(11) + 5$

$a + 7d = 22 + 5$

$a + 7d = 27 \quad ...(2)$

On subtracting equation (2) from equation (1) we get–

$4d = 16$

$d = 4$

On putting the value of d in equation (1), we get–

$a + 3 \times 4 = 11$

$a + 12 = 11$

$a = 11 - 12$

$a = -1$

Hence, A. P. $= -1, 3, 7, 11...$

$$S_{20} = \frac{20}{2}[2a + (20-1)d]$$

$= 10[2 \times -1 + 19 \times 4]$

$= 10[-2 + 76]$

$= 10 \times 74$

$= 740.$

28. How many terms of the sequence –12, –9, –6, –3... must be taken to make the sum 54 ?

Sol. Given : $a = -12, d = 3, S_n = 54$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$54 = \frac{n}{2}[2\times -12 + (n-1)3]$$

$$54 \times 2 = -24n + 3n^2 - 3n$$

$$-3n^2 + 27n + 108 = 0$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here $a = -3$
$b = 27$
$c = 108$

$$n = \frac{-27 \pm \sqrt{(27)^2 - 4\times -3 \times 108}}{2\times -3}$$

$$= \frac{-27 \pm \sqrt{729 + 1296}}{-6}$$

$$= \frac{-27 \pm \sqrt{2025}}{-6}$$

$$= \frac{-27 \pm 45}{-6}$$

$$= \frac{-27 + 45}{-6}$$

$$= \frac{18}{-6}$$

$$n = -3$$

$$n = \frac{-27 - 45}{-6}$$

$$n = \frac{72}{6}$$

$n = 12.$
$n = 12$ or $-3.$
n can not be negative
$\therefore \quad n = 12.$

29. Find the sum of first *n* natural numbers.

Sol. First n natural numbers = 1, 2, 3, 4, 5 ... n
Sum of first n natural numbers (S) = 1 + 2 + 3 + 4 + 5 ... n
It is an A. P. in which
$a = 1, d = 1, l = n, n = n$

$$S_n = \frac{n}{2}[a + l]$$

$$S_n = \frac{n}{2}(1 + n)$$

$$\therefore \quad S_n = \frac{n(n+1)}{2}.$$

30. Find the sum of first *n* even natural numbers.

Sol. First n even numbers = 2, 4, 6, 8, 10 ... n
Sum of first n even numbers (S) = 2 + 4 + 6 + 8 + 10 + ... + $2n$
This is in the form of A. P. where.
$a = 2, d = 2, l = 2n, n = n$

$$S_n = \frac{n}{2}[a + l]$$

$$S_n = \frac{n}{2}[2 + 2n]$$

$$S_n = \frac{2n}{2}(n + 1)$$

$$S_n = n(n + 1).$$

31. Find the sum of all even numbers between 200 to 500.

Sol. Even numbers are–2, 4, 6, 8, 10 ...
Even numbers between 200 to 500 are–
202, 204, 206 ... 496, 498
sum of even numbers between 200 to 500 :
202 + 204 + 206 ... 496 + 498
This term is in A.P. where :

$$a = 202, d = 2, l = 498 = a_n$$

$$a_n = a + (n-1)d$$

$$498 = 202 + (n-1)2$$

$$\frac{498 - 202}{2} = n - 1$$

$$148 = n - 1$$

$$n = 148 + 1 = 149$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{149}{2}[2 \times 202 +$$

$$= \frac{149}{2}[404 + (148)2]$$

$= \frac{149}{2}\ [404 + 296]$

$= \frac{149}{2}\ [700]$

$S_n = 52150.$

32. The sum of n terms of a progression is $3n^2 + 4n$. Is this progression an A.P. ? If so, find the A. P. and the sum of its r^{th} term.

Sol. Given : $S_n = 3n^2 + 4n$

$S_1 = 3 \times 1^2 + 4 \times 1 = 7$

$\therefore$ 1^{st} term $= 7 \Rightarrow$

$S_2 = 3 \times 2^2 + 4 \times 2 = 20$

$\therefore$ 2^{nd} term $\Rightarrow 20 - 7 = 13$

$S_3 = 3 \times 3^2 + 4 \times 3 = 39$

3^{rd} term $\Rightarrow 39 - 20 = 19$

$S_4 = 3 \times 4^2 + 4 \times 4 = 64$

4^{th} term $\Rightarrow 64 - 39 = 25$

Hence, progression = 7, 13, 19, 25 ...

This is an A.P.

Sum of r^{th} terms= $S_r = 3 \times r^2 + 4r$
$= 3r^2 + 4r.$

33. If the roots of the equation $(b - c)\ x^2 + (c - a)\ x + (a - b) = 0$ are equal, then show that a, b, c are in A. P.

Sol. If linear equation have equal roots then :

$D = b^2 - 4ac$

$= (c - a)^2 - 4\ (b - c)\ (a - b) = 0$

$= c^2 + a^2 - 2ac - 4bc + 4b^2 + 4ac - 4bc = 0$

$= c^2 + a^2 + 4b^2 + 2ac - 4ab - 4bc = 0$

$= (a)^2 + (2b)^2 + (c)^2 + 2(a)\ (-2b) + 2\ (-2b)\ (c) + 2(a)\ (c) = 0$

$= (a - 2b + c) = 0$

$= -2b = -a - c$

$= 2b = a + c$

Hence, a, b, c in an A.P.

34. Determine, the sum of first 35 terms of an A.P., if second term is 2 and the seventh term is 22.

Sol. Given :

$a_2 = 2 \qquad a + d = 2 \qquad ...(1)$

$a_7 = 22 \qquad a + 6d = 22 \qquad ...(2)$

On subtracting equation (1) from equation (2) we get :

$5d = 20$

$d = 4$

On putting the value of d in equation (1) we get

$a + 4 = 2$

$a = -2$

Hence, $a = -2$, $d = 4$, $n = 35$

$S_n = \frac{n}{2}\ [2a + (n - 1)\ d]$

$S_{35} = \frac{35}{2}\ [2 \times -2 + (35 - 1)\ 4]$

$= \frac{35}{2}\ [-4 + (34)\ 4]$

$= \frac{35}{2}\ [-4 + 136]$

$= \frac{35}{2}\ [132]$

$S_{35} = 2310.$

35. The sum of first 7 terms of an A.P. is 10 and that of next 7 terms is 27. Find the progression.

Sol. Let first term $= a$

Common difference $= d$

$S_n = \frac{n}{2}\ [2a + (n - 1)\ d]$

In first seven terms :

$S_7 = \frac{7}{2}\ [2a + (7 - 1)\ d] = 10$

$14a + 42d = 20 \qquad ...(1)$

For next 7 terms it becomes for 14 terms

$S_{14} = \frac{14}{2}\ [2a + (14 - 1)\ d]$
$= 27$

$14a + 91d = 27 \qquad ...(2)$

On subtracting equation (1) from equation (2) we get :

$49d = 7 \qquad d = \frac{1}{7}$

On substituting value of d in equation (1) we get :

$$14a + 42 \times \frac{1}{7} = 20 \quad a = 1$$

Hence, A. P. = $1, 1\frac{1}{7}, 1\frac{2}{7}, 1\frac{3}{7} \ldots$

36. If the first term of an A. P. is 2 and the sum of first five terms is equal to one–fourth of the sum of the next five terms, find the sum of first 30 terms.

Sol. Given : $a = 2$

$$S_5 = \frac{1}{4}(S_{10} - S_5)$$

$$\frac{5}{2}[2 \times 2 + (5-1)d] = \frac{1}{4}\left[\left\{\frac{10}{2}(2 \times 2 + (10-1)d\right\}\right] - \left[\left\{\frac{5}{2}(2 \times 2 + (5-1)d\right\}\right]$$

$$10 + 10d = \frac{1}{4}[(20 + 45d) - (10 + 10d)]$$

$$4(10 + 10d) = 20 + 45d - 10 - 10d$$
$$40 + 40d = 10 + 35d$$
$$40d - 35d = 10 - 40$$
$$5d = -30$$
$$d = \frac{-30}{5}$$
$$d = -6$$
$$S_{30} = \frac{30}{2}[2 \times 2 + (30-1) - 6]$$
$$S_{30} = -2550.$$

37. If the sum of the first n terms of two A.P.s are in the ratio $(7n - 5) : (5n + 17)$, show that the 6^{th} term of the two progressions are equal.

Sol. Let a, d and A, D be the first term and the common ratio of first and the second A.P. respectively.

So, $$\frac{\left(\frac{n}{2}\right)[2a + (n-1)d]}{\left(\frac{n}{2}\right)[2A + (n-1)D]} = \frac{7n - 5}{5n + 17}$$

$$\frac{2a + (n-1)d}{2A + (n-1)D} = \frac{7(n-1) + 2}{5(n-1) + 22}$$

So, $a = 1, d = 7, A = 11, D = 5$

6^{th} term of 1^{st} AP= $a + 5d = 1 + 5 \times 7 = 36$

6^{th} term of 2^{nd} AP= $A + 5D = 11 + 5 \times 5 = 36$.

38. The ratio between the sum of n terms of two arithmetic progression is $(7n + 1) : (4n + 27)$. Find the ratio of their 11^{th} terms.

Sol. Let a, d and A, D be the first term and the common difference of first and second AP respectively.

So, $$\frac{\left(\frac{n}{2}\right)[2a + (n-1)d]}{\left(\frac{n}{2}\right)[2A + (n-1)D]} = \frac{7n + 1}{4n + 27}$$

$$\frac{2a + (n-1)d}{2A + (n-1)D} = \frac{7(n-1) + 8}{4(n-1) + 31}$$

So, $a = 4, d = 7, A = 15.5, D = 4$

11^{th} term of 1^{st} AP= $a + 10d = 4 + 10 \times 7 = 74$

11^{th} term of 2^{nd} AP = $A + 10D = 15.5 + 10 \times 4 = 55.5$

Ratio = 74 : 55.5
= 148 : 111

39. Two cars start together in the same direction from the same place. The first goes with uniform speed of 10 km/hour. The second goes at a speed of 8 km/hour in the first hour and increases the speed by 1/2 km each succeeding hours. After how many hours will the second car overtake the first, if both cars go non-stop ?

Sol. Let the second car overtakes the first car after n hours.

Hence, Distance covered by first car= Distance covered by second car

$$10n = 8 + (8 + 1/2) + (8 + 2/2) + (8 + 3/2) + ... + (8 + n - 1/2)$$

$$10n = 8n + 1/2\ [1 + 2 + 3 + 4 ... (n - 1)]$$

$$2n = (1/2)\ [n\ (n - 1)/2]$$

$$4n = n\ (n - 1)/2$$

$$8n = n^2 - n$$

$$n^2 - 9n = 0$$

$$n = 9$$

Hence, after 9 hours second car will overtake the first car.

40. Kamal buys a washing machine for ₹ 10,000. He pays ₹ 2,000 in cash and agrees to pay the balance in instalment of ₹ 500 plus 10% interest on the unpaid amount. Find the total amount paid for the washing machine.

Sol. Washing machine cost = ₹ 10,000

Down payment = ₹ 2,000

Rest payment = ₹ 8,000

Now interest on first installment=

$$\frac{P \times R \times T}{100}.$$

$$= 8,000 \times 10 \times 1/100$$

$$= ₹\ 800$$

unpaid amount = 8,000 – 500 = 7,500

Interest on second installment = 7,500 × 10 × 1/100

= ₹ 750

unpaid amount = 7,500 – 500 = 7,000

This is in form of A. P.

800, 750, 700 ...

Where a = 800, d = – 50, n = 16

$$S_n = \frac{n}{2}\ [2a + (n - 1)\ d]$$

$$S_{16} = \frac{16}{2}\ [2 \times 800 + (16 - 1) - 50]$$

$$S_{16} = 6,800$$

Total amount of washing machine = 10,000 + 6,800 = 16,800.

41. Ruchi saves ₹ 120 during first month, ₹ 150 in next month, ₹ 180 in the third month. If she continues her saving in this pattern, In how many months will she saves ₹ 1,800 ?

Sol. Saving = 120, 150, 180 ...

This form in A.P. where,

$$a = 120, d = 30,$$

$$S_n = 1,800$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$1,800 = \frac{n}{2}[2 \times 120 + (n-1)30]$$

$$3,600 = 240n + 30n^2 - 30n$$

$$30n^2 + 210n - 3,600 = 0$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-210 \pm \sqrt{(210)^2 - 4 \times 30 - 3,600}}{2 \times 30}$$

Here, a = 30

b = 210

c = – 3600

$$n = \frac{-210 \pm \sqrt{44100 + 432000}}{60}$$

$$n = \frac{-210 \pm \sqrt{476100}}{60}$$

$$n = \frac{-210 \pm 690}{60}$$

$$n = \frac{-210 + 690}{60}$$

$$n = \frac{480}{60}$$

$$n = 8$$

or, $$n = \frac{-210 - 690}{60}$$

$$n = -\frac{900}{60}$$

$$n = -15.$$

n = 8 or – 15

n can not be negative. Hence n = 8 therefore in 8 months she saves ₹ 1,800.

42. Shri kant saves ₹ 32,000 during first year, ₹ 36,000 in the next year and ₹ 40,000 in the 3rd year. If he continues his savings in this pattern. In how many years will he aves ₹ 2,00,000 ?

Sol. Saving = 32,000; 36,000; 40,000 ...

This form is in A.P., where

$a = 32{,}000, d = 4{,}000, S_n = 2{,}00{,}000$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$2{,}00{,}000 = \frac{n}{2}[2\times 32{,}000+(n-1)4{,}000]$$

$$4{,}00{,}000 = 64{,}000n + 4{,}000n^2 - 4{,}000n$$

$$4{,}000n^2 + 60{,}000n - 4{,}00{,}000 = 0$$

$$n = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

Here, $a = 4{,}000$

$b = 60{,}000$

$c = -4{,}00{,}000$

$$n = \frac{-60{,}000 \pm \sqrt{(60{,}000)^2 - 4\times 4{,}000 \times -4{,}00{,}000}}{2\times 4{,}000}$$

$$n = 5 \text{ or } -20$$

n can not be negative. Hence $n = 5$ therefore in 5 years he saves ₹ 2,00,000.

43. A manufacturer of radio sets, produced 600 units in the third year and 700 units in the seventh year. Assuming that the productioin uniformly increases by a fixed number every year, Find (i) the production in first year (ii) the total production in 7 year and (iii) the production in the 10th year.

Sol. Given that the production uniformly increases by fixed number every year. So, the production in each year forms an A.P.

Let the A.P. be a, $a + d$, $a + 2d$ ....

Production in third year $= a_3 = a + 2d = 600$ ...(i)

Production in seventh year $= a_7 = a + 6d = 700$ ...(ii)

on subtracting equation (1) from equation (2) we get :

$$4d = 100; d = 25$$

on substituting the value of d in equation (i) we get :

$$a = 600 - 50 = 550.$$

Thus, production in first year is 550 units.

Total production in 7 years

$$= S_7 = \frac{n}{2}[2a+(n-1)d]$$

$$= \frac{7}{2}[2\times 550+(7-1)25]$$

$$= \frac{7}{2}(1100+150)$$

$$= 4375.$$

Thus total production in 7 years is 4375 units.

Production in 10th year $= a_{10} = a + 9d$

$= 550 + 9 \times 25$

$= 775$ units.

44. Ritu takes a contract to construct a dispensary upto 31st December 2005. Beyond 31 Dec. 2005 a penalty for delay of construction as follows : ₹ 100 for first day, ₹ 150 for second day ₹ 200 for third day etc. How much does a delay of 20 days cost Ritu ?

Sol. Penalty for delay =

₹ 100 for first day, ₹ 150 for second day

₹ 200 for third day .... upto 20 days

This form is in A.P., where

$a = 100, d = 50, n = 20$

$$S_n = \frac{n}{2}[2a(n-1)d]$$

$$S_{20} = \frac{20}{2}[2\times 100+(20-1)50]$$

$$S_{20} = 10(200+950)$$

$$S_{20} = 10 \times 1{,}150$$

$$S_{20} = 11{,}500$$

Hence, cost for 20 days = ₹ 11,500

❑

6 Co-ordinate Geometry

EXERCISE 6.1

Multiple Choice Type Questions

1. The point (– 13, – 14) lies in :

(a) 1st quadrant (b) 2nd quadrant
(c) 3rd quadrant (d) 4th quadrant.

Ans. (c) 3rd quadrant.

2. Point (8, – 8) is in which quadrant?

(a) First quadrant
(b) Second quadrant
(c) Third quadrant
(d) Fourth quadrant.

Ans. (d) Fourth quadrant.

3. The position of the point with abscissa = – 5 and ordinate = + 4 will be :

(a) In the first quadrant
(b) In the second quadrant
(c) In the third quadrant
(d) In the fourth quadrant.

Ans. (b) In the second quadrant.

4. The point (0, – 5) lies on :

(a) *OX* (b) *OX'*
(c) *OY* (d) *OY'*.

Ans. (d) *OY'*.

5. The point (0, 9) lies on :

(a) *OX* (b) *OY*
(c) *OX'* (d) *OY'*.

Ans. (b) *OY*.

6. The point (– 14, 0) lies on :

(a) *OX* (b) *OY*
(c) *OX'* (d) *OY'*.

Ans. (c) *OX'*.

7. A point lies in third quadrant co-ordinate of this point may be :

(a) (5, 5) (b) (5, – 5)
(c) (– 5, – 5) (d) (– 5, 5).

Ans. (c) (– 5, – 5).

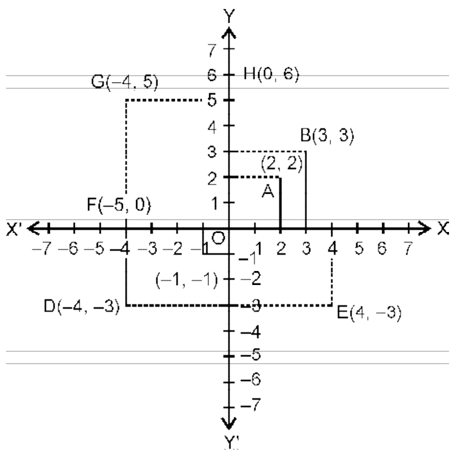
Very Short Answer Type Questions

8. In which quadrant do the following points lie :
(3, 3), (2, 5), (– 5, 2), (– 3, – 4), (4 – 9), (a, b) and (a, – b) ?

Sol. **(i)** Point (3, 3) lies in I quadrant.
(ii) (2, 5) lies in I quadrant.
(iii) (– 5, 2) lies in II quadrant.
(iv) (– 3, – 4) lies in III quadrant.
(v) (4, – 9) lies in IV quadrant.
(vi) (a, b) lies in I quadrant.
(vii) (a, – b) lies in IV quadrant.

9. Show the positions of the following points on the xy-plane :
(2, 2), (3, 3), (– 1, – 1), (– 4, – 3), (4, – 3), and (– 5, 0), (– 4, 5), (0, 6).

Sol. A = (2, 2); B = (3, 3); C = (– 1, – 1); D = (– 4, – 3); E = (4, – 3); F = (– 5, 0); G = (– 4, 5); H = (0, 6).



EXERCISE 6.2

Multiple Choice Type Questions

1. The distance between the points (5, – 3) and (8, 1) is :

(a) 5 units (b) 6 units
(c) 25 units (d) none of these.

Ans. (a) 5 units.

2. The distance between the points (2, 0) and (– 1, 4) is :

(a) 5 units (b) 25 units
(c) – 25 units (d) none of these.

Ans. (a) 5 units.

3. The distance between the points (– 6, 5) and (– 1, 7) is :

(a) 169 units (b) $\sqrt{119}$ units
(c) 13 units (d) none of these.

Ans. (d) none of these.

4. The co-ordinates of a point M are (3, 4). Its distance from the origin is :

(a) 7 units (b) 1 unit
(c) 5 units (d) 12 units.

Ans. (c) 5 units.

5. The distance between points (7, 3) and (– 5, – 2) is :

(a) 10 units (b) 13 units
(c) 15 units (d) 17 units.

Ans. (b) 13 units.

6. The distance between origin and (15, 8) is :

(a) 8 units (b) 15 units
(c) 17 units (d) 23 units.

Ans. (c) 17 units.

7. The distance between (– 1, – 3) and (3, 0) is :

(a) 4 units (b) 5 units
(c) 3 units (d) 6 units.

Ans. (b) 5 units.

8. The distance between the points (– 3, 4) and (0, 0) will be :

(a) 5 units (b) 4 units
(c) 3 units (d) 1 unit.

Ans. (a) 5 units.

9. The third vertex of an equilateral triangle whose other two vertices are (1, 1) and (– 1, – 1) respectively, is :

(a) $(\sqrt{3}, -\sqrt{3})$ (b) $(-\sqrt{3}, \sqrt{3})$
(c) both (a) and (b)
(d) none of these.

Ans. (c) both (a) and (b).

Short Answer Type Questions

10. Find the distance between the following pairs of points :

(i) (7, – 3); (– 5, 2)
(ii) (3, 7); (– 1, 4)
(iii) (– 6, – 5); (– 1, 7)
(iv) (2, 0); (– 1, 4)
(v) $(a\sin\theta, a\cos\theta); (a\cos\theta, -a\sin\theta)$
(vi) $(am_1^2, 2am_1); (am_2^2, 2am_2)$.

Sol. (i) A (7, – 3); B (– 5, 2)

Here, $x_1 = 7, y_1 = -3,$
$x_2 = -5$ and $y_2 = 2$

$$\therefore \quad AB = \sqrt{(-5-7)^2 + (2+3)^2}$$
$$= \sqrt{144 + 25}$$
$$= \sqrt{169} = 13 \text{ units.}$$ **Ans.**

(ii) $A = (3, 7); B = (-1, 4)$

Here, $x_1 = 3, y_1 = 7, x_2 = -1$ and $y_2 = 4$

$$AB = \sqrt{(-1-3)^2 + (4-7)^2}$$
$$= \sqrt{16+9} = \sqrt{25} = 5 \text{ units.}$$ **Ans.**

(iii) A (– 6, – 5); B (– 1, 7)

$$AB = \sqrt{(-1+6)^2 + (7+5)^2}$$
$$= \sqrt{25 + 144}$$
$$= \sqrt{169} = 13 \text{ units.}$$ **Ans.**

(iv) A (2, 0); B (– 1, 4)

$$\therefore \quad AB = \sqrt{(-1-2)^2 + (4-0)^2}$$
$$= \sqrt{9+16} = \sqrt{25}$$
$$= 5 \text{ units.}$$ **Ans.**

(v) $A = (a \sin\theta, a\cos\theta)$;
$B = (a\cos\theta, -a\sin\theta)$

$$AB = \sqrt{(a\cos\theta - a\sin\theta)^2 + (-a\sin\theta - a\cos\theta)^2}$$

$$= \sqrt{a^2(\cos^2\theta + \sin^2\theta - 2\cos\theta.\sin\theta) + a^2(\sin^2\theta + \cos^2\theta + 2\cos\theta\sin\theta)}$$

$$= \sqrt{a^2(1+1)} = a\sqrt{2} \text{ units.}$$ **Ans.**

(vi) $A = (am_1^2, 2am_1); B = (am_2^2, 2am_2)$

$$AB = \sqrt{(am_2^2 - am_1^2)^2 + (2am_2 - 2am_1)^2}$$

$$= \sqrt{a^2(m_2^2 - m_1^2)^2 + 4a^2(m_2 - m_1)^2}$$

$$= \sqrt{a^2(m_2+m_1)^2(m_2-m_1)^2 + 4a^2(m_2-m_1)^2}$$

$$= \sqrt{a^2(m_2-m_1)^2\,[(m_2+m_1)^2 + 4]}$$

$= a\ (m_2 - m_1)\ \sqrt{(m_2 + m_1)^2 + 4}\,.$ **Ans.**

11. (i) If the distance between the points (5, 3) and (*x*, – 1) is 5 units, find the value of *x*.
(ii) If A is (4, 2), B is (1, *y*) find the possible values of y so that AB = 5.

Sol. **(i)** Let A (5, 3) and B (x, – 1). The distance between AB = 5 units.
Here, $x_1 = 5$, $y_1 = 3$, $x_2 = x$ and $y_2 = -1$

$$\therefore\ AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{or}\quad 5 = \sqrt{(x-5)^2 + (-1-3)^2}$$

Squaring both sides, we have

$$(5)^2 = (x - 5)^2 + (-4)^2$$

or $25 = (x - 5)^2 + 16$
or $(x - 5)^2 = 25 - 16 = 9$
or $x - 5 = \pm 3.$
$\therefore$ Either $x = 8$ or $x = 2$. **Ans.**

(ii) Here A (4, 2) and B (1, y).
$AB = 5$, $x_1 = 4$, $y_1 = 2$, $x_2 = 1$ and $y_2 = y$.

$$\therefore\ AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{or}\quad 5 = \sqrt{(1-4)^2 + (y-2)^2}$$

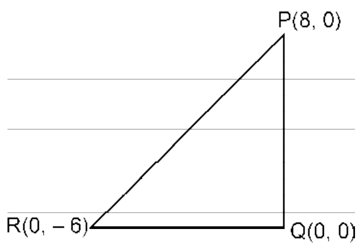
Squaring both sides, we have

$$(5)^2 = (-3)^2 + (y - 2)^2$$

or $(y - 2)^2 = 25 - 9 = 16$
or $y - 2 = \pm 4$
$\therefore$ Either $y = 6$ or $y = -2$. **Ans.**

12. The vertices of a right angled triangle PQR are P (8, 0), Q (0, 0) and R (0, – 6). Find the length of the hypotenuse.

Sol. Length of hypotenuse



$$(PR) = \sqrt{(8-0)^2 + (0+6)^2}$$

$$= \sqrt{64 + 36}$$

$$= 10 \text{ units.}$$

Long Answer Type Questions

13. Show that the four points whose coordinates are (2, – 2), (8, 4), (5, 7) and (– 1, 1) form a rectangle.

Sol. Let A (– 1, 1), B (5, 7), C (8, 4) and D (2, – 2)

$$AB = \sqrt{(5+1)^2 + (7-1)^2} = \sqrt{(6)^2 + (6)^2} = \sqrt{36+36} = \sqrt{72}$$

$$BC = \sqrt{(8-5)^2 + (4-7)^2} = \sqrt{(3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18}$$

$$CD = \sqrt{(2-8)^2 + (-2-4)^2} = \sqrt{(-6)^2 + (-6)^2} = \sqrt{36+36} = \sqrt{72}\,.$$

$$AD = \sqrt{(2+1)^2 + (-2-1)^2} = \sqrt{(3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18}\,.$$

Thus $AB = CD$ and $BC = AD$
Also diagonal (AC)

$$= \sqrt{(8+1)^2 + (4-1)^2} = \sqrt{9^2 + 3^2} = \sqrt{81+9} = \sqrt{90}\,.$$

and diagonal (BD)

$$= \sqrt{(2-5)^2 + (-2-7)^2} = \sqrt{(-3)^2 + (-9)^2} = \sqrt{9+81} = \sqrt{90}\,.$$

Thus diagonal (AC) = diagonal (BD)
Since $AB = CD$; $BC = AD$ and diagonal AC = diagonal BD, therefore $ABCD$ is a rectangle.

Proved.

14. Show that the following four points form a parallelogram : (– 2, – 1), (1, 0), (4, 3) and (1, 2).

Sol. Let A (– 2, – 1); B (1, 0); C (4, 3) and D (1, 2)

$$\therefore \quad AB = \sqrt{(-2-1)^2 + (-1-0)^2} = \sqrt{9+1} = \sqrt{10}\,.$$

$$BC = \sqrt{(4-1)^2 + (3-0)^2} = \sqrt{9+9} = \sqrt{18}\,.$$

$$CD = \sqrt{(4-1)^2 + (3-2)^2} = \sqrt{9+1} = \sqrt{10}\,.$$

$$AD = \sqrt{(-2-1)^2 + (-1-2)^2} = \sqrt{9+9} = \sqrt{18}\,.$$

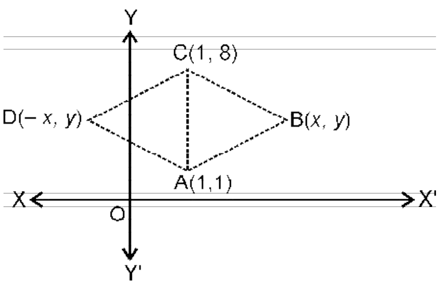
$\because$ $AB = CD$ and $BC = AD$.

$\therefore$ $ABCD$ is a parallelogram.

Proved.

15. Find the other two vertices of that square whose opposite vertices are (1, 1) and (1, 8).

Sol. Let the coordinates of the vertices of the square $ABCD$ be
$A = (1, 1)$; $B = (x, y)$; $C = (1, 8)$ and $D = (-x, y)$



Now $AC^2 = (1-1)^2 + (1-8)^2$
$= 0^2 + 7^2 = 49.$

Let the side of a square be a. Then
$AB^2 + BC^2 = AC^2$
$\Rightarrow a^2 + a^2 = 49$
or $2a^2 = 49$

or $a = \dfrac{7}{\sqrt{2}}$

Now $AB = \sqrt{(x-1)^2 + (y-1)^2}$

$= \dfrac{7}{\sqrt{2}}$ ...(i)

and $BC = \sqrt{(x-1)^2 + (y-8)^2}$

$= \dfrac{7}{\sqrt{2}}$

$\because$ $AB = BC$
$\therefore (x-1)^2 + (y-1)^2$
$= (x-1)^2 + (y-8)^2$
or $(y-1)^2 = (y-8)^2$
or $y^2 + 1 - 2y = y^2 + 64 - 16y$
or $-2y + 16y = 64 - 1$
or $14y = 63$

or $y = \dfrac{63}{14} = \dfrac{9}{2}$.

Substituting the value of y in equation (i), we have

$$(x-1)^2 + (y-1)^2 = \frac{49}{2}$$

or $(x-1)^2 + \left(\dfrac{9}{2} - 1\right)^2 = \dfrac{49}{2}$

or $(x-1)^2 + \left(\dfrac{7}{2}\right)^2 = \dfrac{49}{2}$

or $(x-1)^2 = \dfrac{49}{2} - \dfrac{49}{4} = \dfrac{49}{4}$

or $x - 1 = \dfrac{7}{2}$

or $x = \dfrac{7}{2} + 1 = \dfrac{9}{2}$

Thus, coordinates of B are $\left(\dfrac{9}{2}, \dfrac{9}{2}\right)$.

Ans.

Similarly,
$AD^2 = (1+x)^2 + (1-y)^2$

$= \dfrac{49}{2}$ ...(ii)

and $CD^2 = (1+x)^2 + (8-y)^2$

$= \dfrac{49}{2}$

$\because$ $AD = CD$
$\therefore (1+x)^2 + (1-y)^2$
$= (1+x)^2 + (8-y)^2$
or $(1-y)^2 = (8-y)^2$
or $1 + y^2 - 2y = 64 + y^2 - 16y$
or $63 - 14y = 0$

or $y = \frac{63}{14} = \frac{9}{2}.$

Substituting $y = \frac{9}{2}$ in (ii), we have

$$(1 + x)^2 + \left(1 - \frac{9}{2}\right)^2 = \frac{49}{2}$$

or $(1 + x)^2 + \left(-\frac{7}{2}\right)^2 = \frac{49}{2}$

or $(1 + x)^2 = \frac{49}{2} - \frac{49}{4} = \frac{49}{4}$

or $1 + x = \frac{7}{2}$

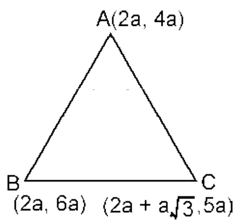
or $x = \frac{7}{2} - 1 = \frac{5}{2}$

$\therefore$ Co-ordinates of D are $(-x, y)$,

i.e., $\left(-\frac{5}{2}, \frac{9}{2}\right)$ **Ans.**

16. Prove that (2*a*, 4*a*), (2*a*, 6*a*) and ($2a + a\sqrt{3}$, 5*a*) are vertices of an equilateral triangle.

Sol. Let the co-ordinates of the vertices of a triangle ABC be A $(2a, 4a)$; B $(2a, 6a)$; C $(2a + a\sqrt{3}, 5a)$



$$AB = \sqrt{(2a - 2a)^2 + (6a - 4a)^2}$$

$$= \sqrt{0 + (2a)^2} = 2a \text{ units.}$$

$$BC = \sqrt{(2a + a\sqrt{3} - 2a)^2 + (5a - 6a)^2}$$

$$= \sqrt{3a^2 + a^2} = 2a \text{ units.}$$

$$CA = \sqrt{(2a - 2a - a\sqrt{3})^2 + (4a - 5a)^2}$$

$$= \sqrt{3a^2 + a^2} = 2a \text{ units.}$$

$\therefore$ $AB = BC = CA$

So, ΔABC is a equilateral triangle.

Proved.

17. Find the third vertex of an equilateral triangle whose other two vertices are (1, 1) and (– 1, – 1).

Sol. Let the third vertex of an equilateral triangle be (x, y).

$A = (x, y)$; $B = (1, 1)$ and $C = (-1, -1)$

$\therefore$ $AB = BC = AC$

Now, $AB^2 = (x - 1)^2 + (y - 1)^2$

$AC^2 = (x + 1)^2 + (y + 1)^2$

and $BC^2 = (-1 - 1)^2 + (-1 - 1)^2$

$= 2^2 + 2^2 = 4 + 4 = 8$

$\therefore$ $(x - 1)^2 + (y - 1)^2 = 8$, $[\because AB = BC]$

or $x^2 + 1 - 2x + y^2 + 1 - 2y = 8$

or $x^2 + y^2 - 2x - 2y = 6$ ...(i)

Also $(x + 1)^2 + (y + 1)^2 = 8$, $[\because AC = BC]$

or $x^2 + 1 + 2x + y^2 + 1 + 2y = 8$

or $x^2 + y^2 + 2x + 2y = 6$ ...(ii)

Subtracting (i) from (ii), we get

$4x + 4y = 0$

or $x = -y$

Substituting $x = -y$ in (i), we get

$\Rightarrow$ $y^2 + y^2 + 2y - 2y = 6$

$2y^2 = 6$

or $y^2 = 3$

or $y = \pm\sqrt{3}$

$\therefore$ $x = \mp\sqrt{3}$

$\therefore$ Co-ordinates of A are $(-\sqrt{3}, \sqrt{3})$ and $(\sqrt{3}, -\sqrt{3})$. **Ans.**

18. For what values of *x* and *y* will the points (0, 0), $(3, \sqrt{3})$ and (*x*, *y*) form an equilateral triangle ?

Sol. Let $A = (x, y)$, $B = (0, 0)$ and $C = (3, \sqrt{3})$

$$AB^2 = \left[\sqrt{(x - 0)^2 + (y - 0)^2}\right]^2$$

$$= \left[\sqrt{x^2 + y^2}\right]^2 = x^2 + y^2$$

$$BC^2 = \left[\sqrt{(3-0)^2 + (\sqrt{3}-0)^2}\right]^2$$

$$= \left[\sqrt{9+3}\right]^2 = \left[\sqrt{12}\right]^2 = 12$$

$$AC^2 = \left[\sqrt{(x-3)^2 + (y-\sqrt{3})^2}\right]^2$$

$$= (x-3)^2 + (y-\sqrt{3})^2$$

$\because$ ABC is an equilateral triangle,

$\therefore$ $AB = BC = AC$

$\Rightarrow$ $x^2 + y^2 = 12$ ...(i)

and $(x-3)^2 + (y-\sqrt{3})^2 = 12$

or $x^2 + 9 - 6x + y^2 + 3 - 2\sqrt{3}y = 12$

or $x^2 + y^2 - 6x - 2\sqrt{3}y = 0$ ...(ii)

$\therefore$ From (i) and (ii), we get

$12 - 6x - 2\sqrt{3}y = 0$ $[\because x^2 + y^2 = 12]$

or $2\sqrt{3}y = 12 - 6x$

or $y = \dfrac{12-6x}{2\sqrt{3}} = \dfrac{6-3x}{\sqrt{3}}$

$\therefore$ $y^2 = \dfrac{(6-3x)^2}{3} = 3(2-x)^2$

$\because$ $x^2 + y^2 = 12$

$\therefore$ $x^2 + 3(2-x)^2 = 12$

or $x^2 + 3(4 + x^2 - 4x) = 12$

or $x^2 + 12 + 3x^2 - 12x = 12$

or $4x^2 - 12x = 0$

or $4x(x-3) = 0$

$\therefore$ Either $x = 0$ or $x = 3$

$\therefore$ $y = \dfrac{6-3(0)}{\sqrt{3}}$ or $y = \dfrac{6-9}{\sqrt{3}}$

$= \dfrac{6}{\sqrt{3}} = 2\sqrt{3}$ or $= -\dfrac{3}{\sqrt{3}} = -\sqrt{3}$

$\therefore$ The points are $(0, 2\sqrt{3})$ and $(3, -\sqrt{3})$. **Ans.**

19. Prove that the point P (2, 2) is the centre of the circle passing through the points A (1, 2), B (2, 1) and C (2, 3).

Sol. Let P (2, 2) be the centre of the circle and the points A (1, 2), B (2, 1), C (2, 3) be the points on the circle.

$\therefore$ $PA = \sqrt{(2-1)^2 + (2-2)^2}$

$= \sqrt{1^2 + 0^2} = 1$

$PB = \sqrt{(2-2)^2 + (2-1)^2}$

$= \sqrt{0^2 + 1^2} = 1$

and $PC = \sqrt{(2-2)^2 + (3-2)^2}$

$= \sqrt{0^2 + 1^2} = 1$

$\because$ $PA = PB = PC$,

$\therefore$ PA, PB, PC are the radii of the circle. Hence, P is the centre of the given circle. **Proved.**

20. If (*x*, *y*) is a point on the circumference of the circle whose centre is (3, – 2) and radius 3 units, prove that $x^2 + y^2 = 6x - 4y - 4$.

Sol. Let $O = (3, -2)$ be the centre of the circle and $P = (x, y)$ be the point on the circumference and $OP = 3$.

$\therefore$ $OP^2 = \left[\sqrt{(x-3)^2 + (y-2)^2}\right]^2$

or $9 = (x-3)^2 + (y+2)^2$, $[\because OP^2 = 3^2 = 9]$

or $9 = x^2 + 9 - 6x + y^2 + 4 + 4y$

or $x^2 + y^2 = 6x - 4y - 4$. **Proved.**

21. Find the centre of the circle on which the points (8, 6), (8, – 2) and (2, – 2) lie.

Sol. Let $O = (x, y)$ be the centre of the circle and $A = (8, 6)$; $B = (8, -2)$ and $C = (2, -2)$

$\therefore$ $OA^2 = (x-8)^2 + (y-6)^2$

$OB^2 = (x-8)^2 + (y+2)^2$

$OC^2 = (x-2)^2 + (y+2)^2$

$\because$ $OA^2 = OB^2$, ($\because$ OA and OB are the radii of the same circle)

$\therefore$ $(x-8)^2 + (y-6)^2 = (x-8)^2 + (y+2)^2$

or $(y-6)^2 = (y+2)^2$

or $y^2 + 36 - 12y = y^2 + 4 + 4y$

or $-16y = -32$
or $y = 2.$
Again $OB^2 = OC^2$
$(x-8)^2+(y+2)^2 = (x-2)^2+(y+2)^2$
or $x^2 + 64 - 16x = x^2 + 4 - 4x$
or $-16x + 4x = 4 - 64$
or $-12x = -60$
$\therefore$ $x = 5.$
$\therefore$ Centre of the circle is (x, y) *i.e.*, (5, 2). **Ans.**

EXERCISE 6.3

1. **The co-ordinates of two points are (6, 0) and (0, 8). The co-ordinates of the mid-point are :**
(a) (3, 4) (b) (6, 8)
(c) (0, 0) (d) (4, 3).

Ans. (a) (3, 4).

2. **The coordinates of two points are (9, 4) and (3, 8). The co-ordinates of the mid-point are :**
(a) (6, 0) (b) (0, 6)
(c) (6, 6) (d) none of these.

Ans. (c) (6, 6).

3. **The co-ordinates of two points are (– 8, 0) and (0, – 8). The coordinates of the mid-point of the line segment joining them will be :**
(a) (– 8, 4) (b) (4, – 8)
(c) (– 4, – 4) (d) (4, 4).

Ans. (c) (– 4, – 4).

4. **The abscissa of the point which divides the line joining points (0, 4) and (0, 8) internally in the ratio of 3 : 1 will be :**
(a) 0 (b) 4
(c) 6 (d) 8.

Ans. (a) 0.

5. **If the vertices of a triangle be (x_1, y_1), (x_2, y_2) and (x_3, y_3), then the co-ordinates of its centroid are :**

(a) $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

(b) $\left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$

(c) $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

(d) none of these.

Ans. (c) $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

6. **The mid-point of the line segment drawn through the points (– 8, 13) and (*x*, 7) is (4, 10). The value of *x* will be :**
(a) 16 (b) 10
(c) 8 (d) 4.

Ans. (a) 16.

7. **The centroid of the triangle whose vertices are (7, 5), (5, 7) and (– 3, 3) is :**
(a) (5, 3) (b) (3, 5)
(c) (– 5, – 3) (d) (– 3, – 5).

Ans. (b) (3, 5).

8. **The co-ordinates of the point which divides the line joining the points (0, 0) and (4, 0) internally in the ratio of 1 : 3 will be :**
(a) (1, 0) (b) (0, 0)
(c) (0, 4) (d) (0, 1).

Ans. (a) (1, 0).

9. **The two vertices of a triangle are (3, 5) and (– 4, – 6); and the co-ordinates of its centroid are (4, 3). Then the co-ordinates of its third vertex are :**
(a) (– 13, 10) (b) (13, – 10)
(c) (– 13, – 10) (d) (13, 10).

Ans. (d) (13, 10).

10. **The co-ordinates of the point which divides the lines segment joining the points (3, 5) and (7, 9) internally in the ratio 2 : 3 are :**

(a) $\left(\frac{23}{5}, \frac{33}{5}\right)$ (b) $\left(\frac{5}{23}, \frac{5}{33}\right)$

(c) $\left(\frac{33}{5}, \frac{23}{5}\right)$ (d) none of these.

Ans. (a) $\left(\frac{23}{5}, \frac{33}{5}\right)$.

11. **The co-ordinates of the point which divides the line segment joining the points (– 3, – 4) and**

(2, 1) externally in the ratio 3 : 2, are :

(a) (11, 12) (b) (12, 11)

(c) (– 11, – 12) (d) (– 12, – 11).

Ans. (b) (12, 11).

Very Short Answer Type Questions

12. Find the co-ordinates of the point P which divides internally the join of :

(i) A (8, 9) and B (– 7, 4) in ratio 2 : 3

(ii) A (1, 2) and B (3, 4) in ratio 5 : 7

(iii) A (2, 0) and B (0, 4) in ratio 3 : 2

(iv) A (5, 7) and B (4, 5) in ratio 2 : 3.

Sol. (i) Let the co-ordinates of the point P be (x, y), where P divides internally the points joining A (8, 9) and B (– 7, 4) in the ratio of 2 : 3.

$$\therefore \quad x = \frac{mx_2 + nx_1}{m+n}$$

$$\text{and} \quad y = \frac{my_2 + ny_1}{m+n}$$

Here $x_1 = 8$, $y_1 = 9$, $x_2 = -7$ and $y_2 = 4$.

$$x = \frac{2 \times (-7) + 3 \times 8}{2+3}$$

$$= \frac{-14 + 24}{5} = \frac{10}{5} = 2$$

$$\text{and} \quad y = \frac{2 \times 4 + 3 \times 9}{2+3} = \frac{8 + 27}{5}$$

$$= \frac{35}{5} = 7$$

∴ The required point is (2, 7). **Ans.**

(ii) $A = (1, 2)$, $B = (3, 4)$, ratio 5 : 7

$x_1 = 1$, $y_1 = 2$, $x_2 = 3$, $y_2 = 4$, $m = 5$ and $n = 7$

$$\therefore \quad x = \frac{5 \times 3 + 7 \times 1}{5+7} = \frac{15 + 7}{12}$$

$$= \frac{22}{12} = \frac{11}{6}$$

$$\text{and } y = \frac{5 \times 4 + 7 \times 2}{5+7} = \frac{20 + 14}{12}$$

$$= \frac{34}{12} = \frac{17}{6}.$$

∴ The required point is $\left(\frac{11}{6}, \frac{17}{6}\right)$. **Ans.**

(iii) $A = (2, 0)$, $B = (0, 4)$, ratio 3 : 2

$x_1 = 2$, $y_1 = 0$, $x_2 = 0$, $y_2 = 4$, $m = 3$ and $n = 2$

$$\therefore \quad x = \frac{3 \times 0 + 2 \times 2}{3+2} = \frac{0 + 4}{5}$$

$$= \frac{4}{5}$$

$$\text{and } y = \frac{3 \times 4 + 2 \times 0}{3+2} = \frac{12 + 0}{5}$$

$$= \frac{12}{5}$$

∴ The required point is $\left(\frac{4}{5}, \frac{12}{5}\right)$. **Ans.**

(iv) $A = (5, 7)$, $B = (4, 5)$, ratio 2 : 3

$x_1 = 5$, $y_1 = 7$, $x_2 = 4$, $y_2 = 5$, $m = 2$ and $n = 3$.

$$x = \frac{2 \times 4 + 3 \times 5}{2+3} = \frac{8 + 15}{5}$$

$$= \frac{23}{5}$$

$$\text{and } y = \frac{2 \times 5 + 3 \times 7}{2+3} = \frac{10 + 21}{5}$$

$$= \frac{31}{5}.$$

∴ The required point is $\left(\frac{23}{5}, \frac{31}{5}\right)$. **Ans.**

Q. 13. Find the co-ordinates of the point Q which divides externally the join of :

(i) A (3, 4) and B (– 6, 2) in ratio 3 : 2

(ii) A (– 3, – 4) and B (2, 1) in ratio 3 : 2

(iii) A (– 2, 5) and B (6, 4) in ratio 1 : 2

(iv) A (– 4, 0) and B (0, 8) in ratio 3 : 4.

Sol. **(i)** Let the co-ordinates of the point Q be (x, y) which divides externally the points joining A (3, 4) and B (– 6, 2) in the ratio of 3 : 2.

$\therefore$ Co-ordinates of Q are given by

$$x = \frac{mx_2 - nx_1}{m - n}$$

and $$y = \frac{my_2 - ny_1}{m - n}$$

Here, $x_1 = 3$, $y_1 = 4$, $x_2 = -6$, $y_2 = 2$, $m = 3$ and $n = 2$

$$\therefore \quad x = \frac{3 \times (-6) - 2 \times 3}{3 - 2}$$

$$= \frac{-18 - 6}{1} = -24$$

and $$y = \frac{3 \times 2 - 2 \times 4}{3 - 2}$$

$$= \frac{6 - 8}{1} = -2.$$

$\therefore$ The required point is (– 24, – 2). **Ans.**

(ii) $A = (-3, -4)$, $B = (2, 1)$, ratio 3 : 2

Here, $x_1 = -3$, $y_1 = -4$, $x_2 = 2$, $y_2 = 1$, $m = 3$ and $n = 2$

$$\therefore \quad x = \frac{3 \times 2 - 2 \times (-3)}{3 - 2}$$

$$= \frac{6 + 6}{1} = 12$$

and $$y = \frac{3 \times 1 - 2 \times (-4)}{3 - 2}$$

$$= \frac{3 + 8}{1} = 11.$$ **Ans.**

$\therefore$ The required point is (12, 11).

(iii) $A = (-2, 5)$, $B = (6, 4)$, ratio 1 : 2

Here, $x_1 = -2$, $y_1 = 5$, $x_2 = 6$, $y_2 = 4$, $m = 1$ and $n = 2$

$$\therefore \quad x = \frac{1 \times 6 - 2 \times (-2)}{1 - 2}$$

$$= \frac{6 + 4}{-1} = -10$$

and $$y = \frac{1 \times 4 - 2 \times 5}{1 - 2} = \frac{4 - 10}{-1}$$

$$= 6.$$

$\therefore$ The required point is (– 10, 6). **Ans.**

(iv) $A = (-4, 0)$, $B = (0, 8)$, ratio 3 : 4

Here, $x_1 = -4$, $y_1 = 0$, $x_2 = 0$, $y_2 = 8$, $m = 3$ and $n = 4$

$$\therefore \quad x = \frac{3 \times 0 - 4 \times (-4)}{3 - 4}$$

$$= \frac{0 + 16}{-1} = -16$$

and $$y = \frac{3 \times 8 - 4 \times 0}{3 - 4} = \frac{24 - 0}{-1}$$

$$= -24.$$

$\therefore$ The required point is (– 16, – 24). **Ans.**

14. In what ratio does the x-axis cut the line segment joining (5, 8) and (7, – 3) ?

Sol. Let the x-axis cut the line AB, where $A = (5, 8)$ and $B = (7, -3)$ in the ratio of $m : n$.

Any line segment through (5, 8) and (7, – 3) will cut the x-axis at the point where $y = 0$.

$$\therefore \quad y = \frac{my_2 + ny_1}{m + n}$$

$$\Rightarrow \quad 0 = \frac{m \times (-3) + n \times 8}{m + n}$$

$$\Rightarrow \quad 3m = 8n \Rightarrow \frac{m}{n} = \frac{8}{3}.$$

$\therefore$ The required ratio is 8 : 3 internally. **Ans.**

15. In what ratio does the point (11, 18) divide the line joining (3, 4) and (7, 11) ?

Sol. Let the point (11, 18) divides the line joining (3, 4) and (7, 11) in the ratio of $m : n$.

$$\therefore \quad 11 = \frac{mx_2 + nx_1}{m + n}$$

or $$11 = \frac{m \times 7 + n \times 3}{m + n}$$

[$\because x_1 = 3, x_2 = 7$]

or $11m + 11n = 7m + 3n$

or $4m = -8n$

or $\frac{m}{n} = -\frac{8}{4} = -\frac{2}{1}$.

$\therefore$ The point (11, 18) divides the line in the ratio of 2 : 1 externally. **Ans.**

16. The middle point of the line AB is (13, 19) and A is (– 9, 30). Find the co-ordinates of B.

Sol. Let the co-ordinates of the point B be (x, y).

$\therefore \quad 13 = \frac{x-9}{2}$ or $26 = x - 9$

or $26 + 9 = x$

$\therefore \quad x = 35$

and $19 = \frac{y+30}{2}$ or $38 = y + 30$

or $38 - 30 = y$

or $y = 8$.

$\therefore$ Co-ordinates of B are (35, 8). **Ans.**

Q. 17. Find the centroid of the triangle whose vertices are (4, 6), (2, – 2) and (0, 2).

Sol. Let the co-ordinates of the centroid be (x, y).

$\therefore \quad x = \frac{x_1 + x_2 + x_3}{3}$,

$y = \frac{y_1 + y_2 + y_3}{3}$

Here, $x_1 = 4$, $y_1 = 6$, $x_2 = 2$, $y_2 = -2$, $x_3 = 0$ and $y_3 = 2$.

$\therefore \quad x = \frac{4+2+0}{3} = \frac{6}{3} = 2$

and $y = \frac{6-2+2}{3} = \frac{6}{3} = 2$.

$\therefore$ Co-ordinates of the centroid are (2, 2). **Ans.**

Long Answer Type Questions

18. The end points of a line segment AB are A (*a*, *b*) and B (*b*, *a*), where a and b are both positive. In what ratio the line segment AB is divided by both axes ?

Sol. For *x*-axis :

The co-ordinates of any point on the x-axis are $(x, 0)$.

Let the required ratio be $m_1 : m_2$.

and $y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$

$\Rightarrow \quad 0 = \frac{am_1 + bm_2}{m_1 + m_2}$

$\Rightarrow am_1 + bm_2 = 0$

$\Rightarrow \quad am_1 = -bm_2$

$\therefore \quad \frac{m_1}{m_2} = -\frac{b}{a}$

Hence, the required ratio is $b : a$. The negative sign represents external division.

For *y*-axis :

The co-ordinates of any point on the y-axis are $(0, y)$

and $x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$

$\Rightarrow \quad 0 = \frac{bm_1 + am_2}{m_1 + m_2}$

$\Rightarrow bm_1 + am_2 = 0$

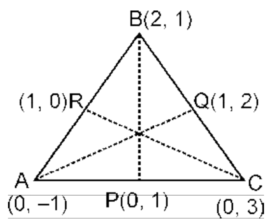
$\Rightarrow \quad bm_1 = -am_2$

$\therefore \quad \frac{m_1}{m_2} = -\frac{a}{b}$

Hence, the required ratio is $a : b$. The negative sign represents external division. **Ans.**

19. Find the lengths of medians of a triangle whose vertices are (0, – 1), (2, 1) and (0, 3).

Sol. The co-ordinates of vertices of a triangle ABC are (0, – 1), (2, 1) and (0, 3) respectively.



Here, $x_1 = 0$, $x_2 = 2$, $x_3 = 0$, $y_1 = -1$, $y_2 = 1$, $y_3 = 3$.

$\therefore$ Co-ordinates of the mid-point P of AC are

$\left(\frac{0+0}{2}, \frac{-1+3}{2}\right)$, *i.e.*, (0, 1) **Ans.**

$\therefore$ Median *BP*

$= \sqrt{(2-0)^2 + (1-1)^2}$

$= \sqrt{4} = 2$ units **Ans.**

Co-ordinates of the mid-point *R* of *AB* are

$\left(\frac{2+0}{2}, \frac{-1+1}{2}\right)$, *i.e.*, (1, 0)

$\therefore$ Median *CR*

$= \sqrt{(1-0)^2 + (0-3)^2}$

$= \sqrt{1^2 + 3^2}$

$= \sqrt{10}$ units. **Ans.**

Co-ordinates of the mid-point *Q* of *BC* are $\left(\frac{2+0}{2}, \frac{1+3}{2}\right)$ *i.e.*, (1, 2)

$\therefore$ Median *AQ*

$= \sqrt{(0-1)^2 + (-1-2)^2}$

$= \sqrt{1^2 + 3^2} = \sqrt{10}$ units. **Ans.**

20. The vertices of a triangle are (5, 1), (1, 5) and (– 3, – 1). Find the lengths of its medians.

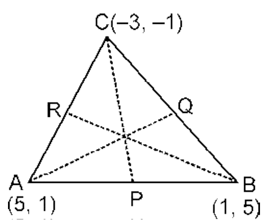
Sol. The co-ordinates of vertices of a triangle *ABC* are respectively, (5, 1), (1, 5) and (– 3, – 1).

Here, $x_1 = 5, y_1 = 1, x_2 = 1, y_2 = 5, x_3 = -3, y_3 = -1$

$\therefore$ Co-ordinates of mid-point *P* of *AB* are

$\left(\frac{5+1}{2}, \frac{1+5}{2}\right)$, *i.e.*, (3, 3)

$\therefore$ Median *CP*



$= \sqrt{(-3-3)^2 + (-1-3)^2}$

$= \sqrt{36+16} = \sqrt{52}$ units. **Ans.**

Co-ordinates of mid-point *Q* of *BC* are $\left(\frac{1-3}{2}, \frac{5-1}{2}\right)$, *i.e.*, (– 1, 2)

$\therefore$ Median *AQ*

$= \sqrt{(5+1)^2 + (1-2)^2}$

$= \sqrt{36+1} = \sqrt{37}$ units. **Ans.**

Co-ordinates of mid-point *R* of *AC* are $\left(\frac{5-3}{2}, \frac{1-1}{2}\right)$, *i.e.*, (1, 0)

$\therefore$ Median *BR*

$= \sqrt{(1-1)^2 + (5-0)^2}$

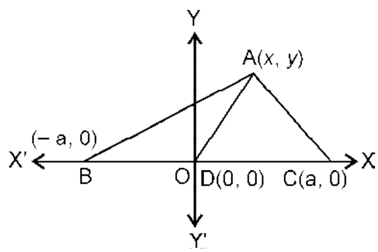
$= \sqrt{0+25} = \sqrt{25} = 5$ units. **Ans.**

21. In ΔABC, D is the mid-point of BC, prove that $AB^2 + CA^2 = 2(AD^2 + DC^2)$.

Sol. In ΔABC, let the co-ordinates of *A*, *B*, *C* be (*x*, *y*), (– *a*, 0), (*a*, 0) respectively.

Hence, Co-ordinates of mid-point *D* of *BC* are

$\left(\frac{-a+a}{2}, \frac{0+0}{2}\right)$ *i.e.*, (0, 0)



$\therefore \quad AB = \sqrt{\{x-(-a)\}^2 + (y-0)^2}$

$= \sqrt{(x+a)^2 + y^2}$

$AC = \sqrt{(x-a)^2 + (y-0)^2}$

$= \sqrt{(x-a)^2 + y^2}$

$AD = \sqrt{(x-0)^2 + (y-0)^2}$

$= \sqrt{x^2 + y^2}$

$CD = \sqrt{(a-0)^2 + (0-0)^2} = a$

$\therefore \quad AB^2 + AC^2$

$= (x + a)^2 + y^2 + (x - a)^2 + y^2$

$= x^2 + a^2 + 2ax + y^2 + x^2 + a^2 - 2ax + y^2$

$= 2(x^2 + a^2 + y^2)$

Now, $AD^2 + CD^2 = (x^2 + y^2) + a^2$

$\therefore \quad 2(AD^2 + CD^2) = 2(x^2 + y^2 + a^2)$

$\therefore \quad AB^2 + AC^2 = 2(AD^2 + CD^2)^2.$

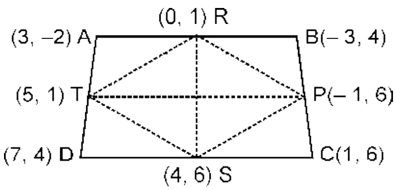
Proved.

22. The vertices of a quadrilateral ABCD are A (3, – 2), B (– 3, 4), C (1, 8) and D (7, 4). Prove that the line joining the mid-points of the sides AB, BC, CD, DA in the same order form a parallelogram.

Sol. The co-ordinates of the vertices of quardilateral are

A (3, – 2), B (– 3, 4), C (1, 8) and D (7, 4).

Here, $x_1 = 3, y_1 = -2, x_2 = -3, y_2 = 4, x_3 = 1, y_3 = 8, x_4 = 7, y_4 = 4.$



Hence, Co-ordinates of the mid-point R of AB are

$$\left(\frac{3-3}{2}, \frac{-2+4}{2}\right), \text{ i.e., } (0, 1)$$

Co-ordinates of mid-point P of BC are

$$\left(\frac{-3+1}{2}, \frac{4+8}{2}\right), \text{ i.e., } (-1, 6)$$

Co-ordinates of mid-point S of CD are

$$\left(\frac{7+1}{2}, \frac{4+8}{2}\right), \text{ i.e., } (4, 6)$$

Co-ordinates of mid-point T of AD are

$$\left(\frac{7+3}{2}, \frac{-2+4}{2}\right), \text{ i.e., } (5, 1)$$

$\therefore$ Co-ordinates of mid-point of TP are

$$\left(\frac{5-1}{2}, \frac{1+6}{2}\right), \text{ i.e., } \left(2, \frac{7}{2}\right)$$

Co-ordinates of mid-point of RS are

$$\left(\frac{0+4}{2}, \frac{1+6}{2}\right), \text{ i.e., } \left(2, \frac{7}{2}\right)$$

Since co-ordinates of the mid-point of the diagonals are the same, hence the diagonals bisect each other. **Proved.**

23. The co-ordinates of the point dividing the line AB in the ratio of 2 : 3 internally are $\left(\frac{1}{5}, \frac{34}{5}\right)$.

If the co-ordinates of A are (3, 6). Find the co-ordinates of B.

Sol. Let the co-ordinate of B be (x_2, y_2)

$\therefore$ According to the question,

$$x = \frac{1}{5}, \qquad y = \frac{34}{5}$$

$$x_1 = 3, \qquad x_2 = x_2$$

$$y_1 = 6, \qquad y_2 = y_2,$$

and $m = 2, \qquad n = 3.$

Now $x = \dfrac{mx_2 + nx_1}{m+n}$

$\Rightarrow \quad \dfrac{1}{5} = \dfrac{2 \times x_2 + 3 \times 3}{2+3}$

or $\quad 1 = 2x_2 + 9$

or $\quad x_2 = \dfrac{-8}{2} = -4$

Again, $y = \dfrac{my_2 + ny_1}{m+n}$

$\Rightarrow \quad \dfrac{34}{5} = \dfrac{2 \times y_2 + 3 \times 6}{2+3}$

$\therefore \quad 34 = 2y_2 + 18$

or $\quad 16 = 2y_2$

or $\quad y_2 = 8.$

$\therefore$ Co-ordinates of B are (x_2, y_2), *i.e.*, (– 4, 8). **Ans.**

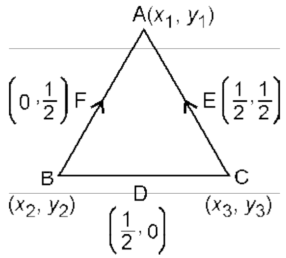
24. The co-ordinates of the mid-points of the sides of a triangle are $\left(0, \frac{1}{2}\right)$, $\left(\frac{1}{2}, \frac{1}{2}\right)$, $\left(\frac{1}{2}, 0\right)$. Find the vertices of the triangle.

Sol. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC. Let $D\left(\frac{1}{2}, 0\right)$, $E\left(\frac{1}{2}, \frac{1}{2}\right)$ and $F\left(0, \frac{1}{2}\right)$ be the mid-points of sides BC, CA and AB respectively.

Since, D is the mid-point of BC

$\therefore \frac{x_2 + x_3}{2} = \frac{1}{2}$ and $\frac{y_2 + y_3}{2} = 0$

$\Rightarrow x_2 + x_3 = 1$ and $y_2 + y_3 = 0$ ...(i)



Similarly, E and F are the mid-points of CA and AB respectively.

$\frac{x_1 + x_3}{2} = \frac{1}{2}$ and $\frac{y_1 + y_3}{2} = \frac{1}{2}$

$\Rightarrow x_1 + x_3 = 1$ and $y_1 + y_3 = 1$ ...(ii)

and $\frac{x_1 + x_2}{2} = 0$ and $\frac{y_1 + y_2}{2} = \frac{1}{2}$

$\Rightarrow x_1 + x_2 = 0$ and $y_1 + y_2 = 1$ ...(iii)

From (i), (ii) and (iii), we get

$(x_2 + x_3) + (x_1 + x_3) + (x_1 + x_2) = 1 + 1 + 0$

$2x_1 + 2x_2 + 2x_3 = 2$

$\therefore x_1 + x_2 + x_3 = 1$ ...(iv)

and $(y_2 + y_3) + (y_1 + y_3) + (y_1 + y_2) = 0 + 1 + 1$

$2y_1 + 2y_2 + 2y_3 = 2$

$\therefore y_1 + y_2 + y_3 = 1$ ...(v)

From (ii) and (iv),

$x_2 + 1 = 1$ and

$y_2 + 1 = 1$

$\therefore x_2 = 0$

$\therefore y_2 = 0$

So, co-ordinate of B are (0, 0)

From (iii) and (iv),

$x_3 + 0 = 1$

and $y_3 + 1 = 1$

$\therefore x_3 = 1$

$\therefore y_3 = 0$

So, co-ordinates of C are (1, 0)

Hence, the vertices of the ΔABC are A (0, 1), B (0, 0) and C (1, 0). **Ans.**

EXERCISE 6.4

Multiple Choice Type Questions

1. Three points are said to be collinear if they lie on a :

(a) line (b) plane
(c) both (a) and (b)
(d) none of these.

Ans. (a) line.

2. The area of the triangle whose vertices are (0, 0), (0, 2) and (2, 0) will be :

(a) 1 (b) 2
(c) 4 (d) 8.

Ans. (b) 2.

3. The area of the triangle whose vertices are (2, 4), (– 3, 7) and (– 4, 5) is :

(a) 7·5 sq. units (b) 7·0 sq. units
(c) 6·5 sq. units (d) none of these.

Ans. (c) 6·5 sq. units.

4. If the area of the triangle whose vertices are (1, – 1), (2, 1) and (4, 5) is :

(a) 0 (b) 1 sq. units
(c) 5 sq. units (d) 7 sq. units.

Ans. (a) 0.

5. If the points (x, y), (2, 3) and (– 3, 4) are collinear, then :

(a) $x + y = 17$ (b) $x - y = 17$
(c) $x - 5y = 17$ (d) $x + 5y = 17$.

Ans. (d) $x + 5y = 17$.

6. If the points (1, 4), (3, y) and (– 3, 16) are collinear, then the value of y is :

(a) 2 (b) – 2
(c) 3 (d) none of these.

Ans. (b) – 2.

7. The area of the quadrilateral whose vertices are (1, 2), (6, 2), (5, 3) and (3, 7) is :

(a) $11\frac{1}{3}$ sq. units (b) $\frac{35}{3}$ sq. units
(c) $11\frac{1}{4}$ sq. units (d) $11\frac{1}{2}$ sq. units.

Ans. (d) $11\frac{1}{2}$ sq. units.

8. The area of the quadrilateral whose vertices are (– 1, 6), (– 3, – 9), (5, – 8) and (3, 9), is :
(a) 94 sq. units (b) 96 sq. units
(c) 48 sq. units (d) none of these.

Ans. (b) 96 sq. units.

9. If the vertices of ΔABC are A (0, 0), B (a, 0), C (0, – a) then area of the triangle will be :
(a) 0 (b) $\frac{1}{2}a^2$
(c) a^2 (d) $2a^2$.

Ans. (b) $\frac{1}{2}a^2$.

10. The equation of the locus of the point, when thrice the distance of the point from y-axis is greater by 7 then the distance of the point from x-axis is :
(a) $3x - y = 7$ (b) $3x + y = 7$
(c) $3x - y + 7 = 0$ (d) none of these.

Ans. (a) $3x - y = 7$.

Very Short Answer Type Questions

11. Find the area of the triangle whose vertices are :
(i) (3, 4), (1, 2), (6, 3)
(ii) (0, 0), (2, 0), (0, 3)
(iii) (2, 0), (11, 6), (– 4, – 4)
(iv) (a, b), (b, c), (c, a).
(v) $\left(at_1, \frac{a}{t_1}\right), \left(at_2, \frac{a}{t_2}\right), \left(at_3, \frac{a}{t_3}\right)$.

Sol. (i) Co-ordinates of the vertices of triangle are (3, 4), (1, 2), (6, 2). Here $x_1 = 3, y_1 = 4, x_2 = 1, y_2 = 2, x_3 = 6$ and $y_3 = 3$.

$$\therefore \text{ Area } = \frac{1}{2}[(x_1y_2 + x_2y_3 + x_3y_1) - (y_1x_2 + y_2x_3 + y_3x_1)]$$
$$= \frac{1}{2}[(3 \times 2 + 1 \times 3 + 6 \times 4) - (4 \times 1 + 2 \times 6 + 3 \times 3)]$$
$$= \frac{1}{2}[(6 + 3 + 24) - (4 + 12 + 9)]$$
$$= \frac{1}{2}[33 - 25] = \frac{8}{2} = 4 \text{ sq. units.}$$
Ans.

(ii) (0, 0), (2, 0), (0, 3)
Here, $x_1 = 0, y_1 = 0, x_2 = 2, y_2 = 0, x_3 = 0$ and $y_3 = 3$.

$$\therefore \text{ Area } = \frac{1}{2}[(0 \times 0 + 2 \times 3 + 0 \times 0) - (0 \times 2 + 0 \times 0 + 3 \times 0)]$$
$$= \frac{1}{2}[(0 + 6 + 0) - (0 + 0 + 0)]$$
= 3 sq. units. **Ans.**

(iii) (2, 0), (11, 6), (– 4, – 4)
Here, $x_1 = 2, y_1 = 0, x_2 = 11, y_2 = 6, x_3 = -4$ and $y_3 = -4$.

$$\therefore \text{ Area } = \frac{1}{2}[\{2 \times 6 + 11 \times (-4) + (-4) \times 0\} - \{2 \times (-4) + 6 \times (-4) + 11 \times 0\}]$$
$$= \frac{1}{2}[(12 - 44 + 0) - (-8 - 24 + 0)]$$
$$= \frac{1}{2}[-32 + 32] = 0 \text{ sq. unit.}$$ **Ans.**

(iv) (a, b), (b, c), (c, a)
Here, $x_1 = a, y_1 = b, x_2 = b, y_2 = c, x_3 = c$ and $y_3 = a$

$$\therefore \text{ Area } = \frac{1}{2}[(a \times c + b \times a + c \times b) - (a \times a + c \times c + b \times b)]$$
$$= \frac{1}{2}[(ac + ba + cb) - (a^2 + c^2 + b^2)]$$
$$= \frac{1}{2}[ac + ba + cb - a^2 - c^2 - b^2]$$
sq. units. **Ans.**

(v) $\left(at_1, \frac{a}{t_1}\right), \left(at_2, \frac{a}{t_2}\right), \left(at_3, \frac{a}{t_3}\right)$

Here, $x_1 = at_1, y_1 = \frac{a}{t_1}, x_2 = at_2, y_2 = \frac{a}{t_2}, x_3 = at_3$ and $y_3 = \frac{a}{t_3}$

$$\therefore \text{ Area } = \frac{1}{2}\left[\left(at_1 \times \frac{a}{t_2} + at_2 \times \frac{a}{t_3} + at_3 \times \frac{a}{t_1}\right) - \left(\frac{a}{t_1} \times at_2 + \frac{a}{t_2} \times at_3 + \frac{a}{t_3} \times at_1\right)\right]$$

$$=\frac{1}{2}\left[a^2\left(\frac{t_1}{t_2}+\frac{t_2}{t_3}+\frac{t_3}{t_1}\right)-a^2\left(\frac{t_2}{t_1}+\frac{t_3}{t_2}+\frac{t_1}{t_3}\right)\right]$$

$$= \frac{1}{2}a^2 \left[\frac{t_1^2t_3+t_2^2t_1+t_3^2t_2}{t_1t_2t_3}-\frac{t_2^2t_3+t_3^2t_1+t_1^2t_2}{t_1t_2t_3}\right]$$

$$= \frac{a^2}{2t_1t_2t_3}[t_1^2t_3+t_2^2t_1+t_3^2t_2 - t_2^2t_3-t_3^2t_1-t_1^2t_2]$$

$$= \frac{a^2}{2t_1t_2t_3}[t_3(t_1^2-t_2^2)-t_1t_2(t_1-t_2)-t_3^2(t_1-t_2)]$$

$$= \frac{a^2}{2t_1t_2t_3}[(t_1-t_2)[t_3(t_1+t_2)-t_1t_2-t_3^2)]$$

$$= \frac{a^2}{2t_1t_2t_3}[(t_1-t_2)(t_3t_1+t_3t_2-t_1t_2-t_3^2)]$$

$$= \frac{a^2}{2t_1t_2t_3}[(t_1-t_2)\{t_2(t_3-t_1)-t_3(t_3-t_1)\}]$$

$$= \frac{a^2}{2t_1t_2t_3}[(t_1-t_2)(t_2-t_3)(t_3-t_1)]$$

sq. units. **Ans.**

12. Prove that the following three points are collinear :
(i) (– 4, – 2), (6, 3), (0, 0)
(ii) (1, 5), (3, 14), (– 1, – 4)
(iii) (4, 3), (5, 1), (1, 9)
(iv) (1, 0), (0, 1), (– 3, 4).

Sol. (i) Co-ordinates of the three points are (– 4, – 2), (6, 3), (0, 0).

Here, $x_1 = -4, y_1 = -2, x_2 = 6, y_2 = 3, x_3 = 0, y_3 = 0$

$\therefore$ Area $= \frac{1}{2}[\{(-4)\times 3+6\times 0+0\times(-2)\}]-\{(-4)\times 0+0\times 3+6\times(-2)\}]$

$= \frac{1}{2}[-12+12]=0.$

$\therefore$ The area of the triangle formed by given points is zero. Hence, the three points are collinear. **Proved.**

(ii) (1, 5), (3, 14), (– 1, – 4)

Here, $x_1 = 1, y_1 = 5, x_2 = 3, y_2 = 14, x_3 = -1$ and $y_3 = -4$

$\therefore$ Area $= \frac{1}{2}[\{1\times 14+3\times(-4)+(-1)\times 5\}-\{1\times(-4)+(-1)\times 14+3\times 5\}]$

$= \frac{1}{2}[(14-12-5)-(-4-14+15)]$

$= \frac{1}{2}[-3+3]=0.$ **Proved.**

$\therefore$ The given points are collinear.

(iii) (4, 3), (5, 1), (1, 9)

Here, $x_1 = 4, y_1 = 3, x_2 = 5, y_2 = 1, x_3 = 1$ and $y_3 = 9$.

$\therefore$ Area $= \frac{1}{2}[\{4\times 1+5\times 9+1\times 3\}-(4\times 9+1\times 1+5\times 3)]$

$= \frac{1}{2}[(4+45+3)-(36+1+15)]$

$= \frac{1}{2}[52-52]=0.$

$\therefore$ The given points are collinear. **Proved.**

(iv) (1, 0), (0, 1), (– 3, 4)

Here, $x_1 = 1, y_1 = 0, x_2 = 0, y_2 = 1, x_3 = -3$ and $y_3 = 4$

$\therefore$ Area $= \frac{1}{2}[\{1\times 1+0\times 4+(-3)\times 0\}-\{1\times 4+(-3)\times 1+0\times 0\}]$

$= \frac{1}{2}[(1+0+0)-(4-3+0)]$

$= 0$

$\therefore$ The given points are collinear. **Proved.**

Short Answer Type Questions

13. (i) For what value of *x* the following three points are collinear ?
(1, – 1), (2, 1), (*x*, 5).
(ii) For what value of y the following three points are collinear ?
(1, 4), (3, y), (– 3, 16).

Sol. (i) (1, – 1), (2, 1), (x, 5).

Here, $x_1 = 1$, $y_1 = -1$, $x_2 = 2$, $y_2 = 1$, $x_3 = x$ and $y_3 = 5$

If the points are collinear, then the area of triangle formed by these vertices is zero.

$$\therefore \frac{1}{2}[\{1 \times 1 + 2 \times 5 + x \times (-1)\} - \{1 \times 5 + x \times 1 + 2 \times (-1)\}] = 0$$

or $\frac{1}{2}[(1 + 10 - x) - (5 + x - 2)] = 0$

or $11 - x - 3 - x = 0$

$\therefore \quad x = 4.$ **Ans.**

(ii) (1, 4), (3, y), (– 3, 16).

Here, $x_1 = 1$, $y_1 = 4$, $x_2 = 3$, $y_2 = y$, $x_3 = -3$ and $y_3 = 16$

$$\therefore 0 = \frac{1}{2}[\{1 \times y + 3 \times 16 + 4 \times (-3)\} - \{1 \times 16 + y \times (-3) + 3 \times 4\}]$$

or $\quad 0 = \frac{1}{2}[(y + 48 - 12) - (16 - 3y + 12)]$

or $\quad 0 = y + 36 - 28 + 3y$

or $\quad -8 = 4y$

or $\quad y = -2.$ **Ans.**

14. If the three points (x, y), (– 5, 7) and (– 4, 5) are collinear, then prove that 2x + y + 3 = 0.

Sol. Co-ordinates of the three points are (x, y), (– 5, 7) and (– 4, 5).

Here, $x_1 = x$, $y_1 = y$, $x_2 = -5$, $y_2 = 7$, $x_3 = -4$ and $y_3 = 5$

$\therefore$ Area of triangle = 0

$$\therefore \frac{1}{2}[\{x \times 7 + 5 \times (-5) \times y \times (-4)\} - \{y \times (-5) + 7 \times (-4) + 5 \times x\}] = 0$$

or $\frac{1}{2}[(7x - 25 - 4y) - (-5y - 28 + 5x)] = 0$

or $7x - 25 - 4y + 5y + 28 - 5x = 0$

or $2x + y + 3 = 0.$ **Proved.**

Long Answer Type Questions

15. Find the area of the quadrilateral whose vertices are :

(i) (1,1), (7,–3), (12,2) and (7,21)

(ii) (–1,6), (–3,–9), (5,–8) and (3,9)

(iii) (1, 2), (6, 2), (5, 3) and (3, 7).

Sol. (i) The co-ordinates of the vertices of a quadrilateral are

(1, 1), (7, – 3), (12, 2) *and* (7, 21)

Here, $x_1 = 1$, $y_1 = 1$, $x_2 = 7$, $y_2 = -3$, $x_3 = 12$, $y_3 = 2$, $x_4 = 7$ and $y_4 = 21$.

$\therefore$ Area of the quadrilateral

$= [(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (y_1x_2 + y_2x_3 + y_3x_4 + y_4x_1)]$

$$= \frac{1}{2}[\{1 \times (-3) + 7 \times 2 + 12 \times 21 + 7 \times 1\} - \{1 \times 7 + 12 \times (-3) + 2 \times 7 + 21 \times 1\}]$$

$$= \frac{1}{2}[\{-3 + 14 + 252 + 7) - (7 - 36 + 14 + 21)]$$

$= \frac{1}{2}[270 - 6] = \frac{1}{2} \times 264 = 132$ sq. units. **Ans.**

(ii) The co-ordinates of the vertices of a quadrilateral are

(– 1, 6), (– 3, – 9), (5, – 8) and (3, 9)

Here, $x_1 = -1$, $y_1 = 6$, $x_2 = -3$, $y_2 = -9$, $x_3 = 5$, $y_3 = -8$, $x_4 = 3$ and $y_4 = 9$.

$\therefore$ Area of the quadrilateral

$$= \frac{1}{2}[(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (y_1x_2 + y_2x_3 + y_3x_4 + y_4x_1)]$$

$$= \frac{1}{2}[(-1) \times (-9) + (-3)(-8) + (5)(9) + (3)(6)\} - \{(6)(-3) + (-9)(5) + (-8)(3) + (9)(-1)\}]$$

$$= \frac{1}{2}[(9 + 24 + 45 + 18) - (-18 - 45 - 24 - 9)]$$

$= \frac{1}{2}[(96) - (-96)] = \frac{1}{2}[192] = 96$ sq. units. **Ans.**

(iii) The co-ordinates of the vertices of a quadrilateral are

(1, 2), (6, 2), (5, 3) and (3, 7).

Here, $x_1 = 1$, $x_2 = 6$, $x_3 = 5$ and $x_4 = 3$

$y_1 = 2$, $y_2 = 2$, $y_3 = 3$ and $y_4 = 7$

$\therefore$ Area of the quadrilateral

$$= \frac{1}{2}[(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (y_1x_2 + y_2x_3 + y_3x_4 + y_4x_1)]$$

$$= \frac{1}{2}[(1 \times 2 + 6 \times 3 + 5 \times 7 + 3 \times 2) - (2 \times 6 + 2 \times 5 + 3 \times 3 + 7 \times 1)]$$

$$= \frac{1}{2}[(2 \times 18 + 35 + 6)$$

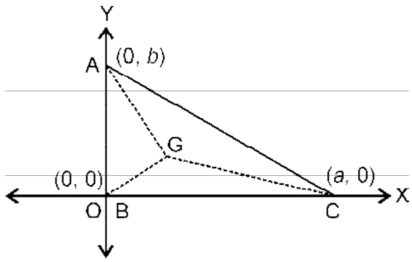
$- (12 + 10 + 9 + 7)]$

$= \frac{1}{2}[61 - 38] = \frac{1}{2} \times 23$

$= 11\frac{1}{2}$ sq. units. **Ans.**

16. If G is the centroid of a triangle ABC, then, Prove analytically that ΔBCG = ΔCAG = ΔABG.

Sol. Let the vertices of ΔABC be $A(0, b)$, $B(0, 0)$ and $C(a, 0)$.



$\therefore$ Co-ordinates of the centroid G. are given by

$$G = \left(\frac{0+0+a}{3}, \frac{b+0+0}{3}\right) = \left(\frac{a}{3}, \frac{b}{3}\right)$$

$\therefore$ Area of ΔABG

$$= \frac{1}{2}\left[\left(0\times 0 + 0\times\frac{b}{3} + \frac{a}{3}\times b\right) - \left(b\times 0 + 0\times\frac{a}{3} + \frac{b}{3}\times 0\right)\right]$$

$$= \frac{1}{2}\left[\frac{ab}{3}\right] = \frac{ab}{6},$$

area of ΔBCG

$$= \frac{1}{2}\left[\left(0\times 0 + a\times\frac{b}{3} + \frac{a}{3}\times 0\right)\left(-0\times a + 0\times\frac{a}{3} + \frac{b}{3}\times 0\right)\right]$$

$$= \frac{1}{2}\left[\frac{ab}{3}\right] = \frac{ab}{6}$$

and area of ΔCAG

$$= \frac{1}{2}\left[\left(a\times b + 0\times\frac{b}{3} + \frac{a}{3}\times 0\right) - \left(0\times 0 + b\times\frac{a}{3} + \frac{b}{3}\times a\right)\right]$$

$$= \frac{1}{2}\left[ab - \frac{ab}{3} - \frac{ab}{3}\right]$$

$$= \frac{1}{6}[3ab - ab - ab] = \frac{ab}{6}$$

$\therefore$ Areas of ΔABG, ΔBCG, ΔCAG are equal,

$\therefore \Delta ABG = \Delta BCG = \Delta CAG$. **Proved.**

17. If three points (a, 0), (0, b) and (x, y) are collinear, then prove that

$$\boldsymbol{\frac{x}{a} + \frac{y}{b} = 1.}$$

Sol. Since the points $(a, 0)$, $(0, b)$ and (x, y) are collinear, hence the area of the triangle formed by them will be zero.

Hence, $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_2) + x_3(y_1 - y_2)] = 0$

$\Rightarrow \frac{1}{2}[a(b - y) + 0(y - b) + x(0 - b)] = 0$

$\Rightarrow ab - ay - bx = 0$

$\Rightarrow bx + ay = ab$

$\Rightarrow \frac{bx}{ab} + \frac{ay}{ab} = \frac{ab}{ab}$

$\Rightarrow \frac{x}{a} + \frac{y}{b} = 1.$

Proved.

18. If the points (x, y), (2, 3) and (–2, 4) are collinear, then prove that x + 2y = 8.

Sol. Since the points (x, y), $(2, 3)$ and $(-2, 4)$ are collinear, hence the area of the triangle formed by them will be zero.

Here $x_1 = x$, $y_1 = y$, $x_2 = 2$, $y_2 = 3$, $x_3 = -2$, $y_3 = 4$

Hence, $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_2) + x_3(y_1 - y_2)] = 0$

$\Rightarrow -x + 2 - 2y + 6 = 0$

$\Rightarrow \frac{1}{2}[x(3 - 4) + 2(4 - 3) + (-2)(y - 3)] = 0$

$\Rightarrow -x + 2 - 2y + 6 = 0$

$\Rightarrow x + 2y = 8$

Proved.

19. The vertices of a triangle ABC are (3, 0), (0, 6) and (6, 9). The line DE divides AB and AC in the ratio of 1 : 2. Prove that

ΔABC = 9 ΔADE.

Sol. The co-ordinates of the point D dividing the line AB in the ratio 1 : 2 are

$$h = \frac{1 \times 0 + 2 \times 3}{1+2} = \frac{0+6}{3} = 2$$

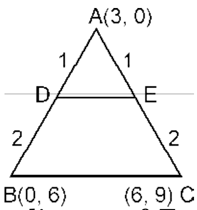
and
$$k = \frac{1 \times 6 + 2 \times 0}{1+2} = \frac{6+0}{3} = 2.$$

∴ Co-ordinates of the point D are (2, 2).

Similarly, co-ordinates of E are :

$$h_1 = \frac{1 \times 6 + 2 \times 3}{1+2} = \frac{6+6}{3} = \frac{12}{3} = 4$$

and
$$k_1 = \frac{1 \times 9 + 2 \times 0}{1+2} = \frac{9+0}{3} = 3.$$



∴ Co-ordinates of E are (4, 3).

∴ Area of $\Delta ABC = \frac{1}{2}$ [(3 × 6 + 0 × 9 + 0 × 6) – (0 × 0 + 6 × 6 + 9 × 3)

$$= \frac{1}{2}[18 - 36 - 27]$$

$$= \frac{-45}{2} = \frac{45}{2} \text{ sq. units.}$$

Also,area of ΔADE

$$= \frac{1}{2}[(3 \times 2 + 2 \times 3 + 4 \times 0) - (0 \times 2 + 2 \times 4 + 3 \times 3]$$

$$= \frac{1}{2}[(6 + 6 + 0) - (0 + 8 + 9)]$$

$$= \frac{1}{2}(12 - 17) = -\frac{5}{2} = \frac{5}{2} \text{ sq. units}$$

$$\therefore \frac{\Delta ABC}{\Delta ADE} = \frac{45}{2} \times \frac{2}{5} = 9.$$

or $\Delta ABC = 9\, \Delta\, ADE$. **Proved.**

20. If the line joining the points (– 1, 3) and (4, – 2) passes through the point (*a*, *b*), then prove that *a* + *b* = 2.

Sol. If the line joining (– 1, 3) and (4, – 2) will pass through (a, b); then the three points are collinear and hence area of Δ = 0

$$\therefore \frac{1}{2}[\{(-1) \times (-2) + 4 \times b + a \times 3\} - \{3 \times 4 + (-2) \times a + b \times (-1)\}] = 0$$

or $2 + 4b + 3a - 12 + 2a + b = 0$

or $5a + 5b = 10$

or $a + b = 2.$

Proved

EXERCISE 6.5

Short Answer Type Questions

1. Find the equation of the locus of a point which moves in such a way that :

(i) its distance from *x*-axis is always 4 units;

(ii) its distance from *y*-axis is 3 units more than its distance from *x*-axis;

(iii) its distance from (3,0) is always 3.

Sol. Let P (X, Y) be a variable point. The distance of the point P from x-axis = Y

The distance of the point P from y-axis = X

(i) According to the condition given, the equation of the locus of such point P is

$$Y = 4$$

or $Y - 4 = 0$ **Ans.**

(ii) According to the condition given, the equation of the locus is

$$X = Y + 3$$

or $X - Y = 3.$ **Ans.**

(iii) Distance of point P (X, Y) from the point (3, 0) is

$$= \sqrt{(X-3)^2 + (Y-0)^2}$$

Now according to the condition, the equation of the locus is.

$$\sqrt{(X-3)^2 + Y^2} = 3$$

$\Rightarrow X^2 - 6X + 9 + Y^2 = 9$

$\Rightarrow X^2 + Y^2 - 6X = 0$

$\Rightarrow X^2 + Y^2 = 6X$. **Ans.**

Long Answer Type Questions

2. Find the equation of the locus of a point such that the sum of their distances from (0, 2) and (0, – 2) is 6.

Sol. Let $P(h, k)$ be any point on the locus and let $A(0, 2)$ and $B(0, -2)$ be the given points.

By the given condition,

$$PA + PB = 6$$

$\Rightarrow \sqrt{(h-0)^2 + (k-2)^2} + \sqrt{(h-0)^2 + (k+2)^2} = 6$

$\Rightarrow \sqrt{h^2 + k^2 - 4k + 4} = 6 - \sqrt{h^2 + k^2 + 4k + 4}$

Let $h^2 + k^2 + 4 = \lambda$

$\therefore \sqrt{\lambda - 4k} = 6 - \sqrt{\lambda + 4k}$

Squaring both sides, we get

$\lambda - 4k = 36 + \lambda + 4k - 12\sqrt{\lambda + 4k}$

$\Rightarrow 12\sqrt{\lambda + 4k} = 36 + 8k$

$\Rightarrow 3\sqrt{\lambda + 4k} = 9 + 2k$

Squaring again, we get

$9(\lambda + 4k) = 81 + 4k^2 + 36k$

$\Rightarrow 9\lambda = 4k^2 + 81$

$\Rightarrow 9(h^2 + k^2 + 4) = 4k^2 + 81$ [as $\lambda = h^2 + k^2 + 4$]

$\Rightarrow 9h^2 + 5k^2 = 81 - 36$

$\Rightarrow 9h^2 + 5k^2 = 45$

$\therefore$ Locus of point $P(h, k)$ is

$9x^2 + 5y^2 = 45$. **Ans.**

3. A point moves so that the sum of its distance from (ae, 0) and (– ae, 0) is $2a$. Prove that the locus of the point is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

where $b^2 = a^2(1 - e^2)$.

Sol. Let $P(h, k)$ be any point on the locus and Let $A(ae, 0)$ and $B(-ae, 0)$ be the given point.

By the given condition.

$$PA + PB = 2a$$

$\Rightarrow \sqrt{(h-ae)^2 + k^2} + \sqrt{(h+ae)^2 + k^2} = 2a$

$\Rightarrow \sqrt{(h-ae)^2 + k^2} = 2a - \sqrt{(h+ae)^2 + k^2}$

Squaring both sides, we get,

$(h - ae)^2 + k^2 = 4a^2 + (h + ae)^2 + k^2 - 4a\sqrt{(h+ae)^2 + k^2}$

$\Rightarrow (h - ae)^2 - (h + ae)^2 - 4a^2 = -4a\sqrt{(h+ae)^2 + k^2}$

$\Rightarrow -4aeh - 4a^2 = -4a\sqrt{(h+ae)^2 + k^2}$

$\Rightarrow (eh + a) = \sqrt{(h+ae)^2 + k^2}$

Squaring both sides again,

$e^2h^2 + a^2 + 2aeh = h^2 + a^2e^2 + 2aeh + k^2$

$\Rightarrow h^2(1 - e^2) + k^2 = a^2(1 - e^2)$

$\Rightarrow \frac{h^2}{a^2} + \frac{k^2}{a^2(1-e^2)} = 1$

$\Rightarrow \frac{h^2}{a^2} + \frac{k^2}{b^2} = 1$

[given $b^2 = a^2(1 - e^2)$]

$\therefore$ Locus of point $P(h, k)$ is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. **Proved.**

4. Let (– a, 0) and B (a, 0) be any two fixed points and let P be a variable point moving in such a way that $PA^2 + PB^2 = 2k^2$, where k is a constant. Find the equation of locus of P.

Sol. Let $P(x, y)$ be a variable point, and $A(-a, 0)$, $B(a, 0)$.

According to the question,

$PA^2 + PB^2 = 2k^2$

$\Rightarrow (x + a)^2 + y^2 + (x - a)^2 + y^2 = 2k^2$

$\Rightarrow x^2 + 2ax + a^2 + y^2 + x^2 - 2ax + a^2 + y^2 = 2k^2$

$\Rightarrow 2[x^2 + y^2 + a^2] = 2k^2$

$\Rightarrow x^2 + y^2 + a^2 = k^2$

Hence the locus of point $P(x, y)$ is

$x^2 + y^2 + a^2 = k^2$. **Ans.**

❑

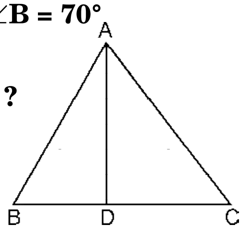
Unit 4 : Geometry

7 Triangles

EXERCISE 7.1

Multiple Choice Type Questions

1. In ΔABC, it is given that $\frac{AB}{AC}=\frac{BD}{DC}$. If $\angle B = 70°$ and $\angle C = 50°$, then $\angle BAD$ = ?



(a) 30°
(b) 40°
(c) 45°
(d) 50°.

Sol. In ΔABD & ΔACD

$AB = AC$ (given)
$BD = DC$ (given)
$AD = AD$ (common)

Hence $\Delta ABD \cong \Delta ACD$ (SSS)

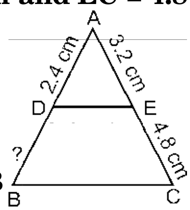
$\angle B = 70°$ (given)
$\angle C = 50°$ (given)
$\angle A = 180° - 70° - 50°$
$= 60°.$

As per congruency rule

$\angle BAD = \angle CAD$

$\therefore \quad \angle BAD = 30°.$ **Ans.**

2. In ΔABC, DE $\|$ BC so that AD = 2.4cm, AE = 3.2cm and EC = 4.8 cm, then AB = ?



(a) 3.6 cm
(b) 6 cm
(c) 6.4 cm
(d) 7.2 cm.

Sol. Given that DE $\|$ AB

$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$$

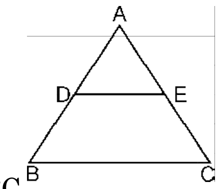
$$\frac{2.4}{DB} = \frac{3.2}{4.8}$$

$$DB = \frac{2.4 \times 4.8}{3.2}$$

$= 3.6.$

$AB = AD + DB$
$= 2.4 + 3.6$
$= 6$ cm. **Ans.**

3. In a ΔABC, if DE is drawn parallel to BC, cutting AB and AC at D and E respectively such that AB = 7.2 cm, AC = 6.4 cm and AD = 4.5 cm. Then AE = ?



(a) 5.4 cm
(b) 4 cm
(c) 3.6 cm
(d) 3.2 cm.

Sol. Given that DE $\|$ BC

$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$$

$AD = 4.5, AB = 7.2,$
$DB = 7.2 - 4.5 = 2.7.$

$AC = 6.4$, let $AE = x$ then $EC = 6.4 - x$

$$\frac{4.5}{2.7} = \frac{x}{6.4 - x}$$

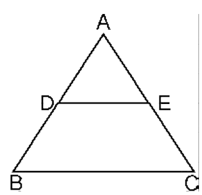
$4.5\,(6.4 - x) = 2.7x$
$28.8 - 4.5x = 2.7x$
$28.8 = 2.7x + 4.5x$
$28.8 = 7.2x$

$$\frac{28.8}{7.2} = x$$

$x = 4.$

$\therefore \quad AE = 4$ cm. **Ans.**

4. In ΔABC, DE $\|$ BC so that AD = $(7x - 4)$cm, AE = $(5x - 2)$cm, DB = $(3x + 4)$ cm and EC = $3x$ cm. Then, we have :



(a) $x = 3$
(b) $x = 5$
(c) $x = 4$
(d) $x = 2.5$.

Sol. Given that DE $\|$ BC

$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{7x-4}{3x+4} = \frac{5x-2}{3x}$$

$$(3x)(7x-4) = (3x+4)(5x-2)$$

$$21x^2 - 12x = 15x^2 - 6x + 20x - 8$$

$$21x^2 - 15x^2 - 12x + 6x - 20x + 8 = 0$$

$$6x^2 - 26x + 8 = 0$$

Here, $a = 6$

$b = -26$

$c = 8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-26) \pm \sqrt{(-26)^2 - 4 \times 6 \times 8}}{2 \times 6}$$

$$x = \frac{26 \pm \sqrt{676 - 192}}{12}$$

$$x = \frac{26 \pm \sqrt{484}}{12}$$

$$x = \frac{26 \pm 22}{12}$$

$$x = \frac{26 + 22}{12}$$

$$x = \frac{48}{12} = 4.$$

Or, $$x = \frac{26 - 22}{12}$$

$$x = \frac{4}{12}$$

$$x = \frac{1}{3}$$

$$x = 0.33.$$

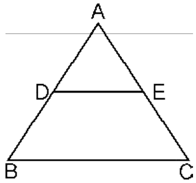
$$x = 4, 0.33$$

The value of x can not be in decimal

$\therefore$ $x = 4$cm.

5. In ΔABC, DE $\parallel$ BC such that $\frac{AD}{DB} = \frac{3}{5}$. If AC = 5.6 cm, then AE = ?

(a) 4.2 cm
(b) 3.1 cm
(c) 2.8 cm
(d) 2.1 cm.



Sol. Given that DE $\parallel$ BC

Let AE $= x$

$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{3}{5} = \frac{x}{5.6 - x}$$

$$3(5.6 - x) = 5x$$

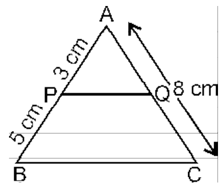
$$3 \times 5.6 = 5x + 3x$$

$$x = \frac{3 \times 5.6}{8}$$

$$x = 2.1 \text{ cm.}$$

6. In ΔABC, P & Q are the points on sides AB and AC respectively, such that PQ $\parallel$ BC. If AP = 3 cm, PB = 5 cm and AC = 8 cm, Find AQ.

Sol. Given that : PQ $\parallel$ BC



$$\therefore \quad \frac{AP}{PB} = \frac{AQ}{QC}$$

Let AQ $= x$ cm

$$\frac{3}{5} = \frac{x}{8 - x}$$

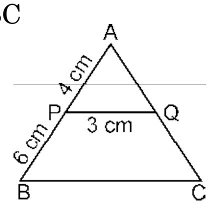
$$3(8 - x) = 5x$$

$$24 = 5x + 3x$$

$$x = \frac{24}{8} = 3 \text{ cm.}$$

7. In a triangle ABC, P and Q are the points on sides AB and AC respectively such that PQ $\parallel$ BC. If AP = 4 cm, PB = 6 cm and PQ = 3 cm, determine BC.

Sol. Given that PQ $\parallel$ BC



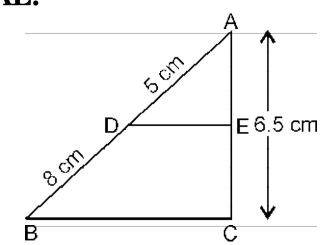
$$\therefore \quad \frac{AP}{AB} = \frac{PQ}{BC}$$

$$\frac{4}{4+6} = \frac{3}{BC}$$

$$BC = \frac{3 \times 10}{4}$$

$$= 7.5 \text{ cm}$$ **Ans.**

8. In ΔABC, D and E are the points on sides AB an AC respectively, such that DE $\parallel$ BC. If AD = 5 cm, DB = 8 cm and AC = 6.5 cm. Find AE.



Sol. Given that DE || BC

$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$$

Let AE $= x$ cm

$$\frac{5}{8} = \frac{x}{6.5 - x}$$

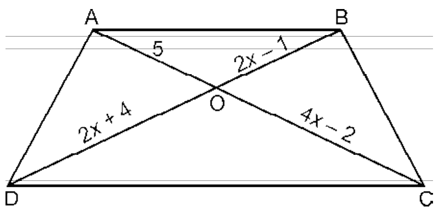
$$5(6.5 - x) = 8x$$

$$5 \times 6.5 = 8x + 5x$$

$$x = \frac{5 \times 6.5}{13}$$

$$AE = x = 2.5 \text{ cm.}$$

9. In fig, if AB || CD, find the value of x.



Sol. Given that AB || CD

$$\therefore \quad \frac{AO}{OC} = \frac{BO}{OD}$$

$$\frac{5}{4x-2} = \frac{2x-1}{2x+4}$$

$$5(2x+4) = (4x-2)(2x-1)$$

$$10x + 20 = 8x^2 - 4x - 4x + 2$$

$$8x^2 - 18x - 18 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $a = 8$, $b = -18$, $c = -18$

$$x = \frac{-(-18) \pm \sqrt{(-18)^2 - 4 \times 8 \times (-18)}}{2 \times 8}$$

$$x = \frac{18 \pm \sqrt{324 + 576}}{16}$$

$$x = \frac{18 \pm \sqrt{900}}{16}$$

$$x = \frac{18 \pm 30}{16}$$

$$x = \frac{18 + 30}{16} = \frac{48}{16} = 3.$$

Or, $$x = \frac{18 - 30}{16}$$

$$x = -\frac{12}{16} = -\frac{6}{8}$$

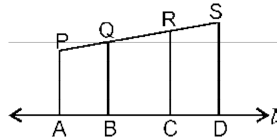
$$x = -0.75$$

$$x = 3 \text{ or } -0.75$$

The value of x can not be negative or in decimal

$$\therefore \quad x = 3 \text{ cm.}$$

10. In fig, PA, QB, RC and SD are all perpendiculars to a line l, AB = 6 cm, BC = 9 cm, CD = 12 cm and SP = 36 cm. Find PQ, QR and RS.



Sol. Given : PA, QB, RC, SD are perpendicular on line l

To Find : PQ, QR and RS

Construction : Produce SP and l to meet each other at E.

Proof : In ΔEDS

AP || BQ || DS || CR (Given)

$\therefore$ PQ : QR : RS = AB : BC : CD

PQ : QR : RS = 6 : 9 : 12

Let PQ $= 6x$, QR $= 9x$, RS $= 12x$

$$PS = PQ + QR + RS$$

$$36 = 6x + 9x + 12x$$

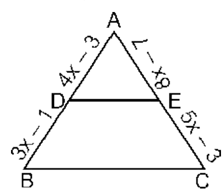
$$x = \frac{36}{27} = \frac{4}{3}$$

$$PQ = 6 \times \frac{4}{3} = 8\text{cm}$$

$$QR = 9 \times \frac{4}{3} = 12\text{cm}$$

$$RS = 12 \times \frac{4}{3} = 16\text{cm.}$$

11. In figure if DE || BC and AD = $4x - 3$, AE = $8x - 7$, BD = $3x - 1$ and CE = $5x - 3$, Find x.



Sol. Given that DE || BC

$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

$(4x-3)(5x-3) = (3x-1)(8x-7)$

$20x^2 - 12x - 15x + 9 = 24x^2 - 21x - 8x + 7$

$20x^2 - 24x^2 - 12x - 15x + 21x + 8x + 9 - 7 = 0$

$-4x^2 + 2x + 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $a = -4$
$b = 2$
$c = 2$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times (-4) \times 2}}{2 \times (-4)}$$

$$x = \frac{-2 \pm \sqrt{4 \times 32}}{-8}$$

$$x = \frac{-2 \pm 6}{-8} = \frac{4}{-8} = -\frac{2}{4} = -\frac{1}{2}$$

$x = -0.5$

$$x = \frac{-2 - 6}{-8}$$

$$x = \frac{-8}{-8}$$

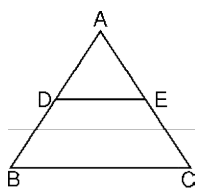
$x = 1.$

The value of x can not be negative or in decimal.

$\therefore$ $x = 1$cm.

12. In a ΔABC, D and E are points on sides AB and AC respectively, such that AD × EC = AE × DB. Prove that DE || BC.

Sol.



Given : AD × EC = AE × BD

To prove : DE || BC

Proof : AD × EC = AE × DB

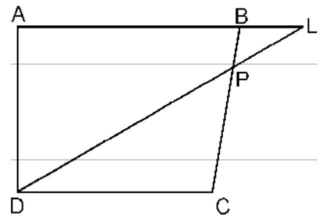
$$\frac{AD}{DB} = \frac{AE}{EC}$$

It meant that sides AB and AC are divided in the same ratio by DE we have, "If a line divides two sides of a triangle in the same ratio then, the line is parallel to the third side." (converse to basic proportionality theorem) therefore, we can say that DE || BC (third side)

Hence, it is proved that DE || BC.

13. ABCD is a parallelogram, P is a point on side BC. DP when produced meets AB produced to L, prove that (i) $\frac{DP}{PL} = \frac{DC}{BL}$ (ii) $\frac{DL}{DP} = \frac{AL}{DC}$.

Sol.



Given : ABCD is a parallelogram.

(i) To Proof : $\frac{DP}{PL} = \frac{DC}{BL}$

Proof : In ΔPCD and ΔBPL

$\angle$DPC = $\angle$BPL(Vertically opposite)

$\angle$C = $\angle$B (Alternate interior)

$\therefore$ $\Delta PCD \sim \Delta PBL$

$$\frac{DP}{PL} = \frac{DC}{BL} \quad \text{(By C.P.C.T.)}$$

Hence, proved

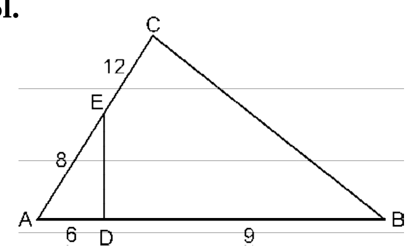
(ii) To Proof : $\frac{DL}{DP} = \frac{AL}{DC}$

Proof : In $\Delta PCD \sim \Delta PBL$ (proved above)

$$\therefore \quad \frac{DC}{DP} = \frac{AL}{DC}$$ Hence, proved.

14. If D and E are respectively, the points on the sides AB and AC of a triangle ABC, such that AD = 6 cm, BD = 9 cm, AE = 8 cm and EC = 12 cm, then show that DE || BC.

Sol.



Given : AD = 6cm, BD = 9cm, AE = 8cm, EC = 12cm

To Proof : DE || BC

Proof : $\frac{AD}{DB} = \frac{6}{9} = \frac{2}{3}$

$\frac{AE}{EC} = \frac{8}{12} = \frac{2}{3}$

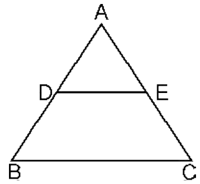
Hence, $\frac{AD}{DB} = \frac{AE}{EC}$

We know that, if a line intersects two sides of a triangle in same ratio of sides, then the intersecting line will be half of thirdside of triangle and parallel to third side. Hence, DE || BD. **Proved.**

15. In ΔABC, ∠B = ∠C, D and E are the points on the sides AB and AC respectively, such that BD = EC. Prove that DE || BC.

Sol.

Given : ΔABC in which D and E are points on sides AB and AC respectively, such that BD = CE



To prove : DE || BC

Proof : In ΔABC, We have

$\angle B = \angle C$

$AC = AB$

$AB = AC$

(sides opposite equal angles are equal)

$AD + DB = AE + EC$

But $BD = CE$

$AD = AE$

Thus, we have

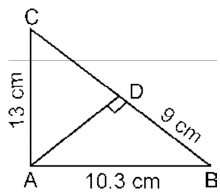
$AD = AE$ and $BD = CE$

$\frac{AD}{BD} = \frac{AE}{CE}$

Therefore, by converse of basic propertionalty theorum, we get DE || BC.

EXERCISE 7.2

1. In the fig. ∠CAB = 90° and AD ⊥ BC. If AC = 13 cm, AB = 10.3 cm and BD = 9cm. Find AD.



Sol. As per pythagoras theorem

$AD = \sqrt{AB^2 - BD^2}$

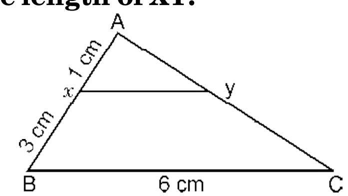
$AD = \sqrt{(10.3)^2 - (9)^2}$

$AD = \sqrt{106.09 - 81}$

$AD = \sqrt{25.09}$

$AD = 5\text{cm}.$

2. In the given fig., XY || BC. Find the length of XY.



Sol. Given : XY || BC

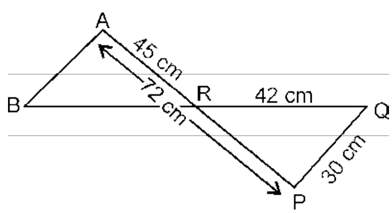
$\therefore \quad \frac{AX}{AB} = \frac{XY}{BC}$

$\frac{1}{3+1} = \frac{XY}{6}$

$\frac{6}{4} = XY$

$XY = 1.5$ cm.

3. In the fig., ΔABR ~ ΔPQR. If PQ = 30cm, AR = 45cm, AP = 72cm and QR = 42cm. Find PR and BR.



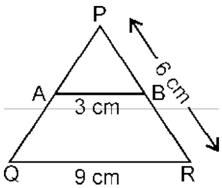
Sol. Given : PQ = 30cm, AR = 45cm, AP = 72cm QR = 42 cm

To find : PR and BR

$PR = AP - AR$

$$PR = 72 - 45$$
$$PR = 27\text{cm.}$$
$$\because \quad \Delta ABR \sim \Delta PQR$$
$$\therefore \quad \frac{AR}{PR} = \frac{BR}{RQ}$$
$$\frac{45}{27} = \frac{BR}{42}$$
$$BR = \frac{45 \times 42}{27}$$
$$= 70\text{cm.}$$

4. In the given figure, AB || QR. Find the length of PB.

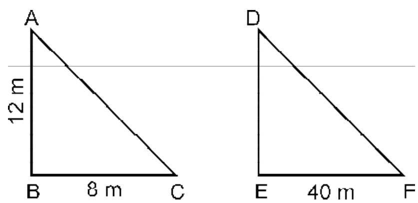


Sol. Given : AB || PQ

$$\therefore \quad \frac{PB}{PR} = \frac{AB}{QR}$$
$$\frac{PB}{6} = \frac{3}{9}$$
$$PB = \frac{3 \times 6}{9}$$
$$PB = 2\text{cm.}$$

5. A vertical stick 12 m long casts a shadow 8 m long on the ground. At the same time, a tower casts the shadow 40m long on the ground. Determine, the height of the tower.

Sol.



ΔABC and ΔDEF

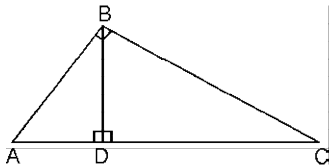
$\angle B = \angle E$ (90°)

$\angle A = \angle D$ (same time same angle)

$\therefore \quad \Delta ABC \sim \Delta DEF$ (AA)

$$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF}$$
$$\frac{12}{DE} = \frac{8}{40}$$
$$DE = \frac{12 \times 40}{8}$$
$$DE = 60 \text{ cm.}$$

6. ΔABC is right angled at B. BD is perpendicular to AC. Prove that ΔBDC is similar to ΔABC.



Sol. Given : $\angle B = 90°$, $BD \perp AC$

To prove : $\Delta BDC \sim \Delta ABC$

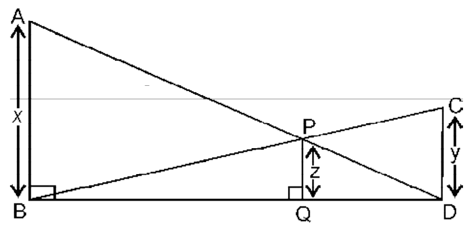
Proof : In ΔBDC and ΔABC

$\angle BDC = \angle ABC$ (90°)

$\angle DCB = \angle BCA$ (common)

$\therefore \quad \Delta BDC \sim \Delta ABC$ (AA similarity)

7. In fig. ∠ABD = ∠CDB = ∠PQB = 90°. If AB = *x* units, CD = *y* units and PQ = *z* units, prove that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$.



Sol. Given : $\angle ABD = \angle CDB = \angle PQB = 90°$

AB = x units, PQ = z units, CD = y units

To prove : $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$

Proof : ΔABD and ΔPQD are similar as the corresponding sides are parallel

$$\frac{x}{z} = \frac{BD}{QD}$$
$$\frac{1}{x} = \frac{QD}{Z \times BD} \quad ...(1)$$

ΔCDB and ΔPQB are similar as the correponding sides are parallel

$$\frac{y}{z} = \frac{BD}{BQ}$$

$$\frac{1}{y} = \frac{BQ}{Z \times BD} \quad ...(2)$$

On adding both the equations we get :

$$\frac{1}{x} = \frac{QD}{Z \times BD}$$

$$\frac{1}{y} = \frac{BQ}{Z \times BD}$$

$$\frac{1}{x} + \frac{1}{y} = \frac{QD}{Z \times BD} + \frac{BQ}{Z \times BD}$$

$$\frac{1}{x} + \frac{1}{y} = \frac{QD + BQ}{Z \times BD}$$

$$\frac{1}{x} + \frac{1}{y} = \frac{BD}{Z \times BD}$$

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$ **Proved.**

8. The perimeters of two similar triangles are 30 cm and 20 cm respectively. If one side of the first triangle is 12 cm, determine the corresponding side of the second triangle.

Sol. Given : $\frac{a}{d} = \frac{b}{e} = \frac{c}{f} = \frac{a+b+c}{d+e+f}$

Therefore, for two similar triangles, if *a, b* and *c* are the sides of first triangle and *d, e and f* are corresponding sides of second triangle then ratio of corresponding sides of the two similar triangles is equal to ratio of their perimeter.

Now, one side of first triangle = 12cm

Let the corresponding side of second triangle = x

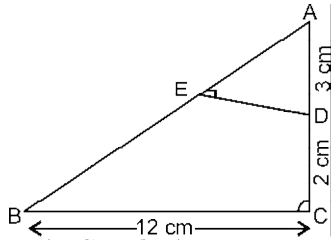
Also, perimeter of first triangle = 30cm and perimeter of second triangle = 20cm

Therefore, $\frac{12}{x} = \frac{30}{20}$

$$x = \frac{12 \times 20}{30}$$

$$x = 8\text{cm.}$$

9. In the given fig, ABC is a right angled triangle at C. Prove that $\Delta ABC \sim \Delta ADE$ and find the lengths of AE and DE.



Sol. In ΔABC and ΔADE

$\angle A = \angle A$ [common angle]

$\angle C = \angle E = 90°$ [given]

$\therefore$ According to AA similarity criterion

$\Delta ABC \sim \Delta ADE$

In ΔABC, $\angle C = 90°$

$\therefore \quad AB^2 = AC^2 + BC^2$

$$AB^2 = \sqrt{5^2 + 12^2}$$

$$AB^2 = \sqrt{25 + 144}$$

$$AB^2 = \sqrt{169}$$

$$AB = 13.$$

$\because \quad \Delta ABC \sim \Delta ADC$

$$\therefore \quad \frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$

$$\frac{13}{3} = \frac{12}{DE} = \frac{9}{AE}$$

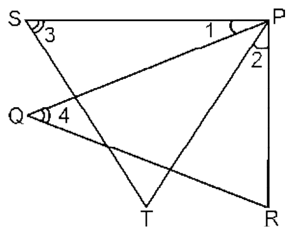
$$DE = \frac{12 \times 3}{13}$$

$$= \frac{36}{13}\text{ cm.}$$

$$AE = \frac{5 \times 3}{13}$$

$$= \frac{15}{13}\text{ cm.}$$

10. In fig, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$. Show that PT. QR = PR. ST.



Sol. Given : $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

To prove : PT. QR = PR.ST

Proof : We have $\angle 1 = \angle 2$

On adding $\angle QPT$ on both sides we get :

$\angle 1 + \angle QPT = \angle 2 + \angle QPT$

$\angle SPT = \angle QPR$ ...(1)

In ΔPST and ΔPQR

$\angle SPT = \angle QPR$ (proved above)

$\angle 3 = \angle 4$ (given)

$\therefore \Delta PST \sim \Delta PQR$ (AA similarity)

$$\frac{PS}{PQ} = \frac{ST}{QR} = \frac{PT}{PR}$$

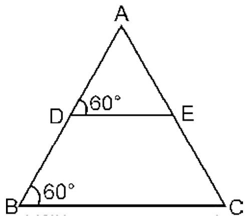
$$\frac{ST}{QR} = \frac{PT}{PR}$$

$$PR \times ST = PT \times QR$$

Hence Proved.

11. In each of the following figures you find two triangles. Indicate, whether the triangles are similar. Give reasons in support of your answer.

(i)



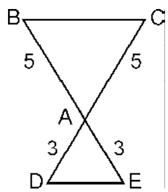
Sol. In ΔABC and ΔADE

$\angle ABC = \angle ADE = 60°$ (Given)

$\angle BAC = \angle DAE$ (common)

$\therefore \Delta ABC \sim \Delta ADE$ (AA similarity)

(ii)



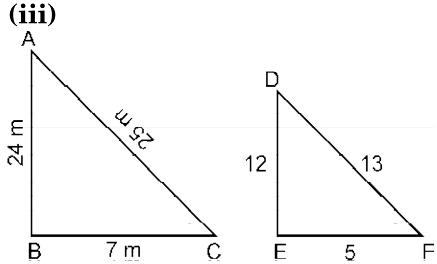
Sol. In ΔABC and ΔADE

$$\frac{AB}{AE} = \frac{AC}{AD} = \frac{5}{3}$$

$\angle BAC = \angle DAE$ (alternative interior angles)

$\therefore \Delta ABC \sim \Delta ADE$ (SAS criterion)

(iii)



Sol. In ΔABC and ΔDEF

$$\frac{AB}{DE} = \frac{24}{12} = \frac{2}{1}$$

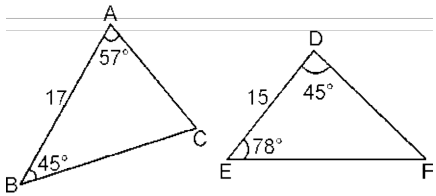
$$\frac{BC}{EF} = \frac{7}{5}$$

$$\frac{AC}{DF} = \frac{25}{13}$$

Here, $\frac{AB}{DE} \neq \frac{BC}{EF} \neq \frac{AC}{DF}$

Hence, ΔABC not similar to ΔDEF.

(iv)



Sol. In ΔABC and ΔDEF

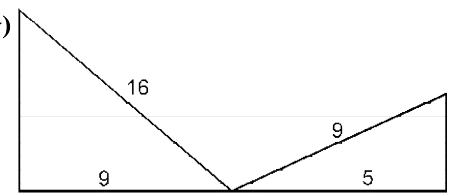
$\angle B = \angle D = 45°$

$\angle A = \angle F = 57°$

$\angle C = \angle E = 78°$

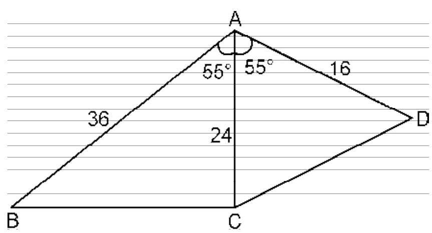
Hence, $\Delta ABC \sim \Delta DEF$ (SSS criterion)

(v)



Sol. Data not sufficient to prove similarity.

(vi)



Sol. In ΔABC and ΔACD

$$\frac{AB}{AC} = \frac{AC}{AD}$$

$$\frac{36}{24} = \frac{24}{16} = \frac{3}{2}$$

$\angle BAC = \angle DAC = 55°$ (given)

$\therefore \Delta ABC \sim \Delta ACD$ (SAS criterion)

12. $\Delta ABC \sim \Delta PQR$, and their perimeters are 20 cm and 30 cm. If AC = 6 cm then find the length of side PR.

Sol. Given : $\Delta ABC \sim \Delta PQR$

To find : PR

For two similar triangles if *a, b, c* and *d, e, f* are the corresponding sides of two similar triangles,then the ratio of corresponding sides of two similar triangles is equal to the ratio of their perimeters.

$$\therefore \quad \frac{a}{d} = \frac{b}{e} = \frac{c}{f} = \frac{a+b+c}{d+e+f}$$

Now, one side of ΔABC = 6 cm let the corresponding side of ΔPQR. PR = x

perimeter of ΔABC = 20 cm

Perimeter of ΔPQR = 30 cm

$$\text{Therefore, } \frac{AC}{PR} = \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR}$$

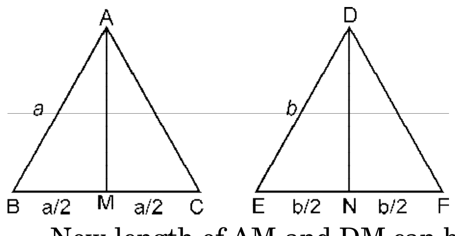
$$\frac{6}{x} = \frac{20}{30}$$

$$x = \frac{6 \times 30}{20}$$

$$PR = 9 \text{ cm.}$$

13. In two triangles are equilateral then prove that the ratio of their respective sides, medians, angles bisectors and altitudes are equal.

Sol. As we know, that for equilateral triangle the median, angle bisecters and altitudes are equal.



Now length of AM and DM can be calculated from the right angle triangle AMB and DNE

$$AM = \sqrt{a^2 - \left(\frac{a}{2}\right)^2}$$

$$= \sqrt{\frac{3a^2}{4}} = \frac{\sqrt{3}}{2}a$$

Similarly

$$DN = \sqrt{b^2 - \left(\frac{b}{2}\right)^2}$$

$$= \sqrt{\frac{3b^2}{4}} = \frac{\sqrt{3}}{2}b$$

Now ratio of sides,

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{a}{b}$$

and as we know all medians, angle bisector and altitude are same so

$$\frac{AM}{DN} = \frac{\frac{\sqrt{3}}{2}a}{\frac{\sqrt{3}}{2}b} = \frac{a}{b}$$

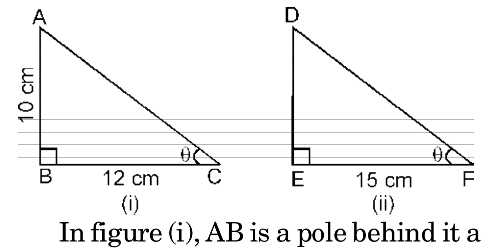
$$\text{Hence, } \frac{AB}{DF} = \frac{AC}{DF} = \frac{BC}{EF}$$

$$= \frac{AM}{DN} = \frac{a}{b}$$

Hence Proved.

14. A pole of height 10 cm cast the shadow of length 12m. Find the length of pole if it cast shadow of length 15 cm at the same point.

Sol.



In figure (i), AB is a pole behind it a sun is risen which casts a shadow of length BC = 12 cm and makes an angle θ to the horizontal and figure it, DE is a height of tower and behind a sun risen which casts a shadow of length EF = 15 cm.

In ΔABC and ΔDEF

$$\angle C = \angle F = \theta$$

$$\angle B = \angle E = 90^\circ$$

$\therefore \quad \Delta ABC \sim \Delta DEF$ (AA similarity)

$$\frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{AB}{BC} = \frac{DE}{EF}$$

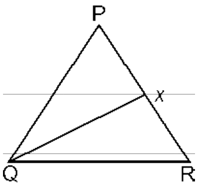
$$\frac{10}{12} = \frac{DE}{15}$$

$$DE = \frac{10 \times 15}{12}$$

$= 12.5$ cm.

15. In a ΔPQR, PQ = PR, X is a point on PR such that $QR^2 = PR \times XR$. Prove that QX = QR.

Sol.



Given : $PQ = PR$

$QR^2 = PR \times xR$

To prove : $Qx = QR$

Proof : It is given that :

$QR \times QR = PR \times xR$

$$\frac{QR}{PR} = \frac{XR}{QR}$$

$\angle R = \angle R$ (Common)

$\therefore \quad \Delta QRP \sim \Delta xRQ$ (By SAS)

$\therefore \quad \angle Q = \angle x$ ...(1)

$PQ = PR$

$\therefore \quad \angle Q = \angle R$ ($\angle$S opposite to equal side) ...(2)

from equation (1) and (2)

$\angle x = \angle R$

$Qx = QR$ (Sides opposite to equal $\angle$S are equal)

Hence Proved.

16. If $\Delta ABC \sim \Delta DEF$ Name 5 other pairs of ΔABC and ΔDEF which are similar in respective order.

Sol. (i) $\Delta ABC \sim \Delta DEF$ (ii) $\Delta BAC \sim \Delta EDF$ (iii) $\Delta BCA \sim \Delta EFD$ (iv) $\Delta CAB \sim \Delta FDE$ (v) $\Delta CBA \sim \Delta FED$.

EXERCISE 7.3

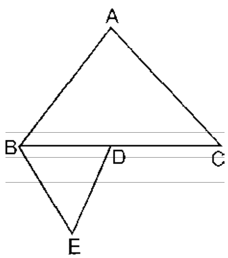
Multiple Choice Type Questions

1. ABC and BDE are two equilateral triangles such that D is the mid point of BC. Ratio of the area of ΔABC and ΔBDE is :

(a) 2 : 1 **(b) 1 : 2**

(c) 4 : 1 **(d) 1 : 4.**

Sol.



Given : ΔABC and ΔBDE are equilateral triangles.

To Find : Area of ΔABC : Area of ΔBDE

Sol. Since, ΔABC and ΔBDE are equilateral their sides would be in the same ratio

$$\frac{AB}{BE} = \frac{AC}{ED} = \frac{BC}{BD}$$

Hence, by SSS similarity

$\Delta ABC \sim \Delta BDE$

And we know that ratio of area of triangle is equal to the ratio of square of corresponding sides.

$$\text{So, } \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta BDE} = \frac{(BC)^2}{(BD)^2}$$

$$= \frac{(BC)^2}{\left(\frac{BC}{2}\right)^2} \quad \left(\text{Since } BD = \frac{1}{2}BC\right)$$

$$= \frac{BC^2}{\frac{BC^2}{4}}$$

$$= \frac{4BC^2}{BC^2}$$

$$= \frac{4}{1}$$

Hence, Area of ΔABC : Area of ΔBDE

4 : 1.

2. Sides of two similar triangles are in ratio 4 : 9. Areas of these triangles are in the ratio :

(a) 2 : 3 **(b) 4 : 9**
(c) 81 : 16 **(d) 16 : 81.**

Sol. If two triangles are similar to each other, then the ratio of the areas of these triangles will be equal to the square of the ratio of the corresponding sides of these triangles.

It is given that the sides are in the ratio 4 : 9.

Therefore ratio between areas of these

$$\text{triangles} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}.$$

Short Answer Type Questions

3. In two similar triangles ABC and DEF, AC = 10 cm and DF = 8 cm. Find the ratio of the areas of the two triangles.

Sol. Given : $\Delta ABC \sim \Delta DEF$

AC = 10 cm,

DF = 8 cm

To Find : Area of ΔABC : Area of ΔDEF

Sol. Ratio of areas of two similar triangles is equal to the ratio of squares of the corresponding sides.

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \left(\frac{AC}{DF}\right)^2$$

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \left(\frac{10}{8}\right)^2$$

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{100}{64}$$

$$= \frac{25}{16}$$

Ratio of areas = 25 : 16.

4. In two similar triangles ABC and PQR, if their corresponding altitudes AD and PS are in the ratio of 4 : 9, find the ratio of the area of ΔABC to that of ΔPQR.

Sol. Given : AD : PS = 4 : 9

To Find : Area of ΔABC : Area of ΔPQR

Sol. Ratio of area of two similar triangles is equal to the ratio of square of their corresponding altitudes.

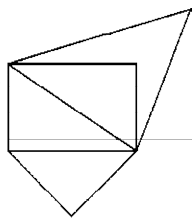
Here, it is given that $\Delta ABC \sim \Delta PQR$

$$\text{So, } \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \left(\frac{4}{9}\right)^2$$

$$= \frac{16}{81}.$$

Hence, the ratio of area of ΔABC : ΔPQR = 16 : 81.

5. If the area of the equilateral triangle described on the side of a square is 48 cm². Find the area of the equilateral triangle described on its diagonal.



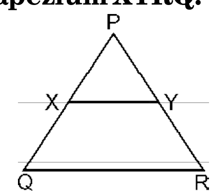
Sol. Given : Area of equilateral triangle = 48 cm²

To Find : Area of equilateral triangle on diagonal.

Sol. ∵ The area of equilateral triangle described on the side of a square is half the area of the equilateral triangle described on its diagonal.

∴ Area = 48 × 2 = 96 cm²

6. In the given fig, ΔPQR, in which XY ∥ QR, PX = 1 cm, XQ = 3 cm, YR = 4·5 cm, QR = 9 cm, find PY and XY. Further, if the area of ΔPXY is 'A' cm², find in terms of A, the area of ΔPQR and the area of trapezium XYRQ.



Sol. Given : XY ∥ QR

QR = 9 cm, PX = 1 cm, XQ = 3 cm, YR = 4·5 cm,

To Find : PY and XY

Sol. ∵ XY is parallel to QR

$$\therefore \quad \frac{PX}{XQ} = \frac{PY}{YR}$$

$$\frac{1}{3} = \frac{PY}{4\cdot5}$$

$$PY = \frac{1 \times 4\cdot5}{3}$$

$= 1{\cdot}5$ cm.

$$\frac{PX}{PQ} = \frac{XY}{QR}$$

$$\frac{1}{1+3} = \frac{XY}{9}$$

$$XY = \frac{9}{4} \text{ cm.}$$

In ΔPXY and ΔPQR

$$\because \quad \frac{PX}{PQ} = \frac{PY}{PR} = \frac{XY}{QR} = \frac{1}{4}$$

$$\therefore \quad \Delta PXY \sim \Delta PQR$$

$$\therefore \quad \frac{\text{Area of } \Delta PXY}{\text{Area of } \Delta PQR} = \left(\frac{1}{4}\right)^2$$

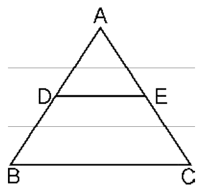
$$\frac{A}{\text{Area of } \Delta PQR} = \frac{1}{16}$$

Area of $\Delta PQR = 16A$ cm^2.

Area of trapezium XYRQ = Area of ΔPQR – Area of ΔPXY

$16A - A = 15A$ cm^2.

7. In the figure, DE is parallel to BC and AD : DB = 2 : 3. Determine Area (ΔADE) : Area (ΔABC).



Sol. Given : DE ∥ BC

AD : DB = 2 : 3

To Find : Area (ΔADE) : Area of (ΔABC)

Sol. Let AD = $2x$ and DB = $3x$

$\therefore$ AB $= 2x + 3x = 5x$

$\therefore$ DB ∥ BC

$$\therefore \quad \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta ABC} = \left(\frac{AD}{AB}\right)^2$$

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta ABC} = \left(\frac{2x}{5x}\right)^2$$

Area of ΔADE : Area of ΔABC

4 : 25.

8. In a ΔABC, AB = BC = CA = 2a and AD ⊥ BC Prove that AD = $a\sqrt{3}$ and area of ΔABC = $\sqrt{3a^2}$.

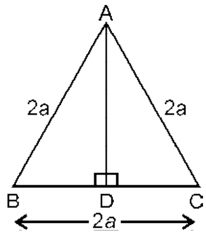
Sol. Given : AB = BC = CA = $2a$

AD ⊥ BC

To Find : AD $= a\sqrt{3}$

Area of ΔABC $= \sqrt{3a^2}$

Sol.



In ΔABD

$$AD = \sqrt{AB^2 - BD^2}$$

$$= \sqrt{(2a)^2 - a^2}$$

$$= \sqrt{4a^2 - a^2}$$

$$= \sqrt{3a^2} = a\sqrt{3} \quad \textbf{Proved.}$$

$$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 2a \times a\sqrt{3}$$

$$= \sqrt{3}a^2 \quad \textbf{Proved.}$$

9. Triangles ABC and DEF are similar. The area of ΔABC is 16 cm² and that of ΔDEF is 25 cm². If BC = 2.8 cm, find EF.

Sol. Given : Area of ΔABC = 16 cm^2

Area of ΔDEF = 25 cm^2

BC = 2.8 cm

To Find : EF

$\because$ $\Delta ABC \sim \Delta DEF$

$$\therefore \quad \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \left(\frac{BC}{EF}\right)^2$$

$$\frac{16}{25} = \frac{(2.8)^2}{EF^2}$$

$$EF = \sqrt{\frac{2.8^2 \times 25}{16}}$$

$$EF = \frac{2.8 \times 5}{4}$$

$= 3{\cdot}5$ cm.

10. The areas of two similar triangles ABC and DEF are 64 cm^2 and 169 cm^2 respectively. If the length of BC is 4 cm, find the length of EF.

Sol. Given : $\Delta ABC \sim \Delta DEF$

To Find : length of EF

$\because \quad \Delta ABC \sim \Delta DEF$

$$\therefore \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \left(\frac{BC}{EF}\right)^2$$

$$\frac{64}{169} = \left(\frac{4}{EF}\right)^2$$

$$\sqrt{\frac{64}{169}} = \frac{4}{EF}$$

$$EF = \frac{4 \times 13}{8}$$

$$= 6{\cdot}5 \text{ cm.}$$

11. Triangles ABC and DEF are similar as shown in the figure. The area of ΔABC is 16 cm^2 and that of ΔDEF = 25 cm^2. If BC = 2·3 cm find EF.

Sol. Given : $\Delta ABC \sim \Delta DEF$

Area of ΔABC = 16 cm^2

Area of ΔDEF = 25 cm^2

BC = 2·3 cm

To Find : EF

Sol. $\because \quad \Delta ABC \sim \Delta DEF$

$$\therefore \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \left(\frac{BC}{EF}\right)^2$$

$$\frac{16}{25} = \left(\frac{2{\cdot}3}{EF}\right)^2$$

$$\sqrt{\frac{16}{25}} = \frac{2{\cdot}3}{EF}$$

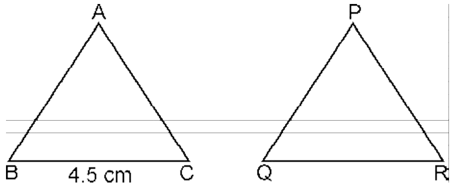
$$\frac{4}{5} = \frac{2{\cdot}3}{EF}$$

$$EF = \frac{2{\cdot}3 \times 5}{4}$$

$$= 2{\cdot}875$$

$$= 2{\cdot}9 \text{ nearly.}$$

12. The areas of two similar triangles ABC and PQR are in the ratio of 9 : 16. If BC = 4·5 cm, find the length of QR.



Sol. Given : $\Delta ABC \sim \Delta PQR$

Area of ΔABC: Area of ΔPQR = 9 : 16

length of BC = 4·5 cm

To Find : length of QR

$\because \quad \Delta ABC \sim \Delta PQR$

$$\therefore \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \left(\frac{BC}{QR}\right)^2$$

$$\frac{9}{16} = \left(\frac{4{\cdot}5}{QR}\right)^2$$

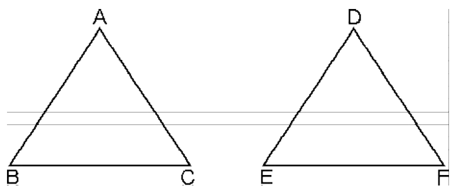
$$\sqrt{\frac{9}{16}} = \frac{4{\cdot}5}{QR}$$

$$\frac{3}{4} = \frac{4{\cdot}5}{QR}$$

$$QR = \frac{4{\cdot}5 \times 4}{3}$$

$$QR = 6 \text{ cm.}$$

13. The areas of two similar triangles are 100 cm^2 and 49 cm^2 respectively. If the altitude of the bigger triangle is 5 cm, find the corresponding altitude of the other.



Sol. Given : $\Delta ABC \sim \Delta DEF$

Area of ΔABC = 100 cm^2

Area of ΔDEF = 49 cm^2

Altitude of ΔABC = 5 cm

To Find : Altitude of ΔDEF

$\because \quad \Delta ABC \sim \Delta DEF$

$$\therefore \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \left(\frac{\text{Altitude of } \Delta ABC}{\text{Altitude of } \Delta DEF}\right)^2$$

$$\frac{100}{49} = \left(\frac{5}{\text{Altitude of } \Delta DEF}\right)^2$$

Altitude of $\Delta DEF = \frac{25 \times 49}{100}$

Altitude of $\Delta DFF = \sqrt{\frac{25 \times 49}{100}}$

Altitude of $\Delta DEF = \frac{5 \times 7}{10}$

$= 3{\cdot}5$ cm.

14. The areas of two similar triangles are 81 cm² and 49 cm² respectively. If the altitude of the first triangle is 6·3 cm, find the corresponding altitude of the other.

Sol. Given : areas of similar triangles = 81 cm² and 49 cm²

altitude of first triangle = 6.3 cm

To find : Altitude of other triangle.

$\because$ Triangles are similar

$$\therefore \frac{\text{Area of first triangle}}{\text{Area of other triangle}} = \left(\frac{\text{Altitude of first triangle}}{\text{Altitude of other triangle}}\right)^2$$

$$\frac{81}{49} = \left(\frac{6{\cdot}3}{\text{Altitude of other triangle}}\right)^2$$

$$\sqrt{\frac{81}{49}} = \frac{6{\cdot}3}{\text{Altitude of other triangle}}$$

Altitude of other triangle = $\frac{6.3 \times 7}{9}$

Altitude of other triangle = 4·9 cm.

15. The areas of two similar triangles are 81 cm² and 49 cm² respectively. If the altitude of bigger triangle is 4·5 cm, find the corresponding altitude of the smaller triangle.

Sol. Corresponding altitude of smaller triangle

$$= \sqrt{\frac{49}{81}} \times 4{\cdot}5$$

$$= \frac{7}{9} \times 4.5$$

$= 3{\cdot}5$ cm.

EXERCISE 7.4

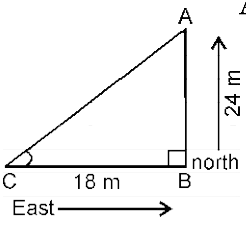
Multiple Choice Type Questions

1. Hari goes 18 m due east and then 24m due north. The distance from the starting point is :

(a) 40m **(b) 30m**

(c) 26m **(d) 42m.**

Sol. According to pythagoras theorem



$AC^2 = AB^2 + BC^2$

$AC = \sqrt{AB^2 + BC^2}$

$AC = \sqrt{24^2 + 18^2}$

$AC = \sqrt{576 + 324}$

$AC = \sqrt{900}$

$AC = 30$

Hence, distance from starting point = 30m.

2. The length of the second diagonal of a rhombus, whose side is 5 cm and one of the diagonals is 6 cm is :

(a) 7 cm **(b) 8 cm**

(c) 9 cm **(d) 12 cm.**

Sol. Length of second diagonal = d_2 of rhombus

Side = 5 cm

d_1 = 6 cm

$$S = \frac{1}{2}\sqrt{d_1^2 + d_2^2}$$

$$5 = \frac{1}{2}\sqrt{6^2 + d_2^2}$$

$(5 \times 2)^2 = 36 + {d_2}^2$

$100 - 36 = {d_2}^2$

$\sqrt{64} = d_2$

d = 8 cm.

3. In ΔABC, AB = $6\sqrt{3}$, AC = 12 cm and BC = 6 cm. The angle B is :

(a) 120° **(b) 60°**

(c) 90° cm **(d) 45°.**

Sol. Given : AB = $6\sqrt{3}$ cm, AC = 12 cm & BC = 6 cm

Converse of pythagoras theorem : In a triangle, if the square of one side is equal to the sum of squares of the other two sides then the angle opposite to the first side is a right angle.

$AC^2 = AB^2 + BC^2$

$12^2 = \left(6\sqrt{3}\right)^2 + 6^2$

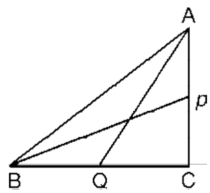
$144 = 108 + 36$

$144 = 144$

$\therefore$ ΔABC is a right angle triangle.

$\therefore \quad \angle B = 90°$.

4. P and Q are the mid points on the sides CA and CB respectively of ΔABC right angled at C. Prove that $4(AQ^2 + BP^2) = 5AB^2$.



Sol. Given : P is mid point of CA and Q is mid point of CB; $\angle C = 90°$.

To Prove : $4(AQ^2 + BP^2) = 5AB^2$

Proof : In ΔACQ

$AQ^2 = AC^2 + QC^2$

$AQ^2 = AC^2 + (BC/2)^2$

$AQ^2 = AC^2 + BC^2/4$ ...(1)

In ΔBPC

$BP^2 = BC^2 + PC^2$

$BP^2 = BC^2 + (AC/2)^2$

$BP^2 = BC^2 + AC^2/4$ ...(2)

On adding equation (1) and (2) we get :

$AQ^2 + BP^2 = AC^2 + BC^2 + AC^2/4 + BC^2/4$

$4(AQ^2 + BP^2) = 5AC^2 + 5BC^2$

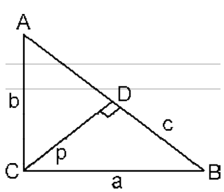
$4(AQ^2 + BP^2) = 5(AC^2 + BC^2)$

$4(AQ^2 + BP^2) = 5(AB^2)$ **Proved.**

5. ABC is a right triangle, right angled at C. If *p* is the length of the perpendicular from C to AB and *a*, *b*, *c* have the usual meaning, then prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

Sol. Given : ΔABC is a right triangle

$\angle C = 90°$



To Prove : $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Proof : ABC is a right angled at C.
Let BC = a, CA = b, AB = c

$$\text{Area}(\Delta ABC) = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times bc \times ab = \frac{1}{2}\, ab \quad ...(1)$$

$$\text{Area}(\Delta ABC) = \frac{1}{2} \times ab \times cd = \frac{1}{2}\, cp \quad ...(2)$$

From equation (1) and equation (2)

$$\frac{1}{2}ab = \frac{1}{2}cp.$$

$\therefore \quad ab = cp$ ...(3)

In ΔABC, by pythagoras theorem :

$AB^2 = BC^2 + AC^2$

$$c^2 = a^2 + b^2 \left[\text{From (3)}, c = \frac{ab}{p}\right]$$

$$\left(\frac{ab}{p}\right)^2 = a^2 + b^2$$

$$= \frac{a^2b^2}{p^2} = a^2 + b^2$$

$$= \frac{1}{p^2} = \frac{a^2 + b^2}{a^2b^2}$$

$$\frac{1}{p^2} = \frac{a^2}{a^2b^2} + \frac{b^2}{a^2b^2}$$

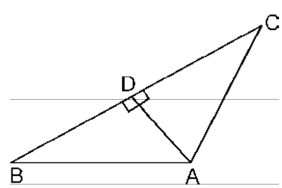
$$\frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{a^2}$$ **Proved.**

6. In a ΔABC, $AD \perp BC$. Prove that $AB^2 + CD^2 = AC^2 + DB^2$.

Sol. Given : $AD \perp BC$

To Prove : $AB^2 + CD^2 = AC^2 + DB^2$

Proof :



Since, $AD \perp BC$

$\angle ADC = \angle ADB = 90°$

So, ΔADB is a right triangle using pythagoras theorem

$(\text{Hypotenuse}) = (\text{Height})^2 + (\text{Base})^2$

$(AB)^2 = (AD)^2 + (BD)^2$ ...(1)

ΔADC is also a right triangle using pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{Height})^2 + (\text{Base})^2$

$(AC)^2 = (AD)^2 + (CD)^2$ ...(2)

on subtracting equation (2) from equation (1) we get

$(AB)^2 - (AC)^2) = (AD^2 + BD^2) - (AD^2 + CD^2)$

$AB^2 - AC^2 = AD^2 + BD^2 - AD^2 - CD^2$

$AB^2 + CD^2 = BD^2 + AC^2$

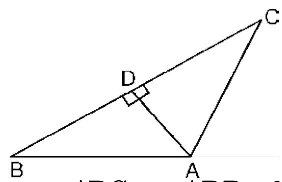
Proved.

7. In a triangle, AD is drawn perpendicular to BC. Prove that $AB^2 - BD^2 = AC^2 - CD^2$.

Sol. Given : $AD \perp BC$

To Prove : $AB^2 - BD^2 = AC^2 - CD^2$

Proof : Since $AD \perp BC$



$\angle ADC = \angle ADB = 90°$

So, ΔADB is right triangle using pythagoras theorem.

$(\text{Hypotenuse})^2 = (\text{Height})^2 + (\text{Base})^2$

$AB^2 = AD^2 + BD^2$ ...(1)

ΔADC is also a right triangle using pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{Height})^2 + (\text{Base})^2$

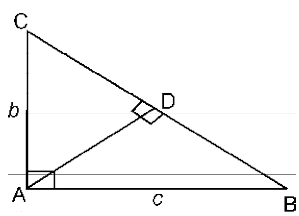
$AC^2 = AD^2 + CD^2$ ...(2)

On subtracting equation (2) from equation (1), we get

$AB^2 - AC^2 = AD^2 + BD^2 - AD^2 - CD^2$

$AB^2 - BD^2 = AC^2 - CD^2$ **Proved.**

8. In a right-angled triangle, the square of the hypotenuse is equal to the sum of squares of the other two sides. Prove using the above theorem, determine the length of AD in terms of *b* and *c*.



Sol. Given : ΔABC is a right triangle.

To Prove : $BC^2 = AC^2 + AB^2$

Proof : We know that. If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then triangle on both sides of the perpendicular are similar to the whole triangle and to each other.

$\therefore \quad \Delta ADB \sim \Delta ABC$

$$\frac{DB}{AB} = \frac{AB}{BC}$$

or $DB \times BC = AB^2$ ...(1)

Also, $\Delta ADC \sim \Delta ABC$

$$\therefore \quad \frac{DC}{AC} = \frac{AC}{BC}$$

$DC \times BC = AC^2$ ...(2)

On adding equation (1) & (2), we get–

$(DB \times BC) + (DC \times BC) = (AB)^2 + (AC)^2$

$BC(DB + DC) = AB^2 + AC^2$

$BC \times BC = AB^2 + AC^2$

$BC^2 = AB^2 + AC^2$

Hence, in a right angle triangle, the square of hypotenuse is equal to the sum of squares of other two sides.

Proved.

In ΔABC

$BC^2 = AC^2 + AB^2$

$BC^2 = b^2 + c^2$

In ΔADC

$AC^2 = AD^2 + CD^2$

$b^2 = AD^2 + CD^2$ ...(1)

In ΔADB

$AB^2 = AD^2 + DB^2$

$c^2 = AD^2 + DB^2$ ...(2)

On Adding the equation (1) and (2)

$b^2 + c^2 = 2AD^2 + CD^2 + DB^2$

$BC^2 = 2AD^2 + CD^2 + DB^2$

$(CD + DB)^2 = 2AD^2 + CD^2 + DB^2$

$CD^2 + DB^2 + 2 \times CD \times DB = 2AD^2 + CD^2 + DB^2$

$2AD^2 = 2.CD.DB$

$AD^2 = CD \times DB$

$AD = \sqrt{b^2 - AD^2} \times \sqrt{c^2 - AD^2}$.

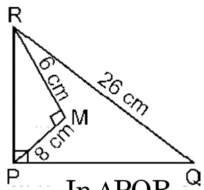
9. M is a point in the interior of a right angled ΔPQR right angled at P such that $\angle RMP = 90°$. If

length of sides RM, PM and QR are 6 cm, 8 cm and 26 cm respectively then find area of ΔPQR.

Sol. Given RM = 6 cm
PM = 8 cm
QR = 26 cm

To Find : Area of ΔPQR

ΔRMP



$$RP = \sqrt{RM^2 + PM^2}$$
$$RP = \sqrt{6^2 + 8^2}$$
$$RP = \sqrt{36 + 64}$$
$$RP = \sqrt{100}$$
$$RP = 10 \text{ cm}$$

In ΔPQR

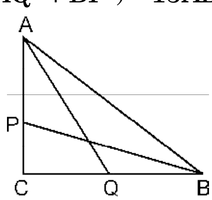
$$RQ^2 = RP^2 + PQ^2$$
$$26^2 = 10^2 + PQ^2$$
$$676 = 100 + PQ^2$$
$$675 - 100 = PQ^2$$
$$576 = PQ^2$$
$$\sqrt{576} = PQ$$
$$PQ = 24 \text{ cm.}$$

$$\text{Area of } \Delta PQR = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times RP \times PQ$$
$$= \frac{1}{2} \times 10 \times 24$$
$$= 120 \text{ cm}^2.$$

10. In a right triangle ABC, right angled at C, P and Q are the points of the sides CA and CB respectively, which divide these sides in the ratio 2 : 1. Prove that :
(i) $9AQ^2 = 9AC^2 + 4BC^2$
(ii) $9BP^2 = 9BC^2 + 4AC^2$
(iii) $9(AQ^2 + BP^2) = 13AB^2$

Sol. Given : ΔABC is a right triangle at C. P and Q are point on CA and CB respectively.

To Prove : (1) $9AQ^2 = 9AC^2 + 4BC^2$
(2) $9BP^2 = 9BC^2 + 4AC^2$
(3) $9(AQ^2 + BP^2) = 13AB^2$



Proof : In a right angle ΔACQ

$$AQ^2 = AC^2 + CQ^2$$

$$\left[\frac{CQ}{QB} = \frac{2}{1}, \frac{CQ}{BC - CQ} = \frac{2}{1}, 3CQ = 2BC, CQ = 2\frac{BC}{3}\right]$$

$$AQ^2 = AC^2 + \left(2\frac{BC}{3}\right)^2$$
$$AQ^2 = AC^2 + \frac{4BC^2}{9}$$
$$9AQ^2 = 9AC^2 + 4BC^2 \quad ...(1)$$

Similarly in a right angle ΔBCP, we get,

$$9BP^2 = 9BC^2 + 4AC^2 \quad ...(2)$$

On adding equation (1) & (2) we get,

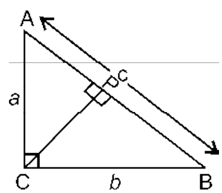
$$9AQ^2 + 9BP^2 = 9AC^2 + 4BC^2 + 9BC^2 + 4AC^2$$
$$9(AQ^2 + BP^2) = 13AC^2 + 13BC^2$$
$$9(AQ^2 + BP^2) = 13(AC^2 + BC^2)$$
$$9(AQ^2 + BP^2) = 13AB^2.$$

11. ABC is a right triangle, right angled at C. If *p* is the length of the perpendicular from C to AB and *a*, *b*, *c* have the usual meaning, then prove that :

(i) $pc = ab$

(ii) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Sol.



Given : $p \perp AB$

To Prove : (i) $pc = ab$

(ii) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Proof : Area of $\Delta ABC = \frac{1}{2} \times AB \times CP$

$$= \frac{1}{2} \times cp \quad ...(1)$$

$$\text{Area of } \Delta ABC = \frac{1}{2} \times BC \times AC$$

$$= \frac{1}{2} \times ab \qquad ...(2)$$

From equation (1) and (2), we get :

$$\frac{1}{2} cp = \frac{1}{2} ab$$

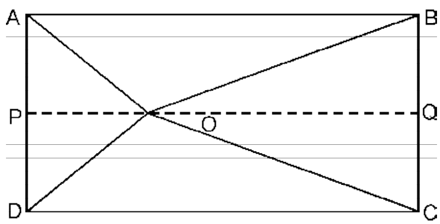
$$\therefore \quad cp = ab \qquad ...(3)$$

In ΔACB

$$AB^2 = AC^2 + BC^2$$
$$c^2 = b^2 + a^2$$
$$\frac{a^2b^2}{p^2} = b^2 + a^2$$
$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$ **Proved.**

12. A point O in the interior of a rectangle ABCD is joined with each of vertices, A, B, C & D. Prove that : $OB^2 + OD^2 = OC^2 + OA^2$.

Sol.



Given : O is interior point on rectangle ABCD.

To Prove : $OB^2 + OD^2 = OC^2 + OA^2$

Proof : We draw PQ || AB || CD as shown in the figure.

ABCD is a rectangle, it means ABPQ and PQDC are also rectangle.

For, ABPQ

AP = BQ[Opposite sides are equal]

For, PQDC

PD = QC[Opposite sides are equal]

Now, for ΔOPD

$$OD^2 = OP^2 + PD^2 \qquad ...(1)$$

For ΔOQB

$$OB^2 = OQ^2 + BQ^2 \qquad ...(2)$$

On adding equation (1) and (2), we get–

$$OB^2 + OD^2 = (OP^2 + PD^2) + (OQ^2 + BQ^2)$$
$$= (OP^2 + CQ^2) + (OQ^2 + AP^2)$$

ΔOPA and ΔOQC are also right angled triangle,

For $\quad \Delta OCQ = OQ^2 + CQ^2 = OC^2$

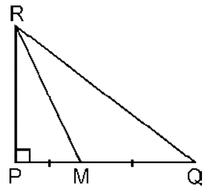
For $\quad \Delta OPA = OP^2 + AP^2 = OA^2$

Now, $OB^2 + OD^2 = OC^2 + OA^2$

Proved.

13. In a right angled triangle PQR, $\angle P = 90°$. If M is the mid point of PQ, prove that $QR^2 = 4MR^2 - 3PR^2$.

Sol. Given : In ΔPQR, $\angle P = 90°$; M is the mid point of PQ



$$PM = MQ = \frac{1}{2} PQ$$

To Prove : $QR^2 = 4MR^2 - 3PR^2$

In ΔRPM

$$MR^2 = PR^2 + PM^2$$
$$MR^2 = PR^2 + \left(\frac{1}{2} PQ\right)^2$$
$$MR^2 = (PR^2 + PR^2Q^2)/4$$
$$4MR^2 = 4PR^2 + PQ^2$$
$$PQ^2 = 4RM^2 - 4PR^2 \quad ...(1)$$

In ΔPQR

$$QR^2 = PR^2 + PQ^2$$
$$QR^2 = PR^2 + (4RM^2 - 4PR^2)$$
$$QR^2 = PR^2 + 4RM^2 - 4PR^2$$
$$QR^2 = 4RM^2 - 3PR^2$$

Hence, it is prove that

$$QR^2 = 4RM^2 - 3PR^2.$$

14. In a quadrilateral ABCD, $\angle B = 90°$ and $AD^2 = AB^2 + BC^2 + CD^2$. Prove that $\angle ACD = 90°$.

Sol. Given : $\angle B = 90°$

$$AD^2 = AB^2 + BC^2 + CD^2$$

To Prove : $\angle ACD = 90°$

Proof : In quadrilateral ABCD

$$\angle ABC = 90°$$

Hence, $AD^2 + BC^2 = AC^2 \qquad ...(1)$

$$AD^2 = (AB^2 + BC^2) + CD^2$$

(Given)

$$AD^2 = AC^2 + CD^2 \qquad ...(2)$$

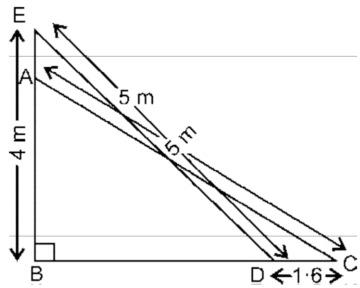
In ΔACD, AD has to be hypotenuse and angle ACD must be 90°.

Hence, it is proved that

$$\angle ACD = 90°.$$

15. A 5m long ladder is placed learning towards a vertical wall such that it reaches the wall at a point 4m high. If the foot of the ladder is moved 1.6 m towards the wall, then find the distance by which the top of the ladder would slide upwards on the wall.

Sol.



In ΔABC

$$AC^2 = AB^2 + BC^2$$
$$5^2 = 4^2 + BC^2$$
$$\sqrt{5^2 - 4^2} = BC$$
$$BC = 3 \text{ cm}$$
$$BD = BC - DC$$
$$BD = 3 - 1{\cdot}6 = 1{\cdot}4 \text{ cm}$$

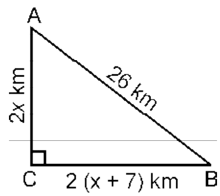
In ΔEBD

$$ED^2 = EB^2 + BD^2$$
$$5^2 = EB^2 + 1{\cdot}4^2$$
$$EB = \sqrt{25 - 1{\cdot}96}$$
$$EB = 4{\cdot}8$$
$$EA = EB - AB$$
$$EA = 4{\cdot}8 - 4 = 0{\cdot}8 \text{ cm.}$$

Hence the distance by which the top of the ladder slide upwards the wall $= 0{\cdot}8$ cm.

16. For going to a city B from city A, there is a route via city C such that AC $\perp$ CB, AC = $2x$ km and CB = 2 (x + 7) km. It is proposed to construct a 26 km highway which directly connects the two cities A and B. Find how much distance will be saved in reaching city B from city A after the construction of the highway.

Sol.



In ΔABC

$$AB^2 = AC^2 + BC^2$$
$$26^2 = (2x)^2 + [2\,(x + 7)]^2$$
$$676 = 4x^2 + 4\,(x^2 + 49 + 14x)$$
$$676 = 4x^2 + 4x^2 + 196 + 56x$$
$$676 = 8x^2 + 56x + 196$$
$$8x^2 + 56x - 480 = 0$$
$$8(x^2 + 7x - 60) = 0$$
$$x^2 + 7x - 60 = 0$$
$$x^2 + 12x - 5x - 60 = 0$$
$$x\,(x + 12) - 5\,(x + 12) = 0$$
$$(x + 12)\,(x - 5)$$
$$x = -12,\ x = 5$$

Since distance cannot be negative.

$\therefore$ $x = 5$

The distance covered to reach city B from city A = 34 km. Hence, the required saved distance = 34 – 26 = 8 km.

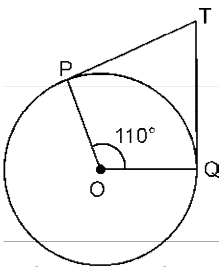
❑

8 Tangent to a Circle

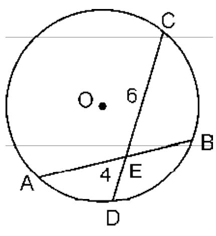
EXERCISE 8.1

Multiple Choice Type Questions

1. **From a point Q, the length of the tangent of a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is ?**
(a) 7 cm
(b) 12 cm
(c) 15 cm
(d) 24.5 cm.



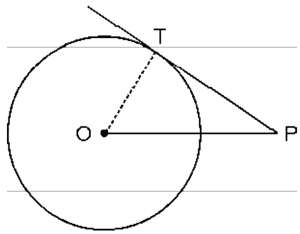
2. **In Fig., if *TP* and *TQ* are the two tangents to a circle with centre *O* so that $\angle POQ = 110°$, then $\angle PTQ$ is equal to :**
(a) 60° (b) 70°
(c) 80° (d) 90°
3. **If tangents *PA* and *PB* from a point *P* to a circle with centre *O* are inclined to each other at angle of 80°, then $\angle POA$ is equal to :**
(a) 50° (b) 60°
(c) 70° (d) 80°
4. **In the adjoining figure, the point *O* is the centre of circle whose *AB* and *CD* are two chords intersecting each other at *E*. If *CE* = 4 cm, *ED* = 4 cm, then the area of rectangle whose adjacent sides are *AE* and *EB* is :**
(a) 27 cm^2
(b) 24 cm^2
(c) 20 cm^2
(d) 18 cm^2.



Ans. 1. (d), **2.** (c), **3.** (a), **4.** (b).

Very Short Answer Type Questions

5. **In the following figure, *PT* is a tangent to a circle whose centre is *O*. If *OP* = 17 cm and *OT* = 8 cm, find the length of the tangent segment *PT*.**



Sol. Since, the tangent at a point on a circle is perpendicular to the radius through the points of contact, we have $OT \perp PT$.

$$\therefore \quad PT = \sqrt{OP^2 - OT^2}$$

$$= \sqrt{(17)^2 - (8)^2},$$

[$\because$ *OP* = 17 cm and *PT* = 8 cm]

$$= \sqrt{289 - 64}$$

$$= \sqrt{225}$$

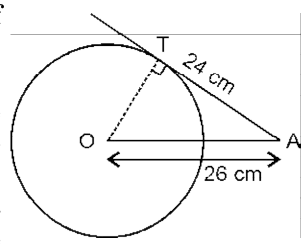
i.e., $PT = 15$ cm

$\therefore$ Length of tangent segment,

PT = 15 cm. **Ans.**

6. **A point *A* is 26 cm away from the centre of a circle and the length of tangent drawn from *A* to the circle is 24 cm. Find the radius of the circle.**

Sol. Let *OA* = 26 cm, where *O* is the centre of the circle and *AT* is the tangent to the circle. Also, let *OT* is the radius of the circle.



Here, *AT* = 24 cm

In right ΔOTA, we have

$$OT = \sqrt{OA^2 - AT^2}$$

$$= \sqrt{(26)^2 - (24)^2} = \sqrt{676 - 576}$$

$$= \sqrt{100} = 100 \text{ cm}$$

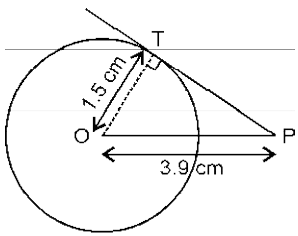
Hence, the radius of the circle is 10 cm. **Ans.**

7. In the adjoining figure, *O* is the centre of the circle and its radius is 1.5 cm. *P* is a point at a distance of 3.9 cm from *O* and *PT* is a tangent drawn to the circle from the point P. Find the length of *PT*.

Sol. Here, $OP = 3.9$ cm

and $OT = 1.5$ cm

In right ΔOTP, we have



$$PT = \sqrt{OP^2 - OT^2}$$

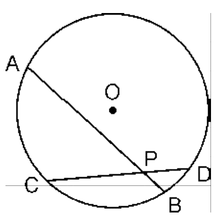
$$= \sqrt{(3.9)^2 - (1.5)^2}$$

$$= \sqrt{15.21 - 2.25}$$

$$= \sqrt{12.96} = 3.6 \text{ cm}$$

Hence, the length of *PT* is 3.6 cm. **Ans.**

8. In the following figure, if *AP* = 8 cm, *CP* = 6 cm and *PD* = 4 cm. Find the length of *AB*.



Sol. Here, $AP = 8$ cm, $CP = 6$ cm and $PD = 4$ cm.

$\therefore$ $PA.PB = PC.PD$

or $8 \times PB = 6 \times 4$

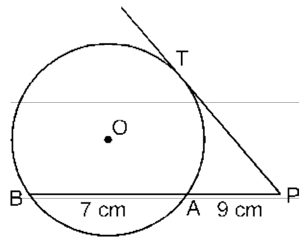
i.e., $PB = \dfrac{6 \times 4}{8} = 3$ cm

$\therefore$ $AB = AP + PB$

$= 8 + 3 = 11$ cm

Hence, $AB = 11$ cm. **Ans.**

9. In the following figure, *PT* is a tangent to circle at point *T* and *PAB* is secant of the circle. If *PA* = 9 cm and *AB* = 7 cm, find *PT*.



Sol. Given, $PA = 9$ cm

and $AB = 7$ cm

$\therefore$ $PB = PA + AB$

$= 9 + 7 = 16$ cm.

Since *PT* is tangent to circle at point T and PAB is a secant to circle,

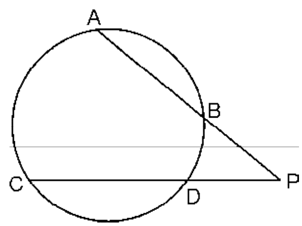
$\therefore$ $PT^2 = PA.PB$

or $PT^2 = 9 \times 16 = 144$

i.e., $PT = \sqrt{144}$

$= 12$ cm. **Ans.**

10. If *AB* and *CD* are two chords of a circle which, when produced meet at a point *P* and if *PA* = *PC*, show that *AB* = *CD*.



Sol. Given, $PA = PC$

We know that whenever two chords of a circle intersect inside or outside the circle when produced, then the rectangle formed by the two segments of one chord is equal in area to the rectangle formed by the two segments of another chord.

$\therefore$ $PA.PB = PC.PD$

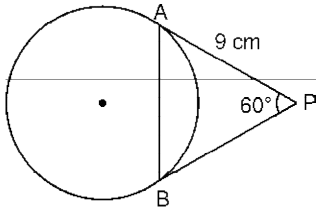
But given, $PA = PC$

$\therefore \quad PB = PD$

i.e., $PA - PB = PC - PD$

Hence, $AB = CD$. **Ans.**

11. In the following figure, *PA* and *PB* are tangents such that *PA* = 9 cm and $\angle APB$ = 60°. Find the length of chord *AB*.



Sol. $PB = PA$

$= 9$ cm

So, ΔAPB is isosceles

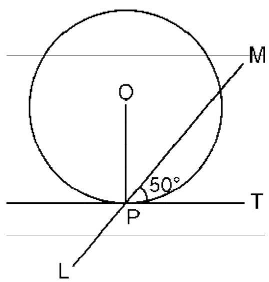
$\therefore \quad \angle PAB = \angle PBA = 60°$

But gven that $\angle APB = 60°$

Thus, ΔABP is equilateral

Hence, $AB = 9$ cm. **Ans.**

12. In the following figure, *OP* is the radius of the circle. At the point P, a secant *LM* is drawn making an angle of 50° with the tangent *PT* drawn at *P* to the circle. Find the measure of $\angle OPM$.



Sol. $OP \perp PT$

Since, $\angle OPT = 90°$

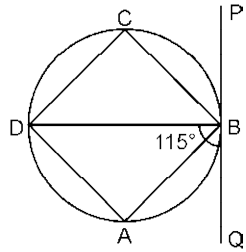
But given, $\angle MPT = 50°$

$\therefore \quad \angle OPM = \angle OPT - \angle MPT$

$= 90° - 50°$

i.e., $\angle OPM = 40°$. **Ans.**

13. In the figure, *ABCD* is a cyclic quadrilateral. The tangent *PBQ* is drawn at the point *B* of the circle. If $\angle DBQ$ = 115° then find the value of $\angle DAB$.



Sol. $\angle DCB = \angle DBQ = 115°$

Now $\square ABCD$ is a cyclic quadrilateral.

$\angle DCB + \angle DAB = 180°$

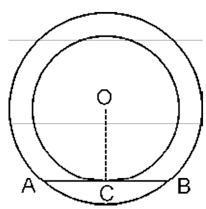
$\therefore \quad 115° + \angle DAB = 180°$

$\therefore \quad \angle DAB = 180° - 115°$

$\therefore \quad \angle DAB = 65°$. **Ans.**

14. In two concentric circles, prove that a chord of larger circle which is tangent to smaller circle is bisected at the point of contact.

Sol. Let there be two concentric circles with the same centre O. Let AB be a chord of the larger circle touching the smaller circle at C. Join OC.



Since, OC is the radius of the smaller circle and AB is a tangent to this circle at the point C, so, $OC \perp AB$.

Now, $OC \perp AB$ and AB is a chord of the larger circle. So, C is the mid-point of AB, *i.e.*, $AC = CB$.

15. A circle touches all the four sides of a quadrilateral *PQRS*. Prove that *PQ* + *RS* = *QR* + *SP*.

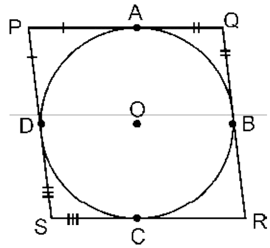
Sol. Let the circle touches the sides PQ, QR, RS and SP at the points A, B, C and D respectively. Then by Theorem 3, we have

$PA = PD$

$QA = QB$

$SC = SD$

and $RC = RB$



Adding all these, we get

$PA + QA + SC + RC = PD + QB + SD + RB$

or $(PA + AQ) + (SC + CR) = (QB + BR) + (PD + DS)$

i.e., $PQ + RS = QR + SP$. **Proved.**

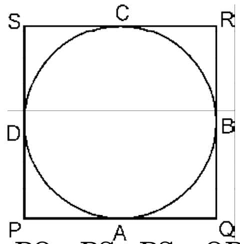
16. If all the sides of a parallelogram touch a circle, show that the parallelogram is a rhombus.

Sol. Let the sides PQ, QR, RS and SP of a parallelogram PQRS touch a given circle at the points A, B, C and D respectively. Since the lengths of two tangents drawn from an external point to a circle are equal, we have

PA = PD, QA = QB, RC = RB

and SC = SD

$\therefore$ PQ + RS = PA + QA + RC + SC
= PD + QB + RB + SD
= (PD + DS) + (QB + BR)
= PS + QR



Now, PQ + RS = PS + QR

or PQ + PQ = PS + PS, [$\because$ RS = PQ and QR = PS]

or 2PQ = 2PS

i.e., PQ = PS

But PQ = RS and QR = PS

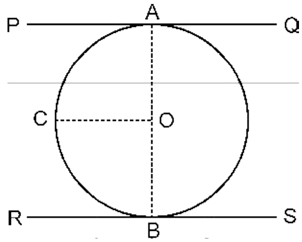
$\therefore$ PQ = QR = RS = SP

Hence, PQRS is a rhombus.

Proved.

17. Prove that the line segment joining the points of contact of two parallel tangents passes throgh the centre.

Sol. Let PAQ and RBS be two parallel tangents to a circle with centre O. Joint OA and OB.



Draw OC || PQ, Now, PA || CO

$\angle PAO + COA = 180^\circ$

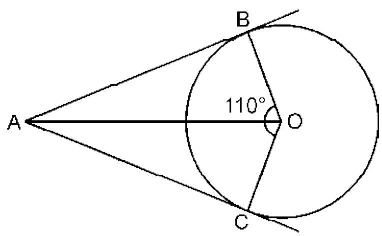
But, $\angle PAO = 90^\circ$

So, $\angle COA = 90^\circ$

Similarly, $\angle COB = 90^\circ$

Hence, AOB is a straight line.

18. In figure, O is the centre of circle. If $\angle BOC = 110^\circ$ and AB, AC are tangents to the circle. Find the measure of $\angle OAB$.



Sol. In quadrilateral ABOC $90^\circ + 110^\circ + 90^\circ + \angle A = 360^\circ$

$\therefore \quad \angle A = 360^\circ - 290^\circ \quad (\because \angle B = \angle C = 90^\circ)$

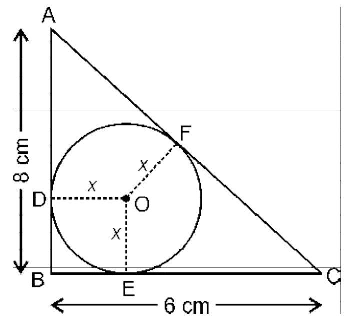
$= 70^\circ$

$\therefore \quad \angle OAB = \frac{1}{2} \angle A$

$= \frac{1}{2} (70^\circ)$

$= 35^\circ$. **Ans.**

19. ABC is a right angled triangle with BC = 6 cm and AB = 8 cm. A circle with centre O and radius *x* cm has been inscribed in $\triangle$ABC. Find the value of *x*.



Sol. Here, AF = AD = AB – DB

∴ AF = $(8 - x)$ cm

and CF = CE = BC – BE

∴ CF = $(6 - x)$ cm.

Now, in right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

or $(14 - 2x)^2 = (8)^2 + (6)^2$

[∵ AC = AF + CF = $8 - x + 6 - x = 14 - 2x$]

or $196 + 4x^2 - 56x = 64 + 36$

or $4x^2 - 56x + 96 = 0$

or $x^2 - 14x + 24 = 0$

or $x^2 - 12x - 2x + 24 = 0$

or $x(x - 12) - 2(x - 12) = 0$

or $(x - 12)(x - 2) = 0$

i.e., $x = 12$

or $x = 2$.

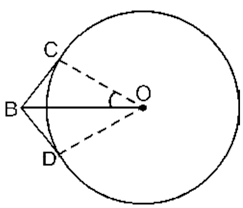
Neglecting $x = 12$ cm,

we have $x = 2$ cm. **Ans.**

20. Two tangent segments BC and BD are drawn to a circle with centre O such that ∠CBD = 120°. Prove that OB = 2BC.

Sol. Join OB, OC and OD.

Given, ∠CBD = 120°



∵ BO is the angle bisector of ∠CBD,

∴ ∠OBC = ∠OBD = 60°

Here, BC and BD are the tangent segments

∴ ∠OCB = 90°

and ∠ODB = 90°

Now, in right ΔBOC, we get

∠BOC + ∠OCB + ∠CBO = 180°

or ∠BOC + 90° + 60° = 180°

i.e., ∠BOC = 180° – 90° – 60° = 30°

Also, $\sin 30° = \dfrac{BC}{OB}$

or $\dfrac{1}{2} = \dfrac{BC}{OB}$ $\left[\because \sin 30° = \dfrac{1}{2}\right]$

i.e., OB = 2BC. **Proved.**

Long Answer Type Questons

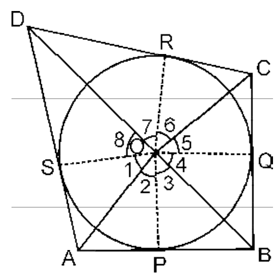
21. A circle touches the sides of a quadrilateral ABCD at the points P, Q, R and S respectively. Show that the angles subtended at the centre by a pair of opposite sides are supplementary.

Sol. Given. A circle with centre O touches the sides AB, BC, CD and DA of quadrilateral ABCD at P, Q, R and S respectively.

To Prove. ∠AOB + ∠COD = 180°

and ∠AOD + ∠BOC = 180°

Construction. Join OP, OQ, OR and OS. Also join AO, BO, CO and DO.



Proof. Since, the two tangents drawn from an external point of a circle subtend equal angles at the centre, we have

∠1 = ∠2, ∠3 = ∠4, ∠5 = ∠6

and ∠7 = ∠8 ...(i)

But ∠1 + ∠2 + ∠3 + ∠4 + ∠5 + ∠6 + ∠7 + ∠8 = 360°

[Sum of the angles at a pont is 360°]

∴ 2 (∠2 + ∠3 + ∠6 + ∠7) = 360°

and 2 ($\angle 1 + \angle 8 + \angle 4 + \angle 5$) = 360°

[Using (i)]

$\therefore$ $\angle 2 + \angle 3 + \angle 6 + \angle 7 = 180°$

and $\angle 1 + \angle 8 + \angle 4 + \angle 5 = 180°$

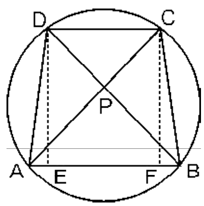
i.e., $\angle AOB + \angle COD = 180°$

[$\because$ $\angle 2 + \angle 3 = \angle AOB$ and $\angle 6 + \angle 7 = \angle COD$]

and $\angle AOD + \angle BOC = 180°$

[$\because$ $\angle 1 + \angle 8 = \angle AOD$ and $\angle 4 + \angle 5 = \angle BOC$]

22. In a trapezium ABCD, AB || CD and AD = BC. If P is the point of intersection of diagonals AC and BD, prove that PA × PC = PB × PD.



Sol. Draw DE $\perp$ AB and CF $\perp$ AB.

Now, in right triangles DEA and CFB, we have

AD = BC and DE = CF [$\because$ DE and CF are the $\perp$ distances between the same two || lines AB and CD]

$\therefore$ $\Delta DEA \cong \Delta CFB$

[By R.H.S. congruence rule]

$\therefore$ $\Delta DAE = \angle CBE$ [c.p.c.t.c.]

or $\angle DAB = \angle CBA$

Now AB and CD are two parallel lines and they cut out by the transversal AD.

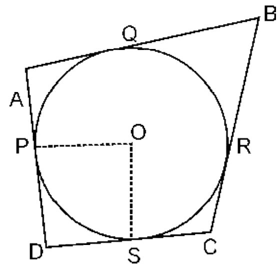
$\therefore$ $\angle ADC + \angle DAB = 180°$, [$\because$ Sm of the interior angles on the same side of the transversal is 180°]

But $\angle DAB = \angle CBA$ [Proved above]

$\therefore$ $\angle ADC + \angle CBA = 180°$

$\therefore$ ABCD s a cyclic quadrlateral.

[$\because$ Sum of one pair of opposite $\angle s$, of a cyclic quadrilateral is 180°.]



Thus, AC and BD are the two chords of a circle passing through A, B, C and D. These chords intersect at P.

Hence, PA.PC = PB.PD.

23. In the given, a circle is inscribed within a quadrilateral ABCD. Given that BC = 38 cm, BQ = 27 cm and DC = 25 cm and that AD is perpendicular to DC find the radius of the circle.

Sol. Since, the lengths of tangents drawn from an external point to a circle are equal,

$\therefore$ BQ = BR = 27 cm.

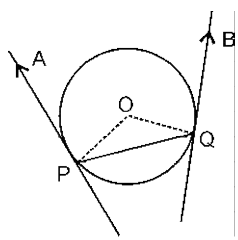
Let x be the radius of the circle.

$\therefore$ OP = OS = PD = x cm,

CR = BC – BR = 38 – 27 = 11 cm

Now, CS = CR = 11 cm

[$\because$ Lengths of tangents from an external point to a circle are equal]



$\therefore$ DS = CD – CS = 25 – 11 = 14 cm

But, DS = OP = x = 14 cm

Hence, the radius of the circle is 14 cm. **Ans.**

24. Show that the tangents at the extremities of a chord of a circle make equal angles with the chord.

Sol. Let PQ be a chord of a circle with centre O and let AP and BQ be the tangents at P and Q respectively. Join OP and OQ.

Now, OP = OQ

$\therefore$ $\angle OPQ = \angle OQP$

Also, $OP \perp AP$ and $OQ \perp BQ$

$\therefore$ $\angle APQ = 90^\circ + \angle OPQ$

$= 90^\circ + \angle OQP = \angle BQP.$

25. The circle of a ΔABC touches the sides AB, BC and CA at the points P, Q, R respectively. Show that :

AP + BQ + CR = BP + QC + RA

$= \frac{1}{2}$ (Perimeter of ΔABC)

Sol. Since, the lengths of two tangents drawn from an external point to a circle are equal we have

AP = RA, BQ = PB and CR = QC.

On adding all these, we get

AP + BQ + CR = RA + PB + QC ...(i)

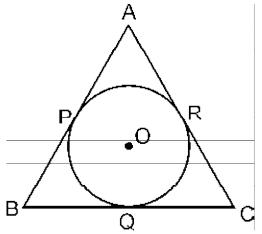
and, perimeter of ΔABC

= AB + BC + CA

= AP + PB + BQ + QC + CR + RA

= (AP + BQ + CR) + (PB + QC + RA)

= 2 (AP + BQ + CR) [Using (i)]



$\therefore$ AB + BQ + CR =

$\frac{1}{2}$ (perimeter of ΔABC)

Hence,

AP + BQ + CR = PB + QC + RA

$= \frac{1}{2}$ (perimeter of ΔABC)

❑

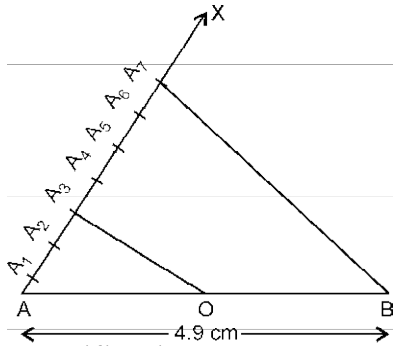
9 Constructions

EXERCISE 9.1

1. Draw a line segment of length 4.9 cm and divide it internally in the ratio 3 : 4. Measure the two parts.

Sol. Steps of Constructions :

1. Draw a line segment AB = 4·9 cm.
2. Draw a ray AX, making an acute ∠BAX.
3. Along AX mark 3 + 4 = 7 points A_1, A_2, A_3...A_7 such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7 = AA_1$.
4. Join A_7B.
5. From A_3 draw O||A_7B meeting AB at O Then, O is the point on AB which divides it in the ratio 3 : 4. So, AO : OB = 3 : 4



Justification :

Let $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7 = x$

In ΔABA_7, we have

$$A_3O \parallel A_7B$$

$$\frac{AO}{OB} = \frac{A_1A_3}{A_3A_7} = \frac{3x}{4x} = \frac{3}{4}$$

AO : OB = 3 : 4

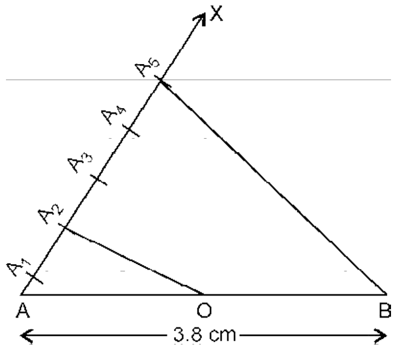
On measuring we find that AO = 2.2 cm

and OB = 2.7 cm.

2. Draw a line segment of length 3.8 cm and divide it internally in the ratio 2 : 3. Measure the two parts.

Sol. Steps of Construction :

1. Draw a line segment AB = 3.8 cm
2. Draw a ray AX, making an acute ∠BAX.
3. Along AX mark 2 + 3 = 5 points A_1, A_2, A_3, A_4, A_5
4. Join A_5B
5. From A_2 draw $A_2O \parallel A_5B$ meeting AB at O. Then O is the point on AB which divide it in the ratio 2 : 3. So AO : OB = 2 : 3



Justification :

It $AA_1 = AA_2 = AA_3 = AA_4 = AA_5 = x$

In ΔABA_5, we have

$$A_2O \parallel A_5B$$

$$\frac{AO}{OB} = \frac{AA_2}{A_2A_5} = \frac{2x}{3x} = \frac{2}{3}$$

Hence AO : OB = 2 : 3

On measuring use find that

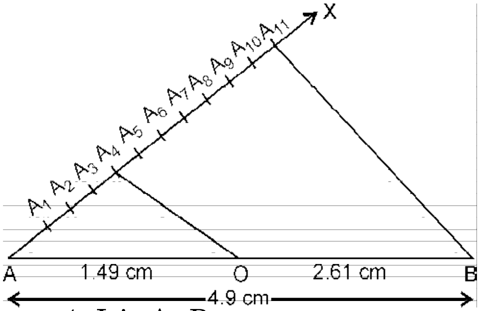
AO = 1.52 cm

and OB = 2.28 cm.

3. Draw a line segment of length 4.1 cm and divide it internally in the ratio 4 : 7. Measure the two points.

Sol. Steps of Construction :

1. Draw a line segment AB = 4.1 cm
2. Draw a ray AX making an acute ∠BAX
3. Along AX mark 4 + 7 = 11 points A_1, A_2...A_{11} such that $AA_1 = AA_2 = AA_3 \dots = AA_{11}$



4. Join $A_{11}B$

5. From A_4 draw $A_4O \parallel A_{11}B$ meeting AB at O. Then O is the point on AB which divides it in the ratio 4 : 7.

So, AO : OB = 4 : 7.

Justification :

Let $AA_1 = AA_2 = AA_3 \ldots\ldots AA_{11} = x$

In ΔABA_{11}, we have

$A_4O \parallel A_{11}B$

$$\frac{AO}{OB} = \frac{AA_4}{A_4A_{11}} = \frac{4x}{7x} = \frac{4}{7}$$

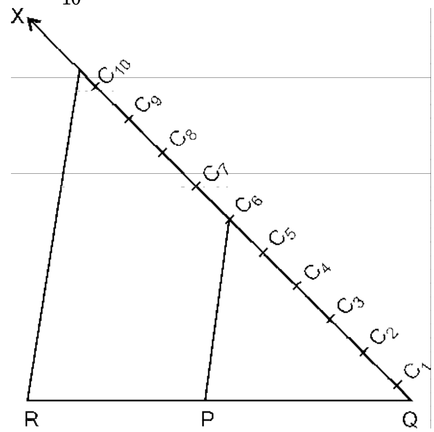
$\therefore$ AO : OB = 4 : 7

on measuring, we find that AO = 1.49cm and OB = 2.61cm.

4. **Draw a line segment of length 3.5cm and divide it externally in the ratio 7 : 3. Measure the two parts.**

Sol. Steps of construction :

1. Draw a line segment PQ 3.5 cm
2. Draw an acute angle PQX at Q
3. Along QX mark the point $C_1, C_2 \ldots . C_{10}$



4. Joint C_6 to P
5. From C_{10} draw $C_{10}R \parallel C_6P$ product QP to R. Then R dividing line externally RQ in 7 : 3.

So, RP : PQ = 7 : 3.

Justification :

Let $C_1 = C_2 = C_3 = C_4 \ldots C_{10} = x$

In RQC_{10}, we have

$C_6PQ \parallel C_{10}R$

$$\frac{RP}{PQ} = \frac{C_6P}{C_6C_{10}} = \frac{7x}{3x} = \frac{7}{x}$$

$\therefore$ RP : PQ = 7 : 3

On measuring, we find that

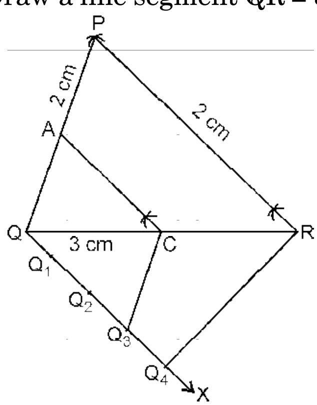
RP = 1.5 cm and PQ = 3.5 cm.

EXERCISE 9.2

1. **Construct a triangle AQC similar to a given triangle PQR with QR = 3cm, PQ = 2 cm, PR = 2.6 cm such that each of its sides is 3/4th of the corresponding sides of ΔPQR.**

Sol. Steps of construction :

1. Draw a line segment QR = 3cm



2. With Q as center and radius equal 2 cm draw an arc.
3. With R as center and radius equal to 2.6 cm, draw another arc cutting the previous arc at P.
4. Joint PQ and PR. So ΔPQR is constructed.
5. Now below QR draw a ray QX making an acute.
6. Along QX, mark 4 points Q_1, Q_2, Q_3, Q_4 such that $QQ_1 = QQ_2 = QQ_3 = QQ_4$

Sol.

7. Join Q_4R
8. From Q_3, draw $Q_3C \parallel Q_4R$ meeting QR at C. (By making an angle equal to $\angle QQ_4R$)
9. From C, draw $AC \parallel PR$ meeting PQ at A. (By making an angle equal to $\angle QRP$.)

Hence, ΔAQC is required triangle.

Each of whose sides is $\frac{3}{4}$ of the corresponding sides of ΔPQR.

Justification :

By construction $Q_3C \parallel Q_4R$

$$\therefore \quad \frac{QC}{CR} = \frac{3}{1}$$

$$\frac{QR}{QC} = \frac{QC+CR}{QC} = 1+\frac{CR}{QC}$$

$$= 1+\frac{1}{3}$$

$$= \frac{4}{3} \text{ i.e., } \frac{QC}{QR} = \frac{3}{4}$$

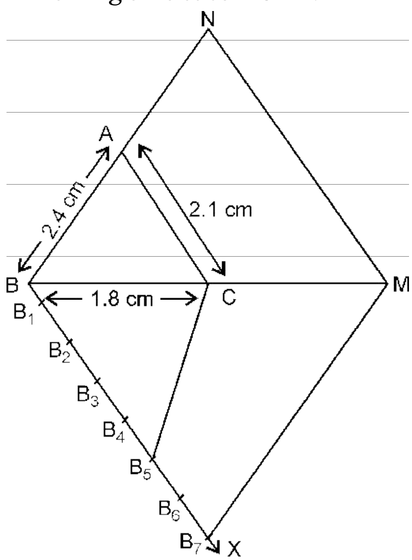
Also, $CA \parallel RP$, we have $\Delta PQR \sim \Delta AQC$

$$\frac{AQ}{PQ} = \frac{AC}{PR} = \frac{QC}{QR} = \frac{3}{4}.$$

2. Construct a triangle with sides 2.4cm, 1.8cm and 2.1cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.

Sol. Steps of construction :

1. Draw a line segment BC = 1.8 cm
2. Taking B andC as center draw two arcs of radii 2.4 cm and 2.1cm each other at A.
3. Join BA and CA, ΔABC is the required triangle.
4. From B draw any ray BX downwards making an acute $\angle CBX$.



5. Locate seven points $B_1, B_2, B_3, B_4, B_5, B_6, B_7$ on BX such tht $BB_1 = BB_2 = BB_3 = BB_4 = BB_5 = BB_6 = BB_7$.
6. Join B_5C from B_7 draw a line $B_7M \parallel B_5C$ intersecting the extendes line segment BN at A. Then ΔNBM is required triangle whose sides are $\frac{7}{5}$ of the corresponding sides of ΔABC.

Justification :

By construction,

$$B_7M \parallel B_5C$$

$$\frac{BC}{CM} = \frac{5}{2}$$

$$\text{Now, } \frac{BM}{BC} = \frac{BC+CM}{BC} = 1+\frac{CM}{BC} = 1+\frac{2}{5} = \frac{7}{5}$$

$$\therefore \quad \frac{BM}{BC} = \frac{7}{5}$$

Also, $MN \parallel CA$

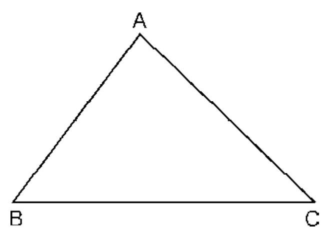
$\Delta ABC \sim \Delta NBA$

$$\text{and } \frac{NB}{AB} = \frac{BM}{BC} = \frac{MN}{CA} = \frac{7}{5}.$$

3. Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{3}{4}$ of the corresponding sides of the triangle ABC (*i.e.*, of scale factor $\frac{3}{4}$).

Sol. Here, we are given ΔABC and scale factor $\frac{3}{4}$

$\therefore$ Scale factor < 1

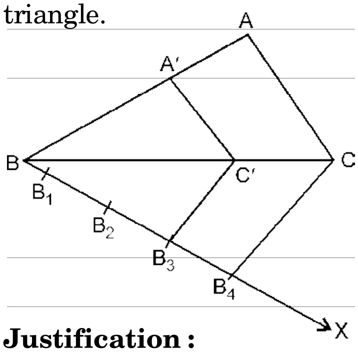


we need to construct triangle similar to ΔABC.

Steps of construction :

1. Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.
2. Mark 4(the greater of 3 & 4 in 3/4) points. B_1, B_2, B_3, B_4 on BX so that $BB_1 = BB_2 = BB_3 = BB_4$.
3. Join B_4C. Draw a line through B_3C' parallel to B_4C, to intersect BC at C′.
4. Draw a through C′A′ parallel to the live CA to intersect BA at A′.

Thus, ΔA′BC′ is the required triangle.



Justification :

Since scale factor is $\frac{3}{4}$

we need to prove $\frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC} = \frac{3}{4}$

By construction,

$$\frac{BC'}{BC} = \frac{BB_3}{BB_4} = \frac{3}{4} \quad ...(1)$$

Also, A'C' is parallel to AC

So, they will make the same angle with line BC.

$\therefore \quad \angle A'C'B = \angle ACB$

(corresponding angles) ...(2)

Now,

In ΔA'BC' and ΔABC

$\angle B = \angle B$ (common)

$\angle A'C'B = \angle ACB$ [from (2)]

$\Delta A'BC' \sim \Delta ABC$ [AAA similarity]

since corresponding sides of similar triangles are in the same ratio.

$$\frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC}$$

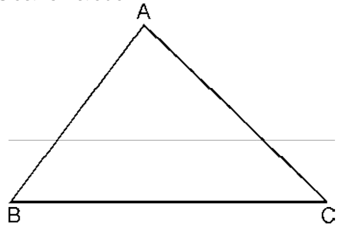
So, $\frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC} = \frac{3}{4}$ ...[from (1)]

Thus, our construction is justified.

4. Construct a triangle similar to given triangle ABC with its sides equal to $\frac{5}{3}$ of the corresponding sides of the triangle ABC. (*i.e.*, of scale factor $\frac{5}{3}$).

Sol. Here we are given ΔABC, and scale factor $\frac{5}{3}$

$\therefore$ Scale factor > 1

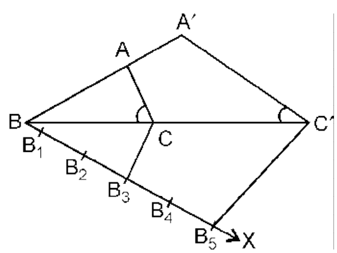


we need to construct a triangle similar to ΔABC.

Steps of construction :

1. Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.
2. Mark 5 (greater then 5 & 3 in $\frac{5}{3}$) points B_1, B_2, B_3, B_4, B_5 on BX so that $BB_1 = BB_2 = BB_3 = BB_4 = BB_5$
3. Join B_3C (3rd point as 3 is smaller in $\frac{5}{3}$) and draw a line through B_5 parallel to B_3C, to intersect BC extended at C'.
4. Draw a line through C' parallel to the line CA to intersect BA extended at A'.

Thus ΔA'BC' is the required triangle.



Justification :

Since scale factor is $\frac{5}{3}$

we need to prove $\frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC} = \frac{5}{3}$

By construction :

$$\frac{BC'}{BC} = \frac{BB_5}{BB_3} = \frac{5}{3} \quad ...(1)$$

Also, A′C′ is parallel to AC

So, they will make the same angle with line BC

$\therefore \quad \angle A'C'B = \angle ACB$

Now,

In ΔA'BC' and ΔABC

$\angle B = \angle B$ (common)

$\angle A'BC' = \angle ABC$

$\angle A'BC' \sim \Delta ABC$ (AA)

since corresponding sides of similar triangles are in same ratio.

$$\frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC}$$

So, $\frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC} = \frac{5}{3}$.

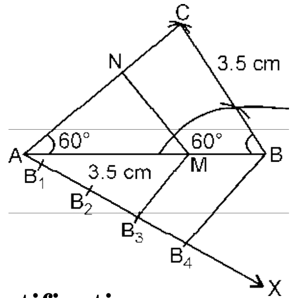
Thus our construction is justified.

5. Draw a triangle ABC with side BC = 3.5cm, AB = 3.5cm and ∠ABC = 60°. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.

Sol. Steps of construction :

1. Draw a line segment AB = 3.5cm.
2. From point B draw ∠ABC = 60° on which take BC = 3.5 cm.
3. Join AC So, ABC is the required triangle.
4. From A, draw any ray AX downwards making an acute angle ∠BAX.
5. Locate 4 points B_1, B_2, B_3 and B_4 on AX such that $AB_1 = AB_2 = AB_3 = AB_4$.
6. Join B_4B and from B_3 draw $B_3M \parallel B_4B$ intersects AB at M.
7. From point M, draw MN ∥ BC intersect AC at N.

Then ΔAMN is the required triangle whose sides are $\frac{3}{4}$ of the corresponding side of ΔABC.



Justification :

By construction :

$B_3M \parallel B_4B$

$\therefore \quad \frac{AM}{MB} = \frac{3}{4}$

Now, $\frac{AB}{AM} = \frac{AM + MB}{AM}$

$= 1 + \frac{MB}{AM} = 1 + \frac{1}{3}$

$= \frac{4}{3}$

$\therefore \quad \frac{AM}{AB} = \frac{3}{4}$

Also, $NM \parallel CB$

$\Delta AMN \sim \Delta ABC$

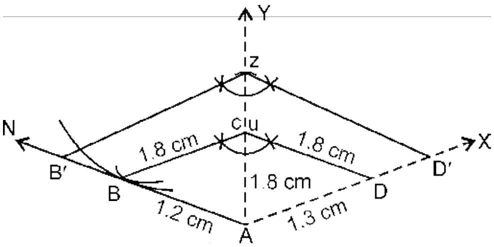
$$\frac{AM}{AB} = \frac{AN}{AC} = \frac{MN}{BC} = \frac{3}{4}.$$

6. Construct a quadrilateral ABCD, in which AB = 1.2 cm, BC = 1.8cm, CD = 1.8cm and AD = 1.3 cm. Construct another quadrilateral AB'C'D', with diagonal AC' = 3cm, such that it is similar to quadrilateral ABCD.

Sol. Construction : To construct a quadrilateral AB′C′D′, similar to a given quadrilateral ABCD.

Given : A quadrilateral ABCD, Scale factor AB = 1·2 cm, BC = 1·8 cm, CD = 1·8 cm and AD = 1·3 cm.

Required : To construct another quadrilateral AB′C′D′ similar to given quadrilateral ABCD with diagonal AC′ = 3 cm.



Steps of Construction :

1. Draw a diagonal AC' of length 1·8 cm.
2. Taking 1·2 cm as radius and A as centre, draw an arc on top of AC.
3. Taking 1·8 cm as radius and C as centre, draw an arc on top of AC.
4. Taking 1·3 cm as radius and A as centre, draw an arc on bottom of AC.
5. Taking 1·8 cm as radius and C as centre, draw an arc on bottom of AC.
6. Join AB, BC, AC, CD. ABCD is required quadrilateral.
7. To draw a similar quadrilateral taking 3 cm as radius cut an arc from point A to C and draw a diagonal AC.
8. From diagonal AC draw a quadrilateral AB'C'D' similar to ABCD.

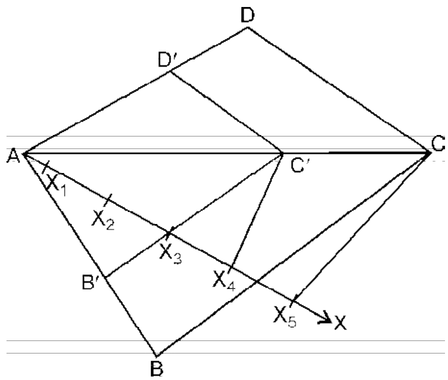
7. Construct a quadrilateral, similar to a given quadrilateral ABCD with its sides $\frac{4}{5}$th of the corresponding sides of ABCD. Also, write the steps of construction.

Sol. We draw a quadrilateral ABCD and also similar quadrilateral to AB'C'D' with scale factor $\frac{4}{5}$, As :

Step 1 : Draw any quadrilateral ABCD.

Step 2 : Join AC. Below AC, draw a line AX as ∠CAX is any acute angle and draw five arcs to equal radii on the line AX that intersect at X_1, X_2, X_3, X_4 & X_5 such that :
$AX_1 = X_1X_2 = X_2X_3 = X_3X_4 = X_4X_5$

Step 3 : Join X_5 to C and then draw a line from X_4 as parallel to X_5C that intersect our line AC at C'.



Step 4 : Draw line from C' as parallel to CD that intersect line AD at D'.

Step 5 : Draw line from C' as parallel to BC that intersect line AB to B'.

Now we can say that quadrilateral AB'C'D' is similar to quadrilateral ABCD as each side $\frac{4}{5}$th of the corresponding sides.

8. Construct a triangle AQC similar to a given triangle PQR with QR = 4.5cm, PR = 3.1cm, PQ = 3.3cm, such that each of its sides is 5/7th of the corresponding sides of the ΔPQR.

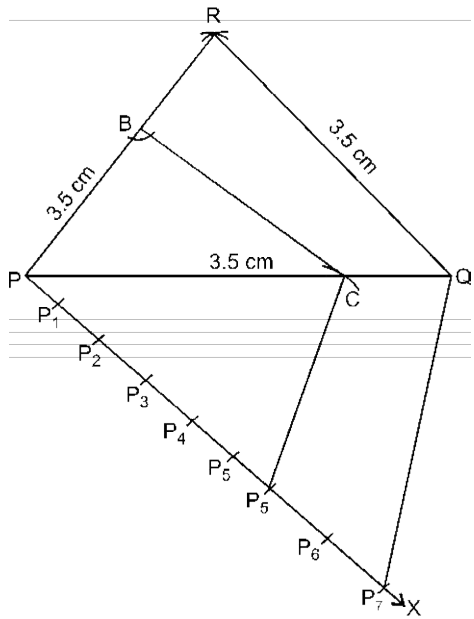
Sol. Same as question (1)

9. Construct a triangle PBC similar to a given equilateral triangle PQR with sides 3.5cm, such that each of its sides is 6/7th of the corresponding sides of ΔPQR.

Sol. Steps of construction :

1. Draw a line segment PQ = 3.5cm
2. With P as center and radius PR = 3.5cm, draw an arc.
3. With Q as center and radius = QR = 3.5cm draw another arc meeting the drawn in step 2 at the point R.
4. Join PR and QR to obtain ΔPQR
5. Below PQ, construct an acute ∠QPX
6. Along PX, mark off seven points P_1, P_2, P_3, P_4, P_5, P_6, P_7 such that $PP_1 = PP_2 = PP_3 = PP_4 = PP_5 = PP_6 = PP_7$.
7. Join P_7Q
8. Draw $P_5C \parallel P_7Q$
9. From C draw CB ∥ QR

   Hence, PBC is the required triangle.



EXERCISE 9.3

1. Draw a circle of radius 2.2 cm. Take two points P and Q on one of its extended diameter each at a distance of 4.3 cm from its centre. Draw tangents to the circle from these two points P and Q.

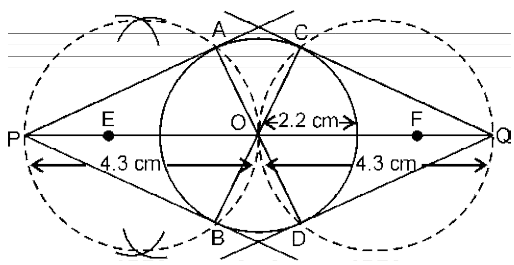
Sol. Given two points P and Q on the diameter of a circle with radius = 2.2 cm and OP = OQ = 4.3 cm

We have to construct the tangents to the circle from the given points P and Q.

Steps of construction :

1. Draw a circle of radius 2.2 cm with center at O.
2. Produce its diameter on both sides and take points P and Q on diameter such that OP = OQ = 4.3.
3. Bisect OP and OQ. Let E and F be the mid points of OP and OQ respectively.
4. Take E as center and OE as radius draw a circle (0, 2.2) at two point A and B. Again, taking F as centre and OF as radius draw a circle which intersect the given circle (0, 2.2) at two points C and D.
5. Join PA, PB, QC, QD. These are the required tangents. From P and Q to the given circle. (0, 2.2).

   Justification : Join OA and OB. The ∠OAP is the angle lies in the semi-circle and therefore ∠OA = 90°. Since OA is radius of the circle. So, AP has to be tangent to the circle. Similarly PB, QC and QD are also tangents to the given circle.



2. Draw a poor tangents to a circle of radius 2cm which are inclimed to each other at an angle of 90°.

Sol.

3. Draw a circle of radius 1.8 cm. Take a point P on it. Draw a tangent to circle at point P.

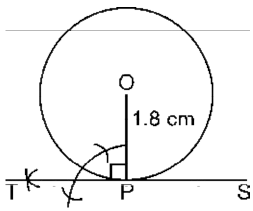
Sol. Given : A circle with radius 1.8 cm and a point P on it.

Required : To draw the tangent to the circle at point P.

Steps of construction :

1. Draw a circle with O as center and radius = 1.8 cm.
2. Take a point P on circle.
3. Join OP.
4. Draw a line TS perpendicular to OP at a point P.

TPS is the required tangent P.



4. From the point P on the circle of radius 2.2cm, draw a tangent to the circle, write steps of construction.

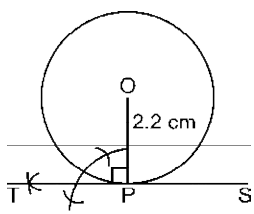
Sol. Given : A circle with radius 2.2 cm and a point P on it.

Required : To draw the tangent to the circle at point P.

Steps of construction :

1. Draw a circle with O as center and radius = 2.2 cm.
2. Take a point P on the circle
3. Join OP.
4. Draw a line TS perpendicular to OP at a point P.

TPS is the required tangent at P.



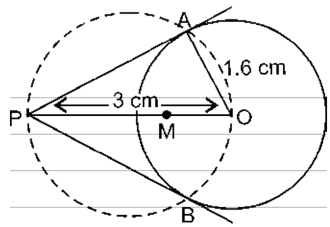
5. Draw a circle of radius 1.6 cm. Take a point P at a distance of 3cm from the centre of the circle. From the point P, draw two tangent to the circle.

Sol. Given : A circle of radius 1.6 cm and a point P at a distance of 3 cm from the centre.

Required : Two tangents to the circle.

1. Draw a line segment OP = 3 cm.
2. From the point O draw a circle of radius = 1.6 cm.
3. Draw a perpendicular bisector of OP. Let M be the mid point of OP.
4. Taking M as centre and OM as radius draw a circle.
5. Let this circle intersects the given circle at point A and B.
6. Join PA and PB.

PA and PB are the required tangents.



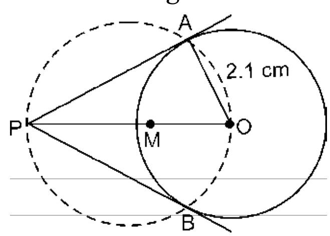
6. Draw a circle of radius 2.1cm. Take a point P outside the circle. Draw two tangents to the circle from point P.

Sol. Given : A circle of radius = 2.1cm and a point P on the circle.

Required : Two tangents to the circle from point P.

Steps of construction :

1. Draw a line segment OP.



2. Let the mid-point of OP is M, from OM as radius draw a circle.
3. Draw a perpendicular bisector of OP is OA.
4. OA = 2.1cm.
5. From OA as radius draw a circle.

7. Draw a line segment AB of length 4.4cm. Taking A as centre, draw a circle of radius 2cm and taking B as centre, draw another circle of radius 2.2cm. Construct tangents to

each circle from the centre of the other circle.

Sol. Given : A line segment AB = 4.4cm two circles with centres A and B of radii 2 cm and 2.2 cm, respectively.

Required : We have to construct two tangents to each circle from the centre of the other circle.

Steps of construction :

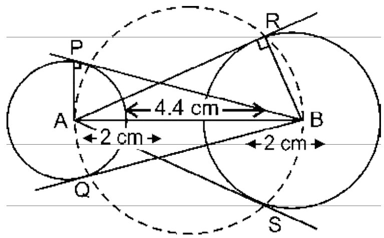
1. Draw a line segment AB = 4.4 cm
2. Draw a circle with centre A and radius 2 cm and another circle with centre B and radius 2.2 cm.
3. Now, bisect AB. Let O be the mid-point of AB. Take O as centre and AO as radii draw a circle which intersects the two circles at P, Q, R and S.
4. Join AP, AQ, BR and BS. These are the required tangents.

Justification :

Join AP and AQ so, $\angle APB = 90°$ ($AP \perp BP$)

Since, AP is radius of the circle with centre A. So, BP has to be tangent of the circle with centre A. Similarly BQ is also a tangent of the circle with centre A.

Again join BP and BQ. Then $\angle APB$ is the angle lie in semi-circle, so $\angle BRA = 90°$ and BR is radius of the circle with centre B. So, AR has to be tangent of the circle B. Similarly, AR is also tangent of the circle with centre B.



Question 8 and 9 (same as question 5 and 6)

❑

Unit 5 : Trigonometry

10 Introduction to Trigonometry

EXERCISE 10.1

Multiple Choice Type Questions

1. If $\sin\theta = \frac{3}{5}$, then $\cos\theta$ will be :

(a) $\frac{4}{5}$ (b) $\frac{3}{4}$

(c) $\frac{5}{3}$ (d) $\frac{5}{4}$.

Ans. (a) $\frac{4}{5}$.

2. If $\tan\theta = \frac{3}{4}$, then the value of $\sin\theta$ will be :

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$

(c) $\frac{4}{3}$ (d) $\frac{5}{3}$.

Ans. (b) $\frac{3}{5}$.

3. If $\cos\theta = \frac{21}{29}$, then the value of $\sin\theta$ will be :

(a) $\frac{29}{50}$ (b) $\frac{21}{50}$

(c) $\frac{20}{29}$ (d) $\frac{29}{8}$.

Ans. (c) $\frac{20}{29}$.

Very Short Answer Type Questions

4. ΔABC is right angled at A. In each of the following, write down the values of sin B, cos C and tan B :
(i) AB = 20, AC = 21, BC = 29;
(ii) BC = $\sqrt{2}$, AB = AC = 1;
(iii) AB = 12, AC = 5, BC = 13;
(iv) AB = 15, AC = 8, BC = 17.

Sol. **(i)** AB = 20, AC = 21, BC = 29.
In ΔABC, right angled at A, we have

$$\sin B = \frac{AC}{BC}$$

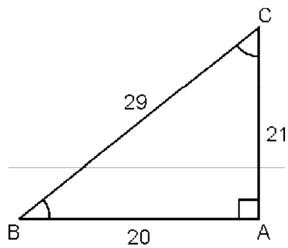
or $$\sin B = \frac{21}{29}.$$

$$\cos C = \frac{AC}{BC}$$

or $$\cos C = \frac{21}{29}.$$

and $$\tan B = \frac{AC}{AB}$$

or $$\tan B = \frac{21}{20}.$$



(ii) BC = $\sqrt{2}$, AB = AC = 1.
In ΔABC, right angled at A, we have

$$\sin B = \frac{AC}{BC}$$

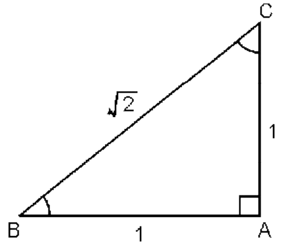
or $$\sin B = \frac{1}{\sqrt{2}}$$

$$\cos C = \frac{AC}{BC}$$

or $$\cos C = \frac{1}{\sqrt{2}}.$$

and $\tan B = \frac{AC}{AB}$

or $\tan B = \frac{1}{1} = 1.$



(iii) AB = 12, AC = 5, BC = 13.
In ΔABC, right angled at A, we have

$$\sin B = \frac{AC}{BC}$$

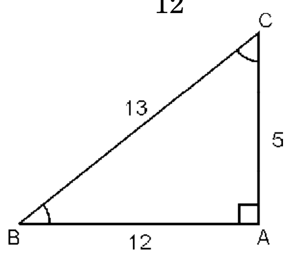
or $\sin B = \frac{5}{13}$

$$\cos C = \frac{AC}{BC}$$

or $\cos C = \frac{5}{13}.$

and $\tan B = \frac{AC}{AB}$

or $\tan B = \frac{5}{12}.$



(iv) AB = 15, AC = 8, BC = 17.
In ΔABC, right angled at A, we have

$$\sin B = \frac{AC}{BC}$$

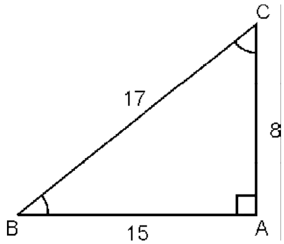
or $\sin B = \frac{8}{17}.$

$$\cos C = \frac{AC}{BC}$$

or $\cos C = \frac{8}{17}.$

and $\tan B = \frac{AC}{AB}$

or $\tan B = \frac{8}{15}.$



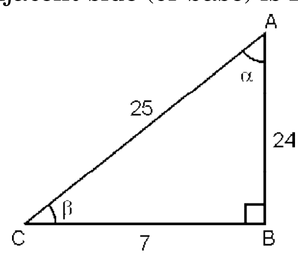
5. In ΔABC, B is a right angle, AB = 24 and AC = 25. If ∠A = α, ∠C = β, find sin α, cos α, sin β, cos β and tan β.

Sol. In the adjoining figure, right angled at B, we have by Pythagoras Theorem,

$$BC^2 = AC^2 - AB^2$$
$$= (25)^2 - (24)^2$$
$$= 625 - 576 = 49$$

$\therefore$ BC = 7.

For angle α, the opposite side (or perpendicular) is BC and the **adjacent side** (or base) is AB; and for angle β, the opposite side (or **perpendicular**) is AB and the adjacent side (or base) is BC.



$$\therefore \quad \sin \alpha = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$= \frac{BC}{AC} = \frac{7}{25}, \quad \textbf{Ans.}$$

$$\cos \alpha = \frac{\text{base}}{\text{hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{24}{25}, \quad \textbf{Ans.}$$

$$\sin\beta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{24}{25}, \quad \textbf{Ans.}$$

$$\cos\beta = \frac{\text{base}}{\text{hypotenuse}}$$

$$= \frac{BC}{AC} = \frac{7}{25}, \quad \textbf{Ans.}$$

and $$\tan\beta = \frac{\text{perpendicular}}{\text{base}}$$

$$= \frac{AB}{BC} = \frac{24}{7}. \quad \textbf{Ans.}$$

EXERCISE 10.2

Very Short Answer Type Questions

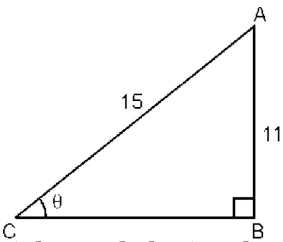
In each of the following one of three trigonometric ratios is given, find the other two ratios :

1. $\sin\theta = \dfrac{11}{15}$.

Sol. $$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{11}{15}$$

$\therefore$ AB = 11 and AC = 15



In right angled triangle ABC, we have

$$BC^2 = AC^2 - AB^2$$
$$= (15)^2 - (11)^2$$
$$= 225 - 121 = 104$$

$\therefore$ $BC = \sqrt{104} = 2\sqrt{26}$.

$\therefore$ $$\cos\theta = \frac{BC}{AC} = \frac{2\sqrt{26}}{15} \quad \textbf{Ans.}$$

and $$\tan\theta = \frac{AB}{BC} = \frac{11}{2\sqrt{26}}. \quad \textbf{Ans.}$$

2. $\cos B = \dfrac{1}{2}$.

Sol. $$\cos B = \frac{\text{base}}{\text{hypotenuse}}$$

$$= \frac{BC}{AB} = \frac{1}{2}$$

$\therefore$ BC = 1 and AB = 2.

In right angled triangle ACB, we have

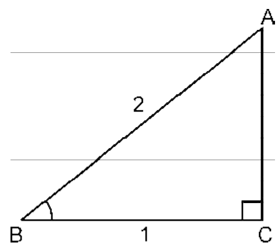
$$AC^2 = AB^2 - BC^2$$
$$= (2)^2 - (1)^2$$
$$= 4 - 1 = 3$$

$\therefore$ $AC = \sqrt{3}$.

$\therefore$ $$\sin B = \frac{AC}{AB} = \frac{\sqrt{3}}{2} \quad \textbf{Ans.}$$

and $$\tan B = \frac{AC}{BC}$$

$$= \frac{\sqrt{3}}{1} = \sqrt{3}. \quad \textbf{Ans.}$$

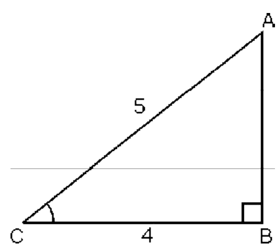


3. $\cos C = \dfrac{4}{5}$.

Sol. $$\cos C = \frac{\text{base}}{\text{hypotenuse}}$$

$$= \frac{BC}{AC} = \frac{4}{5}$$

$\therefore$ BC = 4 and AC = 5.



In right angled triangle ABC, we have

$$\begin{aligned} AB^2 &= AC^2 - BC^2 \\ &= (5)^2 - (4)^2 \\ &= 25 - 16 = 9. \end{aligned}$$

$\therefore$ $AB = \sqrt{9} = 3.$

$\therefore$ $\sin C = \dfrac{AB}{AC} = \dfrac{3}{5}.$ **Ans.**

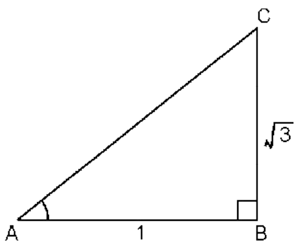
and $\tan C = \dfrac{AB}{BC} = \dfrac{3}{4}.$ **Ans.**

4. tan A = $\sqrt{3}$.

Sol. $\tan A = \dfrac{\text{perpendicular}}{\text{base}}$

$= \dfrac{BC}{AB} = \dfrac{\sqrt{3}}{1}$

$\therefore$ $BC = \sqrt{3}$ and $AB = 1.$



In right angled triangle ABC, we have

$$\begin{aligned} AC^2 &= BC^2 + AB^2 \\ &= (\sqrt{3})^2 + (1)^2 \\ &= 3 + 1 = 4 \end{aligned}$$

$\therefore$ $AC = \sqrt{4} = 2.$

$\therefore$ $\sin A = \dfrac{BC}{AC} = \dfrac{\sqrt{3}}{2}.$ **Ans.**

and $\cos A = \dfrac{AB}{AC} = \dfrac{1}{2}.$ **Ans.**

5. sin α = $\dfrac{2}{3}$.

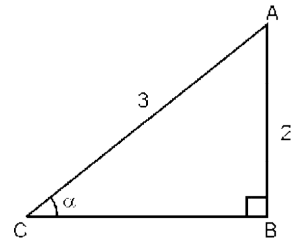
Sol. $\sin \alpha = \dfrac{\text{perpendicular}}{\text{hypotenuse}}$

$= \dfrac{AB}{AC} = \dfrac{2}{3}$

$\therefore$ $AB = 2$ and $AC = 3$

In right angled triangle ABC, we have

$$\begin{aligned} BC^2 &= AC^2 - AB^2 \\ &= (3)^2 - (2)^2 \\ &= 9 - 4 = 5. \end{aligned}$$



$\therefore$ $BC = \sqrt{5}.$

$\therefore$ $\cos \alpha = \dfrac{BC}{AC} = \dfrac{\sqrt{5}}{3}.$ **Ans.**

and $\tan \alpha = \dfrac{AB}{BC} = \dfrac{2}{\sqrt{5}}.$ **Ans.**

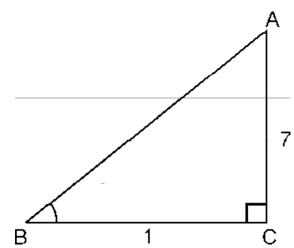
6. tan B = 7.

Sol. $\tan B = \dfrac{\text{perpendicular}}{\text{base}}$

$= \dfrac{AC}{BC} = \dfrac{7}{1}$

$\therefore$ $AC = 7$ and $BC = 1$

In right angled triangle ACB, we have



$$\begin{aligned} AB^2 &= AC^2 + BC^2 \\ &= (7)^2 + (1)^2 \\ &= 49 + 1 = 50 \end{aligned}$$

$\therefore$ $AB = \sqrt{50} = 5\sqrt{2}.$

$\therefore$ $\sin B = \dfrac{AC}{AB} = \dfrac{7}{5\sqrt{2}}.$ **Ans.**

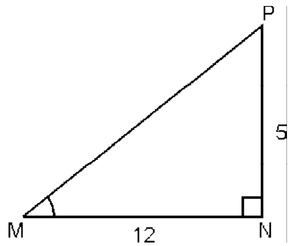
and $\cos B \ \dfrac{AC}{AB} = \dfrac{7}{5\sqrt{2}}.$ **Ans.**

7. tan M = $\frac{5}{12}$.

Sol. $\tan M = \frac{\text{perpendicular}}{\text{base}}$

$= \frac{PN}{MN} = \frac{5}{12}$.

$\therefore$ PN = 5 and MN = 12



In right angled triangle PNM, we have

$PM^2 = PN^2 + MN^2$
$= (5)^2 + (12)^2$
$= 25 + 144 = 169$

$\therefore \quad PM = \sqrt{169} = 13.$

$\therefore \quad \sin M = \frac{PN}{PM} = \frac{5}{13}$. **Ans.**

and $\cos M = \frac{MN}{PM} = \frac{12}{13}$. **Ans.**

8. sin A = $\frac{12}{13}$.

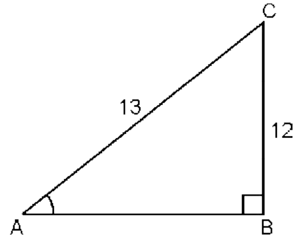
Sol. $\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}}$

$= \frac{BC}{AC} = \frac{12}{13}$

$\therefore$ BC = 12 and AC = 13.

In right angled triangle ABC, we have

$AB^2 = AC^2 - BC^2$
$= (13)^2 - (12)^2$
$= 169 - 144 = 25$



$\therefore \quad AB = \sqrt{25} = 5.$

$\therefore \quad \cos A = \frac{AB}{AC} = \frac{5}{13}$. **Ans.**

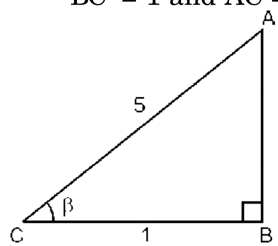
and $\tan A = \frac{BC}{AB} = \frac{12}{5}$. **Ans.**

9. cos β = $\frac{1}{5}$.

Sol. $\cos \beta = \frac{\text{base}}{\text{hypotenuse}}$

$= \frac{BC}{AC} = \frac{1}{5}$

$\therefore$ BC = 1 and AC = 5.



In right triangle ABC, we have

$AB^2 = AC^2 - BC^2$
$= (5)^2 - (1)^2$
$= 25 - 1 = 24$

$\therefore \quad AB = \sqrt{24} = 2\sqrt{6}$.

$\therefore \quad \sin \beta = \frac{AB}{AC} = \frac{2\sqrt{6}}{5}$ **Ans.**

and $\tan \beta = \frac{AB}{AC} = \frac{2\sqrt{6}}{1}$

$= 2\sqrt{6}$. **Ans.**

EXERCISE 10.3

Multiple Choice Type Questions

1. If in a right angle triangle tan θ = 1, then the value of sin θ, will be :

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{2}}{1}$

(c) 2 (d) $\frac{1}{2}$.

Ans. (a) $\frac{1}{\sqrt{2}}$.

2. If cos θ = $\frac{\sqrt{3}}{2}$, then the value of 4 cos³θ – 3 cos θ will be :

(a) – 1 (b) 0

(c) 1 (d) 2.

Ans. (b) 0.

3. If $\sin\theta = \frac{4}{5}$, then the value of 3 $\sec\theta - 5\cos\theta$ will be :

(a) 8 (b) 4

(c) 2 (d) 0.

Ans. (c) 2.

4. If 4 tan A = 3, then the value of $\frac{4\sin A + 3\cos A}{8\sin A + 5\cos A}$ will be :

(a) $\frac{3}{4}$ (b) $\frac{6}{7}$

(c) $\frac{6}{9}$ (d) $\frac{6}{11}$.

Ans. (d) $\frac{6}{11}$.

5. If $\sqrt{3}$ tan θ = 1, then the value of $\frac{2\tan\theta}{1-\tan^2\theta}$ will be :

(a) $\sqrt{3}$ (b) 0

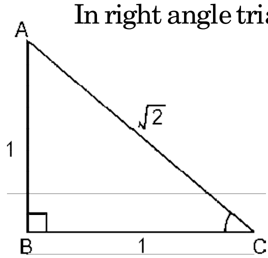
(c) ∞ (d) None of these.

Ans. (a) $\sqrt{3}$.

6. If $\sin A = \frac{1}{\sqrt{2}}$ find the value of cot A.

Sol. $\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}}$

In right angle triangle ABC, we have



$BC^2 = AC^2 - AB^2$

$BC^2 = (\sqrt{2})^2 - (1)^2$

$BC^2 = 2 - 1$

$BC^2 = 1$

$BC = \sqrt{1}$

$BC = 1.$

$\therefore \quad \cot A = \frac{\text{base}}{\text{berpendicular}}$

$\cot A = \frac{1}{1}$

$\cot = 1.$

Very Short Answer Type Questions

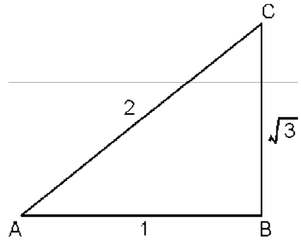
7. If $\cos A = \frac{1}{2}$, find the value of tan A :

Sol. $\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{1}{2}$

In right angled triangle ABC,

$BC^2 = AC^2 - AB^2$

$= (2)^2 - (1)^2 = 3$



$\Rightarrow \quad BC = \sqrt{3}$

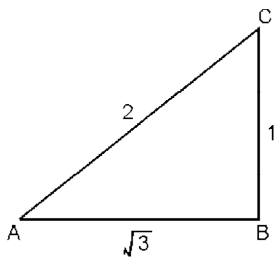
$\therefore \quad \tan A = \frac{BC}{AB} = \frac{\sqrt{3}}{1}$

$= \sqrt{3}$. **Ans.**

8. If $\tan A = \frac{1}{\sqrt{3}}$, find the value of sin A.

Sol. $\tan A = \frac{\text{perpendicular}}{\text{base}}$

$= \frac{1}{\sqrt{3}}$



In right angled triangle ABC,

$AC^2 = AB^2 + BC^2$

$= (\sqrt{3})^2 + (1)^2$

$= 3 + 1 = 4$

$\Rightarrow \quad AC = \sqrt{4} = 2$

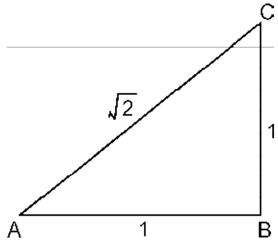
$\therefore \quad \sin A = \frac{BC}{AC} = \frac{1}{2}$. **Ans.**

9. If sec A = $\sqrt{2}$, find the value of tan A.

Sol. $\sec A = \frac{\text{hypotenuse}}{\text{base}} = \frac{\sqrt{2}}{1}$

In right angled triangle ABC,

$$BC^2 = AC^2 - AB^2$$



$$= (\sqrt{2})^2 - (1)^2$$
$$= 2 - 1$$
$$= 1$$

$\Rightarrow \quad BC = \sqrt{3}$

$\therefore \quad \tan A = \dfrac{BC}{AB} = \dfrac{1}{1} = 1.$ **Ans.**

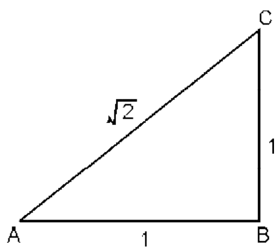
10. If cosec A = $\sqrt{2}$, find the value of cot A.

Sol. $\text{cosec } A = \dfrac{\text{hypotenuse}}{\text{perpendicular}}$

$$= \frac{\sqrt{2}}{1}$$

In right angled triangle ABC.

$$AB^2 = AC^2 - BC^2$$
$$= (\sqrt{2})^2 - (1)^2$$
$$= 2 - 1$$
$$= 1$$



$\Rightarrow \quad AB = \sqrt{1} = 1$

$\therefore \quad \cot A = \dfrac{AB}{BC} = \dfrac{1}{1} = 1.$ **Ans.**

Short Answer Type Questions

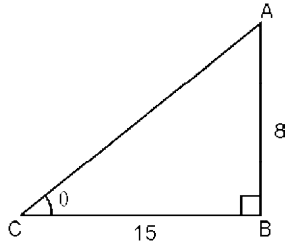
11. If tan θ = $\dfrac{8}{15}$, find the values of other five trigonometric ratios.

Sol. Given, $\tan\theta = \dfrac{\text{perpendicular}}{\text{base}}$

$$= \frac{AB}{BC} = \frac{8}{15}.$$

$\therefore \quad AB = 8$ and $BC = 15$.

In right angled triangle ABC, we have



$$AC^2 = AB^2 + BC^2$$
$$= (8)^2 + (15)^2$$
$$= 64 + 225 = 289$$

$\therefore \quad AC = \sqrt{289} = 17.$

$\therefore \quad \sin\theta = \dfrac{AB}{AC} = \dfrac{8}{17}$ **Ans.**

$\cos\theta = \dfrac{BC}{AC} = \dfrac{15}{17}$ **Ans.**

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{15}{8}$ **Ans.**

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{17}{15}$ **Ans.**

and $\text{cosec }\theta = \dfrac{1}{\sin\theta} = \dfrac{17}{8}.$ **Ans.**

12. If cosec θ = $\dfrac{2}{\sqrt{3}}$, find the values of other five trigonometric ratios.

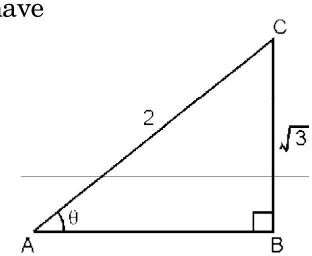
Sol. Given, $\text{cosec }\theta = \dfrac{2}{\sqrt{3}}$

But, $\text{cosec }\theta = \dfrac{\text{hypotenuse}}{\text{perpendicular}}$

$$= \frac{AC}{BC}$$

$\therefore \quad AC = 2$ and $BC = \sqrt{3}$.

In right angled triangle ABC, we have



$$AB^2 = AC^2 - BC^2$$
$$= (2)^2 - (\sqrt{3})^2$$

$= 4 - 3 = 1$

$\therefore \quad AB = \sqrt{1} = 1.$

$\therefore \quad \cos\theta = \frac{AB}{AC} = \frac{1}{2}.$ **Ans.**

$\tan\theta = \frac{BC}{AB}$

$= \frac{\sqrt{3}}{1} = \sqrt{3}$ **Ans.**

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\sqrt{3}}$ **Ans.**

$\sec\theta = \frac{1}{\cos\theta}$

$= \frac{2}{1} = 2$ **Ans.**

and $\quad \sin\theta = \frac{1}{\operatorname{cosec}\theta}$

$= \frac{\sqrt{3}}{2}.$ **Ans.**

13. If $\tan\theta = \frac{5}{12}$, find the values of other five trigonometric ratios.

Sol. Given, $\tan\theta = \frac{5}{12}$

$\because \quad \tan\theta = \frac{\text{perpendicular}}{\text{base}}$

$= \frac{AB}{BC}$

$\therefore \quad AB = 5$ and $BC = 12.$

In right angled triangle ABC, we have

$AC^2 = AB^2 + BC^2$

$= (5)^2 + (12)^2$

$= 25 + 144 = 169$

$\therefore \quad AC = \sqrt{169} = 13.$

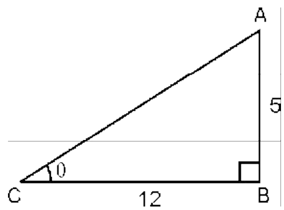
$\therefore \quad \sin\theta = \frac{AB}{AC} = \frac{5}{13}$ **Ans.**

$\cos\theta = \frac{BC}{AC} = \frac{12}{13}$ **Ans.**

$\sec\theta = \frac{1}{\cos\theta} = \frac{13}{12}$ **Ans.**

$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{13}{5}$ **Ans.**

and $\quad \cot\theta = \frac{1}{\tan\theta} = \frac{12}{5}.$ **Ans.**



14. If $\sin A = \frac{1}{\sqrt{10}}$, find the values of other five trigonometric ratios.

Sol. Given, $\sin A = \frac{1}{\sqrt{10}}$

$\because \quad \sin A = \frac{\text{perpendicular}}{\text{hypotenuse}}$

$= \frac{BC}{AC}$

$\therefore \quad BC = 1$ and $AC = \sqrt{10}.$

In right angled triangle ABC, we have

$AB^2 = AC^2 - BC^2$

$= (\sqrt{10})^2 - (1)^2$

$= 10 - 1 = 9$

$\therefore \quad AB = \sqrt{9} = 3.$

$\therefore \quad \cos A = \frac{AB}{AC} = \frac{3}{\sqrt{10}}$ **Ans.**

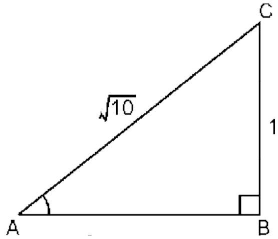
$\tan A = \frac{BC}{AB} = \frac{1}{3}$ **Ans.**

$\cot A = \frac{1}{\tan A} = \frac{3}{1} = 3$ **Ans.**

$\sec A = \frac{1}{\cos A} = \frac{\sqrt{10}}{3}$ **Ans.**

and $\quad \operatorname{cosec} A = \frac{1}{\sin A}$

$= \frac{\sqrt{10}}{1} = \sqrt{10}.$ **Ans.**



15. If tan A = 2, evaluate sec A. sin A + tan² A – cosec² A.

Sol. Given, tan A = 2

$\because \quad \tan A = \dfrac{\text{perpendicular}}{\text{base}}$

$= \dfrac{BC}{AB}$

$\therefore \quad BC = 2$ and $AB = 1$.

In right angled triangle ABC, we have

$AC^2 = BC^2 + AB^2$
$= (2)^2 + (1)^2$
$= 4 + 1 = 5$

$\therefore \quad AC = \sqrt{5}$.

$\therefore \quad \sin A = \dfrac{BC}{AC} = \dfrac{2}{\sqrt{5}}$

$\text{cosec } A = \dfrac{1}{\sin A} = \dfrac{\sqrt{5}}{2}$

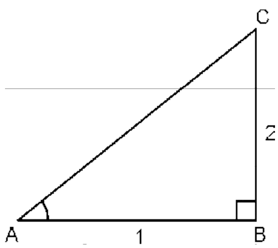
and $\quad \sec A = \dfrac{AC}{AB} = \dfrac{\sqrt{5}}{1} = \sqrt{5}$.

$\therefore$ sec A. sin A + tan² A – cosec² A

$= \sqrt{5} \times \dfrac{2}{\sqrt{5}} + (2)^2 - \left(\dfrac{\sqrt{5}}{2}\right)^2$

$= 2 + 4 - \dfrac{5}{4} = \dfrac{8 + 16 - 5}{4}$

$= \dfrac{19}{4}$. **Ans.**



16. If sin B = $\dfrac{1}{2}$, prove that 3 cos B – 4 cos³ B = 0.

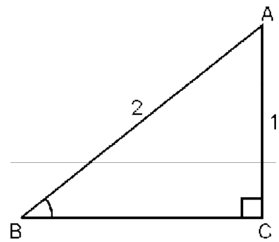
Sol. Given, $\sin B = \dfrac{1}{2}$.

But, $\quad \sin B = \dfrac{\text{perpendicular}}{\text{hypotenuse}}$

$= \dfrac{AC}{AB}$

$\therefore \quad AC = 1$ and $AB = 2$

In right angled triangle ACB, we have



$BC^2 = AB^2 - AC^2$
$= (2)^2 - (1)^2$
$= 4 - 1 = 3$

$\therefore \quad BC = \sqrt{3}$.

$\therefore \quad \cos B = \dfrac{BC}{AB} = \dfrac{\sqrt{3}}{2}$.

Now, L.H.S. = 3 cos B – 4 cos³ B

$= 3 \times \dfrac{\sqrt{3}}{2} - 4 \times \left(\dfrac{\sqrt{3}}{2}\right)^3$

$= \dfrac{3\sqrt{3}}{2} - \dfrac{12\sqrt{3}}{8}$

= 0 = R.H.S. **Proved.**

Long Answer Type Questions

17. If cos α = $\dfrac{4}{5}$, prove that

$$\frac{\tan \alpha}{1 + \tan^2 \alpha} = \frac{\sin \alpha}{\sec \alpha}.$$

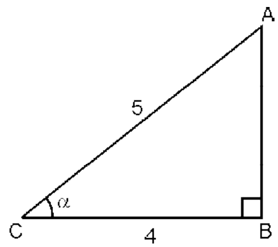
Sol. Given, $\cos \alpha = \dfrac{4}{5}$.

But, $\quad \cos \alpha = \dfrac{\text{base}}{\text{hypotenuse}}$

$$= \frac{BC}{AC}$$

$\therefore$ BC = 4 and AC = 5.

In right angled triangle ABC, we have



$$AB^2 = AC^2 - BC^2$$
$$= (5)^2 - (4)^2$$
$$= 25 - 16 = 9$$

$\therefore$ $AB = \sqrt{9} = 3.$

$\therefore$ $\sin \alpha = \frac{AB}{AC} = \frac{3}{5}$

$\tan \alpha = \frac{AB}{BC} = \frac{3}{4}$

and $\sec \alpha = \frac{1}{\cos \alpha} = \frac{5}{4}.$

Now, $\text{L.H.S.} = \frac{\tan \alpha}{1 + \tan^2 \alpha}$

$$= \frac{3/4}{1 + (3/4)^2} = \frac{3/4}{25/16}$$

$$= \frac{12}{25}$$

and $\text{R.H.S.} = \frac{\sin \alpha}{\sec \alpha}$

$$= \frac{3/5}{5/4} = \frac{12}{25}.$$

$\therefore$ L.H.S. = R.H.S. **Proved.**

18. If $\cot B = \frac{12}{5}$, prove that :

$\tan^2 B - \sin^2 B = \sin^4 B. \sec^2 B.$

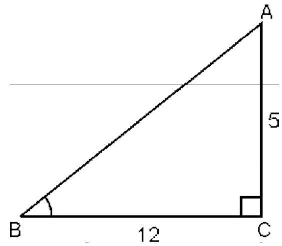
Sol. Given, $\cot B = \frac{12}{5}$

But, $\cot B = \frac{\text{base}}{\text{perpendicular}}$

$$= \frac{BC}{AC}$$

$\therefore$ BC = 12 and AC = 5

In right angled triangle ACB, we have



$$AB^2 = AC^2 + BC^2$$
$$= (5)^2 + (12)^2$$
$$= 25 + 144 = 169$$

$\therefore$ $AB = \sqrt{169} = 13.$

$\therefore$ $\sin B = \frac{AC}{AB} = \frac{5}{13}$

$\sec B = \frac{AB}{BC} = \frac{13}{12}.$

and $\tan B = \frac{1}{\cot B} = \frac{5}{12}.$

Now, $\text{L.H.S.} = \tan^2 B - \sin^2 B$

$$= \left(\frac{5}{12}\right)^2 - \left(\frac{5}{13}\right)^2$$

$$= \frac{25}{144} - \frac{25}{169}$$

$$= \frac{625}{24336}.$$

$\text{R.H.S.} = \sin^4 B. \sec^2 B$

$$= \left(\frac{5}{13}\right)^4 \times \left(\frac{13}{12}\right)^2$$

$$= \frac{625}{(13)^2} \times \frac{1}{(12)^2}$$

$$= \frac{625}{24336}.$$

$\therefore$ L.H.S. = R.H.S. **Proved.**

19. If $\tan \theta = \frac{p}{q}$, show that :

$$\frac{p \sin \theta - q \cos \theta}{p \sin \theta + q \cos \theta} = \frac{p^2 - q^2}{p^2 + q^2}.$$

Sol. Given, $\tan\theta = \frac{p}{q}$,

But, $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

$= \frac{AC}{BC}$

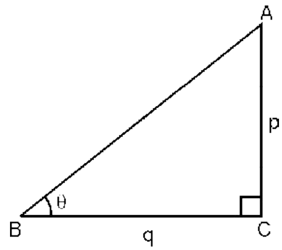
$\therefore$ AC = p and BC = q.

In right angled triangle ACB, we have

$AB^2 = AC^2 + BC^2$

$= p^2 + q^2$

$\therefore$ $AB = \sqrt{p^2 + q^2}$



$\therefore$ $\sin\theta = \frac{AC}{AB} = \frac{p}{\sqrt{p^2+q^2}}$

and $\cos\theta = \frac{BC}{AB} = \frac{q}{\sqrt{p^2+q^2}}$

Now, L.H.S. $= \frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta}$

$$= \frac{p \times \frac{p}{\sqrt{p^2+q^2}} - q \times \frac{q}{\sqrt{p^2+q^2}}}{p \times \frac{p}{\sqrt{p^2+q^2}} + q \times \frac{q}{\sqrt{p^2+q^2}}}$$

$$= \frac{p^2 - q^2}{p^2 + q^2}$$

= R.H.S. **Proved.**

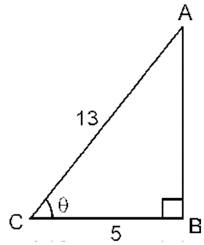
20. If $\cos\theta = \frac{5}{13}$, prove that $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = 3$.

Sol. Given, $\cos\theta = \frac{5}{13}$.

But, $\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$

$= \frac{BC}{AC}$

$\therefore$ BC = 5 and AC = 13.



In right angled triangle ABC, we have

$AB^2 = AC^2 - BC^2$

$= (13)^2 - (5)^2$

$= 169 - 25 = 144.$

$\therefore$ $AB = \sqrt{144} = 12.$

$\therefore$ $\sin\theta = \frac{AB}{AC} = \frac{12}{13}$.

Now, L.H.S. $= \frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$

$$= \frac{2 \times \frac{12}{13} - 3 \times \frac{5}{13}}{4 \times \frac{12}{13} - 9 \times \frac{5}{13}}$$

$$= \frac{\frac{24-15}{13}}{\frac{48-45}{13}} = \frac{9}{3} = 3$$

= R.H.S. **Proved.**

21. If $\cos\theta = \frac{12}{13}$, verify that $\sin\theta(1 - \tan\theta) = \frac{35}{156}$.

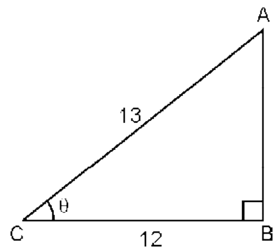
Sol. Given, $\cos\theta = \frac{12}{13}$.

But, $\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$

$= \frac{BC}{AC}$

$\therefore$ BC = 12 and AC = 13.

In right angled triangle ABC, we have



$$AB^2 = AC^2 - BC^2$$
$$= (13)^2 - (12)^2$$
$$= 169 - 144 = 25$$

$\therefore$ $AB = \sqrt{25} = 5.$

$\therefore$ $\sin\theta = \dfrac{AB}{AC} = \dfrac{5}{13}$

and $\tan\theta = \dfrac{AB}{BC} = \dfrac{5}{12}.$

Now, L.H.S. = $\sin\theta\,(1 - \tan\theta)$

$$= \frac{5}{13} \times \left(1 - \frac{5}{12}\right)$$

$$= \frac{5}{13} \times \frac{7}{12} = \frac{35}{156}$$

= R.H.S.

Hence, $\sin\theta\,(1 - \tan\theta) = \dfrac{35}{156}.$

Proved.

22. If $\tan A = \sqrt{2} - 1$, prove that $\sin A \cos A = \dfrac{\sqrt{2}}{4}$.

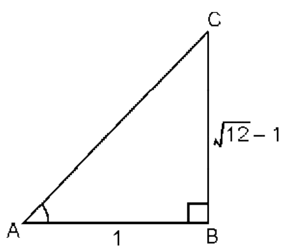
Sol. Given, $\tan A = \sqrt{2} - 1$

But, $\tan A = \dfrac{\text{perpendicular}}{\text{base}}$

$$= \frac{BC}{AB}$$

$\therefore$ $BC = \sqrt{2} - 1$ and $AB = 1$.

In right angled triangle ABC, we have



$$AC^2 = BC^2 + AB^2$$
$$= (\sqrt{2} - 1)^2 + (1)^2$$
$$= 2 + 1 - 2\sqrt{2} + 1$$
$$= 4 - 2\sqrt{2}.$$

$\therefore$ $AC = \sqrt{4 - 2\sqrt{2}}$

$\therefore$ $\sin A = \dfrac{BC}{AC} = \dfrac{\sqrt{2} - 1}{\sqrt{4 - 2\sqrt{2}}}$

and $\cos A = \dfrac{AB}{AC} = \dfrac{1}{\sqrt{4 - 2\sqrt{2}}}.$

Now, L.H.S. = $\sin A \cos A$

$$= \frac{\sqrt{2} - 1}{\sqrt{4 - 2\sqrt{2}}} \times \frac{1}{\sqrt{4 - 2\sqrt{2}}}$$

$$= \frac{\sqrt{2} - 1}{4 - 2\sqrt{2}}$$

$$= \frac{\sqrt{2} - 1}{4 - 2\sqrt{2}} \times \frac{4 + 2\sqrt{2}}{4 + 2\sqrt{2}}$$

$$= \frac{(\sqrt{2} - 1)(4 + 2\sqrt{2})}{(4)^2 - (2\sqrt{2})^2}$$

$[\because (a - b)(a + b) = a^2 - b^2]$

$$= \frac{4\sqrt{2} + 4 - 4 - 2\sqrt{2}}{16 - 8}$$

$$= \frac{2\sqrt{2}}{8} = \frac{\sqrt{2}}{4}$$

= R.H.S. **Proved.**

23. If $\sec\theta = \dfrac{5}{3}$, find the value of $\dfrac{\sin\theta - \dfrac{1}{\tan\theta}}{2\tan\theta}$.

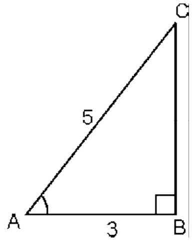
Sol. Given, $\sec\theta = \dfrac{5}{3}$

But, $\sec\theta = \dfrac{\text{hypotenuse}}{\text{base}}$

$$= \frac{AC}{BC}$$

∴ AC = 5 and BC = 3.

In right angled triangle ABC, we have



$$AB^2 = AC^2 - BC^2$$
$$= (5)^2 - (3)^2$$
$$= 25 - 9 = 16$$

∴ $AB = \sqrt{16} = 4,$

∴ $\sin\theta = \dfrac{AB}{AC} = \dfrac{4}{5}$

and $\tan\theta = \dfrac{AB}{BC} = \dfrac{4}{3}.$

Now, $\dfrac{\sin\theta - \dfrac{1}{\tan\theta}}{2\tan\theta} = \dfrac{\dfrac{4}{5} - \dfrac{1}{(4/3)}}{2 \times \dfrac{4}{3}}$

$$= \frac{\frac{4}{5} - \frac{3}{4}}{\frac{8}{3}} = \frac{\frac{1}{20}}{\frac{8}{3}} = \frac{1}{20} \times \frac{3}{8}$$

$= \dfrac{3}{160}.$ **Ans.**

24. If 4 cot A = 3, prove that $\dfrac{\sin A + \cos A}{\sin A - \cos A} = 7$.

Sol. Given, 4 cot A = 3

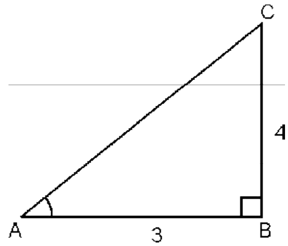
or $\cot A = \dfrac{3}{4}.$

But, $\cot A = \dfrac{\text{base}}{\text{perpendicular}}$

$$= \frac{AB}{BC}$$

∴ AB = 3 and BC = 4.

In right angled triangle ABC, we have



$$AC^2 = AB^2 + BC^2$$
$$= (3)^2 + (4)^2$$
$$= 9 + 16 = 25$$

∴ $AC = \sqrt{25} = 5.$

∴ $\sin A = \dfrac{BC}{AC} = \dfrac{4}{5}$

and $\cos A = \dfrac{AB}{AC} = \dfrac{3}{5}$

Now, L.H.S. $= \dfrac{\sin A + \cos A}{\sin A - \cos A}$

$$= \frac{\frac{4}{5} + \frac{3}{5}}{\frac{4}{5} - \frac{3}{5}} = \frac{\frac{7}{5}}{\frac{1}{5}} = 7$$

= R.H.S. **Proved.**

25. If cosec θ = $\dfrac{5}{3}$, find the value of $\dfrac{\cos\theta - \dfrac{1}{\tan\theta}}{2\tan\theta}$.

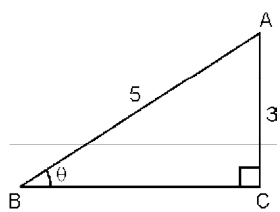
Sol. Given, $\operatorname{cosec}\theta = \dfrac{5}{3}$

But, $\operatorname{cosec}\theta = \dfrac{\text{hypotenuse}}{\text{perpendicular}}$

$$= \frac{AB}{AC}$$

∴ AB = 5 and AC = 3.

In right angle triangle ACB, we have



$$BC^2 = AB^2 - AC^2$$

$= (5)^2 - (3)^2$
$= 25 - 9 = 16$

$\therefore \quad BC = \sqrt{16} = 4.$

$\therefore \quad \cos\theta = \dfrac{BC}{AB} = \dfrac{4}{5}$

$\tan\theta = \dfrac{AC}{BC} = \dfrac{3}{4}$

and $\quad \cot\theta = \dfrac{1}{\tan\theta} = \dfrac{4}{3}.$

$$\therefore \frac{\cos\theta - \dfrac{1}{\tan\theta}}{2\cot\theta} = \frac{\dfrac{4}{5} - \dfrac{4}{3}}{2 \times \left(\dfrac{4}{3}\right)}$$

$$= \frac{-\dfrac{8}{15}}{\dfrac{8}{3}} = -\frac{1}{5}. \qquad \textbf{Ans.}$$

26. If $\cot\theta = \dfrac{x^2 - y^2}{2xy}$, find the values of other trigonometrical ratios.

Sol. Given, $\cot\theta = \dfrac{x^2 - y^2}{2xy}$

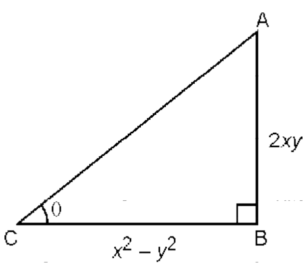
But, $\quad \cot\theta = \dfrac{\text{base}}{\text{perpendicular}}$

$= \dfrac{BC}{AB}$

$\therefore \quad BC = x^2 - y^2$

and $\quad AB = 2xy.$

In right angle triangle ABC, we have



$AC^2 = AB^2 + BC^2$
$= (2xy)^2 + (x^2 - y^2)^2$
$= 4x^2y^2 + x^4 + y^4 - 2x^2y^2$
$= x^4 + y^4 + 2x^2y^2$
$= (x^2 + y^2)^2$

$\therefore \quad AC = (x^2 + y^2).$

$\therefore \quad \sin\theta = \dfrac{AB}{AC} = \dfrac{2xy}{x^2 + y^2}$ **Ans.**

$\cos\theta = \dfrac{BC}{AC} = \dfrac{x^2 - y^2}{x^2 + y^2}$ **Ans.**

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{x^2 + y^2}{x^2 - y^2}$ **Ans.**

$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{x^2 + y^2}{2xy}$ **Ans.**

and $\tan\theta = \dfrac{1}{\cot\theta} = \dfrac{2xy}{x^2 - y^2}.$ **Ans.**

27. If $\cos\theta = \dfrac{4}{5}$, evaluate $\dfrac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$.

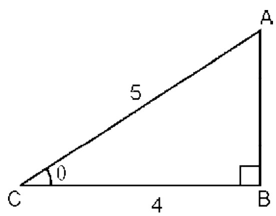
Sol. Given, $\cos\theta = \dfrac{4}{5}$

But, $\quad \cos\theta = \dfrac{\text{base}}{\text{hypotenuse}}$

$= \dfrac{BC}{AC}$

$\therefore \quad BC = 4$ and $AC = 5.$

In right angled triangle ABC, we get



$AB^2 = AC^2 - BC^2$
$= (5)^2 - (4)^2$
$= 25 - 16 = 9$

$\therefore \quad AB = \sqrt{9} = 3$

$\therefore \quad \sin\theta = \dfrac{AB}{AC} = \dfrac{3}{5},$

$\tan\theta = \dfrac{AB}{BC} = \dfrac{3}{4}$

and $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{4}{3}$.

$$\therefore \frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta} = \frac{\frac{3}{5} - 2 \times \frac{4}{5}}{\frac{3}{4} - \frac{4}{3}}$$

$$= \frac{\frac{3}{5} - \frac{8}{5}}{\frac{3}{4} - \frac{4}{3}} = \frac{-\frac{5}{5}}{-\frac{7}{12}}$$

$$= \frac{12}{7}.$$ **Ans.**

28. If $\tan\theta = \dfrac{4}{3}$, prove that $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = 3$.

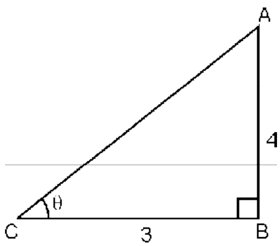
Sol. Given, $\tan\theta = \dfrac{4}{3}$

But, $\tan\theta = \dfrac{\text{perpendicular}}{\text{base}}$

$= \dfrac{AB}{BC}$

$\therefore$ AB = 4 and BC = 3.

In right angle triangle ABC, we get



$$AC^2 = AB^2 + BC^2$$
$$= (4)^2 + (3)^2$$
$$= 16 + 9 = 25$$

$\therefore$ $AC = \sqrt{25} = 5$.

$\therefore$ $\sin\theta = \dfrac{AB}{AC} = \dfrac{4}{5}$

Now, L.H.S. $= \sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}}$

$$= \sqrt{\frac{1+\frac{4}{5}}{1-\frac{4}{5}}} = \sqrt{\frac{\frac{9}{5}}{\frac{1}{5}}}$$

$$= \sqrt{9} = 3$$

= R.H.S. **Proved.**

29. If cosec $\theta = \dfrac{13}{12}$, prove that

$$\mathbf{\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cos\theta} \times \frac{1}{\tan^2\theta} = \frac{595}{3456}.}$$

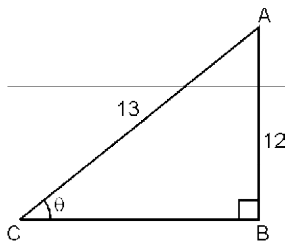
Sol. Given, cosec $\theta = \dfrac{13}{12}$

But, cosec $\theta = \dfrac{\text{hypotenuse}}{\text{perpendicular}}$

$= \dfrac{AC}{AB}$

$\therefore$ AC = 13 and AB = 12.

In right angle triangle ABC, we have



$$BC^2 = AC^2 - AB^2$$
$$= (13)^2 - (12)^2$$
$$= 169 - 144 = 25$$

$\therefore$ $BC = \sqrt{25} = 5$.

$\therefore$ $\cos\theta = \dfrac{BC}{AC} = \dfrac{5}{13}$

$\tan\theta = \dfrac{AB}{BC} = \dfrac{12}{5}$

and $\sin\theta = \dfrac{AB}{AC} = \dfrac{12}{13}$.

Now,

L.H.S. $= \dfrac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cos\theta} \times \dfrac{1}{\tan^2\theta}$

$$= \frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{2 \times \frac{12}{13} \times \frac{5}{13}} \times \frac{1}{(12/5)^2}$$

$$= \frac{\frac{144}{169} - \frac{25}{169}}{\frac{120}{169}} \times \frac{25}{144}$$

$$= \frac{119}{120} \times \frac{25}{144} = \frac{595}{3456}$$

= R.H.S. **Proved.**

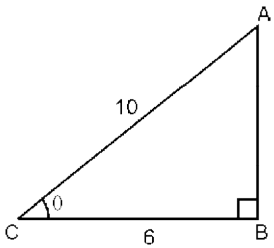
30. If $\cos\theta = \frac{6}{10}$, find the value of $5\sin\theta - 3\tan\theta$.

Sol. Given, $\cos\theta = \frac{6}{10}$

But, $\cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{BC}{AC}$

$\therefore$ BC = 6 and AC = 10.

In right angle triangle ABC, we have



$AB^2 = AC^2 - BC^2$
$= (10)^2 - (6)^2$
$= 100 - 36 = 64$

$\therefore$ $AB = \sqrt{64} = 8.$

$\therefore$ $\sin\theta = \frac{AB}{AC} = \frac{8}{10}$

and $\tan\theta = \frac{AB}{BC} = \frac{8}{6}.$

$\therefore 5\sin\theta - 3\tan\theta = 5 \times \frac{8}{10} - 3 \times \frac{8}{6}$

$= 4 - 4 = 0.$ **Ans.**

31. If $\cot\theta = \frac{1}{\sqrt{7}}$ prove that

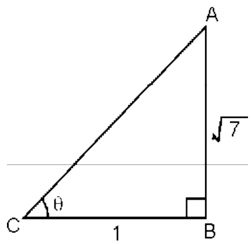
$$\frac{\mathbf{cosec^2\theta - sec^2\theta}}{\mathbf{cosec^2\theta + sec^2\theta}} = -\frac{3}{4}.$$

Sol. Given, $\cot\theta = \frac{1}{\sqrt{7}}$

But, $\cot\theta = \frac{\text{base}}{\text{perpendicular}} = \frac{BC}{AB}$

$\therefore$ BC = 1 and AB = $\sqrt{7}$.

In right angle triangle ABC, we have



$AC^2 = AB^2 + BC^2$
$= (\sqrt{7})^2 + (1)^2$
$= 7 + 1 = 8$

$\therefore$ $AC = \sqrt{8} = 2\sqrt{2}.$

$\therefore$ $\text{cosec}\,\theta = \frac{AC}{AB} = \frac{2\sqrt{2}}{\sqrt{7}}$

and $\sec\theta = \frac{AC}{BC} = \frac{2\sqrt{2}}{1} = 2\sqrt{2}.$

Now,

$$\text{L.H.S.} = \frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$$

$$= \frac{\left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2 - (2\sqrt{2})^2}{\left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2 + (2\sqrt{2})^2}$$

$$= \frac{\frac{8}{7} - 8}{\frac{8}{7} + 8}$$

$$= -\frac{48}{64} = -\frac{3}{4}.$$ **Proved.**

EXERCISE 10.4

Multiple Choce Type Questions

1. The value of sin 60° is :

(a) 1 (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$.

Ans. (b) $\frac{\sqrt{3}}{2}$.

2. The value of tan 30° is :

(a) 0 (b) 1

(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$.

Ans. (d) $\frac{1}{\sqrt{3}}$.

3. The value of sec 60° is :

(a) 0 (b) 1

(c) 2 (d) ∞.

Ans. (c) 2.

4. The value of cosec 45° is :

(a) 2 (b) $\sqrt{2}$

(c) 1 (d) $\frac{1}{\sqrt{2}}$.

Ans. (b) $\sqrt{2}$.

5. The value of cot 45° is :

(a) ∞ (b) 1

(c) 0 (d) $\sqrt{3}$.

Ans. (b) 1.

6. The value of cos 60° is :

(a) 2 (b) 1

(c) $\frac{1}{2}$ (d) 0.

Ans. (c) $\frac{1}{2}$.

7. If cos θ = $\frac{1}{2}$, then θ will be :

(a) 0° (b) 30°

(c) 45° (d) 60°.

Ans. (d) 60°.

8. If sec θ = $\sqrt{2}$, then the value of θ will be :

(a) 90° (b) 60°

(c) 45° (d) 30°.

Ans. (c) 45°.

9. The value of sin 45° × cosec 45° will be :

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) 1 (d) 0.

Ans. (c) 1.

10. The value of sin 60° × cos 30° is :

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{3}{4}$ (d) 1.

Ans. (c) $\frac{3}{4}$.

Very Short Answer Type Questions

11. Find the value of sin 60° × cos 60° × tan 60°.

Sol. sin 60° × cos 60° × tan 60°

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} \times \sqrt{3} = \frac{3}{4}.$$ **Ans.**

12. Find the value of sin 45° × cos 45° × tan 45°.

Sol. sin 45° × cos 45° × tan 45°

$$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times 1$$

$$= \frac{1}{2}.$$ **Ans.**

13. Find the value of sin 30° × cosec 30° × tan 30°.

Sol. sin 30° × cosec 30° × tan 30°

$$= \frac{1}{2} \times 2 \times \frac{1}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}.$$ **Ans.**

14. Find the value of cos 45° × sec 45° × cot 45°.

Sol. cos 45° × sec 45° × cot 45°

$$= \frac{1}{\sqrt{2}} \times \sqrt{2} \times 1$$

= 1. **Ans.**

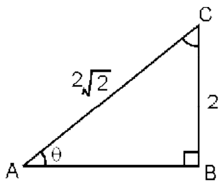
15. Find the value of sin 30° × cot 30° × sec 30°

Sol. sin 30° × cot 30° × sec 30°

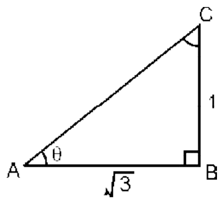
$$= \frac{1}{2} \times \sqrt{3} \times \frac{2}{\sqrt{3}}$$

$= 1.$ **Ans.**

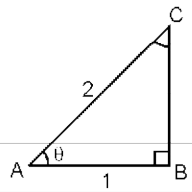
16. Find the values of ∠θ in the following figures :



(i)



(ii)



(iii)

Sol. (i) For figure (i), we have

$$\sin\theta = \frac{BC}{AC} = \frac{2}{2\sqrt{2}}$$

or $\sin\theta = \frac{1}{\sqrt{2}}$

i.e., $\theta = 45°;$

$\left[\because \sin 45° = \frac{1}{\sqrt{2}}\right]$ **Ans.**

(ii) For figure (ii), we have

$$\tan\theta = \frac{BC}{AB}$$

or $\tan\theta = \frac{1}{\sqrt{3}}$

i.e., $\theta = 30°$

$\left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$ **Ans.**

(iii) For figure (iii), we have

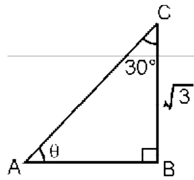
$$\cos\theta = \frac{AB}{AC}$$

or $\cos\theta = \frac{1}{2}$

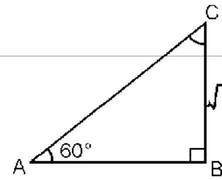
i.e., $\theta = 60°.$

$\left[\because \cos 60° = \frac{1}{2}\right]$ **Ans.**

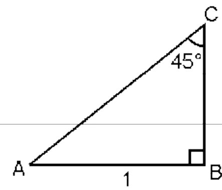
17. Calculate the length of hypotenuse in the following figures :



(i)



(ii)



(iii)

Sol. For figure (i), we have

$$\cos 30° = \frac{BC}{AC}$$

or $\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{AC}$

i.e., AC = 2.

Hence, the length of hypotenuse is 2 units. **Ans.**

For figure (ii), we have

$$\sin 60° = \frac{BC}{AC}$$

or $\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{AC}$

i.e., AC = 2

Hence, the length of hypotenuse is 2 units. **Ans.**

For figure (iii), we have

$$\sin 45° = \frac{AB}{AC}$$

or $\frac{1}{\sqrt{2}} = \frac{1}{AC}$

i.e., AC $= \sqrt{2}$.

Hence, the length of hypotenuse is $\sqrt{2}$ units. **Ans.**

Short Answer Type Questions

18. Evaluate the following expressions :

(a) $\cos 30° \cos 45° - \sin 30° \sin 45°$

(b) $\sin 30° \cos 45° + \cos 30° \sin 45°$

(c) $3 \tan 30° \sec 45° + 3 \tan 60° \sec 30°$

(d) $\cos 60° \cos 45° - \sin 60° \sin 45°$

(e) $\tan 60° \text{cosec}^2 45° + \sec^2 60° \tan 45°$

(f) $\text{cosec}^2 30° \sin^2 45° - \sec^2 60°$

(g) $2 \sin^2 30° - 3 \cos^2 45 + \tan^2 60°$

(h) $4 \cos^2 60° + 4 \sin^2 45° - \sin^2 30°$

(i) $\cot^2 30° - 2 \cos^2 60° - \frac{3}{4} \sec^2 45° - 4 \sec^2 30°$

(j) $2 \sin^2 30° \tan^2 60° - 3 \cos^2 60° \sec^2 30°$.

Sol. (a) $\cos 30° \cos 45° - \sin 30° \sin 45°$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}.$$ **Ans.**

(b) $\sin 30° \cos 45° + \cos 30° \sin 45°$

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{1+\sqrt{3}}{2\sqrt{2}}.$$ **Ans.**

(c) $3 \tan 30° \sec 45° + 3 \tan 60° \sec 30°$

$$= 3\left(\frac{1}{\sqrt{3}} \times \sqrt{2} + \sqrt{3} \times \frac{2}{\sqrt{3}}\right)$$

$$= 3\left(\frac{\sqrt{2}}{\sqrt{3}} + 2\right) = 3\left(\frac{\sqrt{2} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} + 2\right)$$

$$= 3\left(2 + \frac{\sqrt{6}}{3}\right) = 3\left(\frac{6+\sqrt{6}}{3}\right) 6 + \sqrt{6}$$ **Ans.**

(d) $\cos 60° \cos 45° - \sin 60° \sin 45°$

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{1-\sqrt{3}}{2\sqrt{2}} = \frac{1-\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}-\sqrt{6}}{4}.$$ **Ans.**

(e) $\tan 60° \text{cosec}^2 45° + \sec^2 60° \tan 45°$

$$= \sqrt{3} \times (\sqrt{2})^2 + (2)^2 \times 1$$

$$= 2\sqrt{3} + 4$$

$$= 2(\sqrt{3} + 2).$$ **Ans.**

(f) $\text{cosec}^2 30° \, . \, \sin^2 45° - \sec^2 60°$

$$= (2)^2\left(\frac{1}{\sqrt{2}}\right)^2 - (2)^2$$

$$= 4 \times \frac{1}{2} - 4 = 2 - 4 = -2.$$ **Ans.**

(g) $2 \sin^2 30° - 3 \cos^2 45° + \tan^2 60°$

$$= 2 \times \left(\frac{1}{2}\right)^2 - 3 \times \left(\frac{1}{\sqrt{2}}\right)^2 + (\sqrt{3})^2$$

$$= 2 \times \frac{1}{4} - 3 \times \frac{1}{2} + 3$$

$$= \frac{1}{2} - \frac{3}{2} + 3 = \frac{1-3+6}{2}$$

$= 2.$ **Ans.**

(h) $4 \cos^2 60° + 4 \sin^2 45° - \sin^2 30°$

$$= 4 \times \left(\frac{1}{2}\right)^2 + 4 \times \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$= 4 \times \frac{1}{4} + 4 \times \frac{1}{2} - \frac{1}{4}$$

$$= 1 + 2 - \frac{1}{4} = 3 - \frac{1}{4}$$

$$= \frac{11}{4}.$$ **Ans.**

(i) $\cot^2 30° - 2 \cos^2 60° - \frac{3}{4} \sec^2 45° - 4 \sec^2 30°$

$$= (\sqrt{3})^2 - 2 \times \left(\frac{1}{2}\right)^2 - \frac{3}{4} \times \left(\sqrt{2}\right)^2 - 4 \times \left(\frac{2}{\sqrt{3}}\right)^2$$

$$= 3 - 2 \times \frac{1}{4} - \frac{3}{4} \times 2 - 4 \times \frac{4}{3}$$

$$= 3 - \frac{1}{2} - \frac{3}{2} - \frac{16}{3}$$

$$= \frac{18-3-9-32}{6}$$

$= -\frac{26}{6} = -\frac{13}{3}$. **Ans.**

(j) 2 sin² 30° tan² 60° – 3 cos² 60° sec² 30°

$= 2\times\left(\frac{1}{2}\right)^2\times\left(\sqrt{3}\right)^2 - 3\times\left(\frac{1}{2}\right)^2\times\left(\frac{2}{\sqrt{3}}\right)^2$

$= 2\times\frac{1}{4}\times 3 - 3\times\frac{1}{4}\times\frac{4}{3}$

$= \frac{3}{2} - 1 = \frac{1}{2}$. **Ans.**

19. Verify each of the following :

(a) $\cos 60^\circ = 1 - 2\sin^2 30^\circ = 2\cos^2 30^\circ - 1$

(b) $2\sin 30^\circ \cos 30^\circ = \sin 60^\circ$

(c) $\cos 90^\circ = 1 - 2\sin^2 45^\circ = 2\cos^2 45^\circ - 1$

(d) $\cos 30^\circ \cos 60^\circ - \sin 30^\circ \sin 60^\circ = \cos 90^\circ$

(e) $\frac{\tan 60^\circ - \tan 30^\circ}{1+\tan 60^\circ \tan 30^\circ} = \tan 30^\circ$

(f) $\frac{1-\tan 60^\circ}{\cos 30^\circ} = \frac{1-\tan 30^\circ}{1+\tan 60^\circ}$

(g) $\cos 30^\circ = \frac{\cos 30^\circ + \sin 60^\circ}{1+\sin 30^\circ + \cos 60^\circ}$

(h) $\sqrt{\frac{1+\cos 60^\circ}{2}} = \sin 60^\circ$.

Sol. (a) cos 60°= 1 – 2 sin² 30° = 2 cos² 30° – 1

L.H.S. $= \cos 60^\circ = \frac{1}{2}$.

R.H.S. $= 1 - 2\sin^2 30^\circ$

$= 1 - 2\times\left(\frac{1}{2}\right)^2 = \frac{2-1}{2} = \frac{1}{2}$.

Also, $2\cos^2 30^\circ - 1 = 2\times\left(\frac{\sqrt{3}}{2}\right)^2 - 1$

$= 2\times\frac{3}{4} - 1 = \frac{3-2}{2} = \frac{1}{2}$.

Hence, cos 60°= 1 – 2 sin² 30° = 2 cos² 30° – 1. **Proved.**

(b) 2 sin 30° cos 30° = sin 60°

L.H.S. $= 2\sin 30^\circ \cos 30^\circ$

$= 2\times\frac{1}{2}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$.

R.H.S. $= \sin 60^\circ = \frac{\sqrt{3}}{2}$.

Hence, 2 sin 30° cos 30°= sin 60°. **Proved.**

(c) cos 90°= 1 – 2 sin² 45° = 2 cos² 45° – 1.

L.H.S. $= \cos 90^\circ = 0$.

R.H.S. $= 1 - 2\sin^2 45^\circ$

$= 1 - 2\times\left(\frac{1}{\sqrt{2}}\right)^2$

$= 1 - 2\times\frac{1}{2} = 1 - 1 = 0$.

Also,

$2\cos^2 45^\circ - 1 = 2\times\left(\frac{1}{\sqrt{2}}\right)^2 - 1$

$= 2\times\frac{1}{2} - 1 = 1 - 1 = 0$.

Hence, cos 90° = 1 – 2 sin² 45° = 2 cos² 45° – 1. **Proved.**

(d) cos 30° cos 60° – sin 30° sin 60° = cos 90°

L.H.S.

= cos 30° cos 60° – sin 30° sin 60°

$= \frac{\sqrt{3}}{2}\times\frac{1}{2} - \frac{1}{2}\times\frac{\sqrt{3}}{2} = 0$

R.H.S. $= \cos 90^\circ = 0$.

Hence, cos 30° cos 60° – sin 30° sin 60° = cos 90°. **Proved.**

(e) $\frac{\tan 60^\circ - \tan 30^\circ}{1+\tan 60^\circ \tan 30^\circ} = \tan 30^\circ$

L.H.S. $= \frac{\tan 60^\circ - \tan 30^\circ}{1+\tan 60^\circ \tan 30^\circ}$

$= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1+\sqrt{3}\times\frac{1}{\sqrt{3}}}$

$= \frac{\frac{2}{\sqrt{3}}}{2} = \frac{1}{\sqrt{3}}$.

R.H.S. $= \tan 30° = \frac{1}{\sqrt{3}}$.

Hence, $\frac{\tan 60° - \tan 30°}{1 + \tan 60° \tan 30°} = \tan 30°$.

Proved.

(f) $\frac{1-\sin 60°}{\cos 30°} = \frac{1-\tan 30°}{1+\tan 60°}$

L.H.S. $= \frac{1-\sin 60°}{\cos 30°}$

$= \frac{1-\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = \frac{\frac{2-\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = \frac{2-\sqrt{3}}{\sqrt{3}}$.

R.H.S. $= \frac{1-\tan 30°}{1+\tan 60°}$

$= \frac{1-\frac{1}{\sqrt{3}}}{1+\sqrt{3}} = \frac{\sqrt{3}-1}{\sqrt{3}(\sqrt{3}+1)}$

$= \frac{\sqrt{3}-1}{\sqrt{3}(\sqrt{3}+1)} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{3+1-2\sqrt{3}}{\sqrt{3}(3-1)}$

$= \frac{4-2\sqrt{3}}{2\sqrt{3}} = \frac{2-\sqrt{3}}{\sqrt{3}}$.

Hence, $\frac{1-\sin 60°}{\cos 30°} = \frac{1-\tan 30°}{1+\tan 60°}$

Proved.

(g) $\cos 30° = \frac{\cos 30° + \sin 60°}{1 + \sin 30° + \cos 60°}$

L.H.S. $= \cos 30° = \frac{\sqrt{3}}{2}$.

R.H.S. $= \frac{\cos 30° + \sin 60°}{1 + \sin 30° + \cos 60°}$

$= \frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}{1 + \frac{1}{2} + \frac{1}{2}}$

$= \frac{\frac{\sqrt{3}+\sqrt{3}}{2}}{\frac{2+1+1}{2}} = \frac{\frac{2\sqrt{3}}{2}}{\frac{4}{2}} = \frac{\sqrt{3}}{2}$.

Hence,

$\cos 30° = \frac{\cos 30° + \sin 60°}{1 + \sin 30° + \cos 60°}$

Proved.

(h) $\sqrt{\frac{1+\cos 60°}{2}} = \sin 60°$

L.H.S. $= \sqrt{\frac{1+\cos 60°}{2}}$

$= \sqrt{\frac{1+\frac{1}{2}}{2}} = \sqrt{\frac{3}{4}}$

$= \frac{\sqrt{3}}{2}$.

R.H.S. $= \sin 60° = \frac{\sqrt{3}}{2}$.

Hence, $\sqrt{\frac{1+\cos 60°}{2}} = \sin 60°$.

Proved.

20. If A = 45°, prove that
(i) sin 2A = 2 sin A cos A
(ii) cos 2A = cos² **A – sin² A**
(iii) cos 2A = 1 – 2 sin² A
= 2 cos² A – 1.

Sol. (i) $\sin 2A = 2 \sin A \cos A$

L.H.S. $= \sin 2A$
$= \sin (2 \times 45°)$
$= \sin 90°$, [$\because$ A = 45°]
$= 1$.

R.H.S. $= 2 \sin A \cos A$
$= 2 \sin 45° \cos 45°$
$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$.
[$\because$ A = 45°]

$\therefore$ L.H.S. = R.H.S. **Proved.**

(ii) $\cos^2 A = \cos^2 A - \sin^2 A$

L.H.S. $= \cos 2A$

$= \cos(2 \times 45°)$
$= \cos 90°$ [$\because$ A = 45°]
$= 0.$

R.H.S. $= \cos^2 A - \sin^2 A$
$= \cos^2 45° - \sin^2 45°$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 = 0.$$

[$\because$ A = 45°]

$\therefore$ R.H.S. = R.H.S. **Proved.**

(iii) $\cos 2A = 1 - 2\sin^2 A = 2\cos^2 A - 1$

L.H.S. $= \cos 2A$
$= \cos(2 \times 45°)$
$= \cos 90°$ [$\because$ A = 45°]
$= 0.$

R.H.S. $= 1 - 2\sin^2 A$
$= 1 - 2\sin^2 45,$
[$\because$ A = 45°]

$$= 1 - 2 \times \left(\frac{1}{\sqrt{2}}\right)^2$$

$= 1 - 1 = 0.$

Also, $2\cos^2 A - 1 = 2\cos^2 45° - 1$
[$\because$ A = 45°]

$$= 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 1$$

$= 1 - 1 = 0.$

$\because$ L.H.S. R.H.S. **Proved.**

21. If A = 30°, show that

(i) $\mathbf{\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}}$

(ii) $\mathbf{\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}}$

(iii) $\mathbf{\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}}$

Sol. (i) $\sin 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

L.H.S. $= \sin 2A$
$= \sin(2 \times 30°) = \sin 60°,$
[$\because$ A = 30°]

$$= \frac{\sqrt{3}}{2}.$$

R.H.S. $= \dfrac{2\tan A}{1+\tan^2 A}$

$$= \frac{2\tan 30°}{1+\tan^2 30°}$$

[$\because$ A = 30°]

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{2 \times 3}{\sqrt{3} \times 4} = \frac{\sqrt{3}}{2}.$$

$\therefore$ L.H.S. = R.H.S. **Proved.**

(ii) $\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

L.H.S. $= \cos 2A$
$= \cos(2 \times 30°)$
$= \cos 60°$ [$\because$ A = 30°]

$$= \frac{1}{2}.$$

R.H.S. $= \dfrac{1-\tan^2 A}{1+\tan^2 A}$

$$= \frac{1-\tan^2 30°}{1+\tan^2 30°}$$

[$\because$ A = 30°]

$$= \frac{1-\left(\frac{1}{\sqrt{3}}\right)^2}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{3}}{\frac{4}{3}}$$

$$= \frac{2 \times 3}{3 \times 4} = \frac{1}{2}.$$

$\therefore$ R.H.S. = R.H.S. **Proved.**

(iii) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

L.H.S. $= \tan 2A$
$= \tan(2 \times 30°)$
$= \tan 60°$
[$\because$ A = 30°]
$= \sqrt{3}.$

R.H.S. $= \dfrac{2\tan A}{1-\tan^2 A}$

$$= \frac{2\tan 30°}{1-\tan^2 30°}$$

[$\because$ A = 30°]

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$

$$= \frac{2 \times 3}{\sqrt{3} \times 2} = \sqrt{3}.$$

$\therefore$ L.H.S. = R.H.S. **Proved.**

22. If A = 60° and B = 30°, show that
(i) cos (A + B) = cos A cos B – sin A sin B
(ii) sin (A – B) = sin A cos B – cos A sin B
(iii) sin (A + B) = sin A cos B + cos A sin B
(iv) tan (A – B) = $\frac{\tan A - \tan B}{1 + \tan A \tan B}$

Sol. (i) cos (A + B) = cos A cos B – sin A sin B

L.H.S. = cos (A + B)
= cos (60° + 30°) = cos 90° [$\because$ A = 60°, B = 30°]
= 0.

R.H.S. = cos A cos B – sin A sin B
= cos 60° cos 30° – sin 60° sin 30° [$\because$ A = 60°, B = 30°]

$$= \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

= 0.

$\therefore$ L.H.S. = R.H.S. **Proved.**

(ii) sin (A – B) = sin A cos B – cos A sin B

L.H.S. = sin (A – B)
= sin (60° – 30°), [$\because$ A = 60°, B = 30°]

$$= \sin 30° = \frac{1}{2}.$$

R.H.S. = sin A cos B – cos A sin B
= sin 60° cos 30° – cos 60° sin 30° [$\because$ A = 60°, B = 30°]

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{4} - \frac{1}{4} = \frac{1}{2}.$$

$\therefore$ L.H.S. = R.H.S. **Proved.**

(iii) sin (A + B) = sin A cos B + cos A sin B

L.H.S. = sin (A + B)
= sin (60° + 30°), [$\because$ A = 60°, B = 30°]
= sin 90° = 1.

R.H.S. = sin A cos B + cos A sin B
= sin 60° cos 30° + cos 60° sin 30°, [$\because$ A = 60°, B = 30°]

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{4} + \frac{1}{4} = 1.$$

$\therefore$ L.H.S. = R.H.S. **Proved.**

(iv) $\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

L.H.S. = tan (A – B)
= tan (60° – 30°), [$\because$ A = 60°, B = 30°]

$$= \tan 30° = \frac{1}{\sqrt{3}}.$$

$$\text{R.H.S.} = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{\tan 60° - \tan 30°}{1 + \tan 60° \tan 30°}$$

[$\because$ A = 60°, B = 30°]

$$= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}} = \frac{\frac{3-1}{\sqrt{3}}}{2} = \frac{1}{\sqrt{3}}.$$

$\therefore$ L.H.S. = R.H.S. **Proved.**

23. If sin (A – B) = cos (A + B) = $\frac{1}{2}$, Find A and B.

Sol. $\sin (A - B) = \frac{1}{2}$

$\Rightarrow$ (A – B) = 30° ...(i)

and $\cos (A + B) = \frac{1}{2}$

$\Rightarrow$ A + B = 60° ...(ii)

Solving (i) and (ii), we get
A = 45° and B = 15°. **Ans.**

24. Using the formula $\cos\theta = \sqrt{\dfrac{1+\cos 2\theta}{2}}$, find the value of $\cos 45°$.

Sol. To find the value of cos 45°, we substituting θ = 45° in the given formula, we have

$$\cos 45° = \sqrt{\frac{1+\cos 90°}{2}}$$

$$= \sqrt{\frac{1+0}{2}} = \frac{1}{\sqrt{2}}.$$ **Ans.**

25. Prove that $4(\sin^4 30° + \cos^4 60°) - 3(\cos^2 45° - \sin^2 90°) = 2$.

Sol. L.H.S.

$= 4(\sin^4 30° + \cos^4 60°) - 3(\cos^2 45° - \sin^2 90°)$

$$= 4\left[\left(\frac{1}{4}\right)^2 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right]$$

$$= 4\left[\frac{1}{16}+\frac{1}{16}\right] - 3\left[\frac{1}{2}-1\right]$$

$$= 4 \times \frac{2}{16} - 3 \times \left(-\frac{1}{2}\right) = \frac{1}{2}+\frac{3}{2} = 2.$$

= R.H.S. **Proved.**

26. Prove that $\left(\dfrac{1+\cot 60°}{1-\cot 60°}\right)^2 = \dfrac{1+\cos 30°}{1-\cos 30°}$

Sol. L.H.S.$= \left(\dfrac{1+\cot 60°}{1-\cot 60°}\right)^2$

$$= \left(\frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}\right)^2 = \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2$$

$$= \frac{4+2\sqrt{3}}{2-2\sqrt{3}} = \frac{2+\sqrt{3}}{2-\sqrt{3}}.$$

R.H.S. $= \dfrac{1+\cos 30°}{1-\cos 30°}$

$$= \frac{1+\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}} = \frac{2+\sqrt{3}}{2-\sqrt{3}}.$$

∴ L.H.S. = R.H.S. **Proved.**

27. If A = 60°, prove that $\dfrac{1-\sin^4 A}{\cos^4 A} - 2\tan^2 A = 1$.

Sol. L.H.S.$= \dfrac{1-\sin^4 A}{\cos^4 A} - 2\tan^2 A$

$$= \frac{1-\sin^4 60°}{\cos^4 60°} - 2\tan^2 60°,$$

[∵ Given, A = 60°]

$$= \frac{1-\left(\frac{\sqrt{3}}{2}\right)^4}{\left(\frac{1}{2}\right)^4} - 2 \times (\sqrt{3})^2$$

$$= \frac{7/16}{1/16} - 6 = 7 - 6 = 1$$

= R.H.S. **Proved.**

EXERCISE 10.5

Multiple Choice Type Questions

1. The value of sin 37° sec 53° is :

(a) 0 (b) 1
(c) 2 (d) None of these.

Ans. (b) 1.

2. The value of tan 15° tan 75° is :

(a) – 1 (b) 0
(c) 1 (d) None of these.

Ans. (c) 1.

3. The value of $\dfrac{\sin 35°}{\cos 55°}$ is :

(a) 1 (b) 0
(c) – 1 (d) None of these.

Ans. (a) 1.

4. The value of tan 210° is :

(a) $-\dfrac{1}{\sqrt{3}}$ (b) 1
(c) $\dfrac{1}{\sqrt{3}}$ (d) None of these.

Ans. (c) $\dfrac{1}{\sqrt{3}}$.

5. The value of sin 225° is :

(a) $\frac{1}{\sqrt{2}}$ (b) 1

(c) $-\frac{1}{\sqrt{2}}$ (d) None of these.

Ans. (c) $-\frac{1}{\sqrt{2}}$.

Very Short Answer Type Questions

6. tan 25° – cot 65°.

Sol. $\tan 25° - \cot (90° - 25°)$
$= \tan 25° - \tan 25°$
$= 0.$ **Ans.**

7. sec (65° + θ) – cosec (25° – θ)

Sol. $\sec (65° + \theta) - \operatorname{cosec} (25° - \theta)$
$= \sec (65° + \theta) - \operatorname{cosec} [90° - (65° + \theta)]$
$= \sec (65° + \theta) - \sec (65° + \theta)$
$= 0.$ **Ans.**

8. $\frac{\sin 21°}{\cos 69°} + \frac{\cos 59°}{\sin 31°}$ **+ sin 30°.**

Sol. $\frac{\sin 21°}{\cos 69°} + \frac{\cos 59°}{\sin 31°} + \sin 30°$

$= \frac{\sin 21°}{\cos(90° - 21°)} + \frac{\cos 59°}{\sin(90° - 59°)} + \sin 30°$

$= \frac{\sin 21°}{\sin 21°} + \frac{\cos 59°}{\cos 59°} \sin 30°$

$= 1 + 1 + \frac{1}{2}$

$= \frac{5}{2}.$ **Ans.**

9. tan 21°. tan 69°. tan 60°.

Sol. $\tan 21°. \tan 69°. \tan 60°$
$= \tan 21°.\tan (90° - 21°). \sqrt{3}$
$= \tan 21°.\cot 21°$
$= \tan 21°. \frac{1}{\tan 21°} . \sqrt{3}$
$= \sqrt{3}.$ **Ans.**

10. sin (75° + θ) – cos(15° – θ).

Sol. $\sin (75° + \theta) - \cos (15° - \theta)$
$= \sin (75° + \theta) - \cos [90° - (75° + \theta)]$
$= \sin (75° + \theta) - \sin (75° + \theta)$
$= 0.$ **Ans.**

Short Answer Type Questions

11. $\frac{\cot 56°}{\tan 34°} + \frac{\tan 10°}{\cot 80°} = 2$ **.**

Sol. L.H.S.= $\frac{\cot 56°}{\tan 34°} + \frac{\tan 10°}{\cot 80°}$

$= \frac{\cot 56°}{\tan(90° - 56°)} + \frac{\tan 10°}{\cot(90° - 10°)}$

$= \frac{\cot 56°}{\cot 56°} + \frac{\tan 10°}{\tan 10°}$

$= 1 + 1 = 2$
= R.H.S. **Proved.**

12. $\frac{\operatorname{cosec} 37°}{\sec 53°} + \frac{\cos 42°}{\sin 48°} = 2$

Sol. L.H.S.= $\frac{\operatorname{cosec} 37°}{\sec 53°} + \frac{\cos 42°}{\sin 48°}$

$= \frac{\operatorname{cosec} 37°}{\sec(90° - 37°)} + \frac{\cos 42°}{\sin(90° - 42°)}$

$= \frac{\operatorname{cosec} 37°}{\operatorname{cosec} 37°} + \frac{\cos 42°}{\cos 42°}$

$= 1 + 1 = 2$
= R.H.S. **Proved.**

13. $\frac{\sin 39°}{\cos 51°} + \frac{\sin 51°}{\cos 39°} = 2$

Sol. L.H.S.= $\frac{\sin 39°}{\cos 51°} + \frac{\sin 51°}{\cos 39°}$

$= \frac{\sin 39°}{\cos(90° - 39°)} + \frac{\sin 51°}{\cos(90° - 51°)}$

$= \frac{\sin 39°}{\cos 39°} + \frac{\sin 51°}{\sin 51°}$

$= 1 + 1$
$= 2$
= R.H.S. **Proved.**

14. $\mathbf{\dfrac{2\tan 43^\circ}{\cot 47^\circ} - \dfrac{\tan 70^\circ}{\cot 20^\circ} - \tan 45^\circ = 0.}$

Sol. L.H.S.= $\dfrac{2\tan 43^\circ}{\cot 47^\circ} - \dfrac{\tan 70^\circ}{\cot 20^\circ} - \tan 45^\circ$

$= \dfrac{2\tan 43^\circ}{\cot(90^\circ - 43^\circ)} - \dfrac{\tan 70^\circ}{\cot(90^\circ - 70^\circ)} - 1$

$= \dfrac{2\tan 43^\circ}{\tan 43^\circ} - \dfrac{\tan 70^\circ}{\tan 70^\circ} - 1$

= 2 – 1 – 1

= 0

= R.H.S. **Proved.**

15. $\mathbf{\tan 240^\circ + \sin 120^\circ = \dfrac{3\sqrt{3}}{2}}$

Sol. L.H.S.= tan 240° + sin 120°

= tan (180° + 60°) + sin (90° + 30°)

= tan 60° + cos 30°

$= \sqrt{3} + \dfrac{\sqrt{3}}{2}$

$= \dfrac{3\sqrt{3}}{2}$

= R.H.S. **Proved.**

16. tan (90° – θ) = cot (180° + θ)

Sol. L.H.S. = tan (90° – θ)

= cot θ

R.H.S. = cot (180° + θ)

= cot θ

L.H.S. = R.H.S. **Proved.**

17. sin (180° – θ) = cos (90° – θ)

Sol. L.H.S. = sin (180° – θ)

= sin [90° + (90° – θ)]

= cos (90° – θ)

= R.H.S. **Proved.**

18. $\mathbf{\dfrac{\sin(90^\circ+\theta)}{\cos\theta} - \dfrac{\sin(180^\circ+\theta)}{\sin\theta} - \dfrac{\tan\theta}{\cot(90^\circ+\theta)} = 3.}$

Sol. L.H.S.= $\dfrac{\sin(90^\circ+\theta)}{\cos\theta} - \dfrac{\sin(180^\circ+\theta)}{\sin\theta} - \dfrac{\tan\theta}{\cot(90^\circ+\theta)}$

$= \dfrac{\cos\theta}{\cos\theta} + \dfrac{\sin\theta}{\sin\theta} + \dfrac{\tan\theta}{\tan\theta}$

= 1 + 1 + 1

= 3

= R.H.S. **Proved.**

19. $\mathbf{\sin 210^\circ - \tan 225^\circ = -\dfrac{3}{2}}$

Sol. L.H.S.= sin 210° – tan 225°

= sin (180° + 30°) – tan (180° + 45°)

= – sin 30° – tan 45°

$= -\dfrac{1}{2} - 1$

$= -\dfrac{3}{2}$

= R.H.S. **Proved.**

20. $\mathbf{\cot 240^\circ + \sec 210^\circ = -\dfrac{1}{\sqrt{3}}}$.

Sol. L.H.S.= cot 240° + sec 210°

= cot (180° + 60°) + sec (180° + 30°)

= cot 60° – sec 30°

$= \dfrac{1}{\sqrt{3}} - \dfrac{2}{\sqrt{3}} = -\dfrac{1}{\sqrt{3}}$ = R.H.S. **Proved.**

EXERCISE 10.6

Multiple Choice Type Questions

1. The value of

$\mathbf{\sin^2\theta + \dfrac{1}{(1+\tan^2\theta)}}$ **is :**

(a) $\sin^2\theta$ (b) $\cos^2\theta$

(c) $\sec^2\theta$ (d) 1.

2. The value of $\sin^2 18^\circ + \cos^2 18^\circ$ will be :

(a) 0 (b) 1

(c) – 1 (d) 2.

3. $\tan^2\theta.(\text{cosec}^2\theta - 1)$ is equivalent to :

(a) $\tan^2\theta$ (b) $\text{cosec}^2\theta$

(c) $\cot^2\theta$ (d) 1.

4. sec A. cosec A. cot A is equivalent to :

(a) sin A (b) $\sec^2$ A

(c) cosec^2 A (d) $\sec^2$ A.

5. The value of :
cos 54°.cosec 54°.tan54° will be :

(a) – 1 (b) 1

(c) $\sqrt{2}$ (d) $\sqrt{3}$.

6. $(\tan\theta - \sec\theta)(\tan\theta + \sec\theta)$ is equivalent to :

(a) 1 (b) – 1

(c) $2\tan\theta$ (d) $2\sec\theta$.

7. $\sin\theta\sqrt{1+\cot^2\theta}$ is equivalent to :

(a) $\sin\theta$ (b) $\text{cosec}\,\theta$

(c) $\tan\theta$ (d) 1.

8. $\cos\theta.\sqrt{\sec^2\theta-1}$ is equivalent to :

(a) $\sin\theta$ (b) $\cot\theta$

(c) $\sec\theta$ (d) 1.

9. The value of $\cos\frac{9\pi}{4}$ is :

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $-\sqrt{2}$ (d) $-\frac{1}{\sqrt{2}}$.

10. The value of sin 105° + cos 105° is:

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$

(c) $-\frac{1}{\sqrt{2}}$ (d) $-\sqrt{2}$.

11. If cosec A = $\frac{17}{15}$ then the value of sec A is :

(a) $\frac{-8}{17}$ (b) $\frac{8}{17}$

(c) $\frac{17}{8}$ (d) $\frac{-17}{8}$.

Sol. $\text{cosec}\,A = \frac{17}{15}$

$\Rightarrow \quad \sin A = \frac{15}{17}$

$$\cos A = \sqrt{1-\sin^2 A}$$

$$= \sqrt{1-\left(\frac{15}{17}\right)^2}$$

$$= \sqrt{\frac{289-225}{289}}$$

$$= \sqrt{\frac{64}{289}}$$

$$= \frac{8}{17}$$

$$\sec A = \frac{17}{8}.$$

12. If $\sin\theta = \frac{\sqrt{3}}{2}$, then the value of $(\text{cosec}\,\theta + \cot\theta)$ is :

(a) $-\sqrt{3}$ (b) $\sqrt{3}$

(c) $\sqrt{2}$ (d) $-\sqrt{2}$.

Sol. Here $\sin\theta = \frac{\sqrt{3}}{2}$

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

$$= \sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{\frac{4-3}{4}}$$

$$= \frac{1}{2}$$

$$\text{cosec}\,\theta + \cot\theta = \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sqrt{3}/2} + \frac{1/2}{\sqrt{3}/2}$$

$$= \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}$$

$$= \frac{3}{\sqrt{3}} = \sqrt{3}.$$

13. $\frac{1+\cos\theta-\sin^2\theta}{\sin\theta+\sin\theta\cos\theta}$ is equal to :

(a) $\text{cosec}\,\theta$ (b) $\sec\theta$

(c) $\tan\theta$ (d) $\cot\theta$.

Sol. $\dfrac{1+\cos\theta-\sin^2\theta}{\sin\theta+\sin\theta\cos\theta}$

$= \dfrac{(1-\sin^2\theta)+\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \dfrac{\cos^2\theta+\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \dfrac{\cos\theta(\cos\theta+1)}{\sin\theta(1+\cos\theta)}$

$= \cot\theta.$

14. $\sqrt{\dfrac{\mathbf{1+\sin\theta}}{\mathbf{1-\sin\theta}}}$ **is equal to :**

(a) $\sin\theta + \cos\theta$ (b) $\cos\theta - \sin\theta$
(c) $\sec\theta + \tan\theta$ (d) $\sec\theta - \tan\theta$.

15. $\dfrac{\mathbf{\sin\theta}}{\mathbf{1-\cot\theta}} + \dfrac{\mathbf{\cos\theta}}{\mathbf{1-\tan\theta}}$ **is equal to :**

(a) 0 (b) 1
(c) $\tan\theta$ (d) $\sin\theta + \cos\theta$.

16. $\dfrac{\tan\theta}{\sec\theta+1} + \dfrac{\tan\theta}{\sec\theta-1}$ is equal to :

(a) $2\sec\theta$ (b) $2\tan\theta$
(c) $2\,\text{cosec}\,\theta$ (d) $2\tan\theta\sec\theta$.

Ans. **1.** (d), **2.** (b), **3.** (d), **4.** (c), **5.** (b), **6.** (a), **7.** (d), **8.** (a), **9.** (b), **10.** (a), **11.** (c), **12.** (b), **13.** (d), **14.** (c), **15.** (d), **16.** (c).

Very Short Answer Type Questions

Prove the following identities :

17. $\mathbf{\sin\theta(1+\tan\theta)+\cos\theta(1+\cot\theta) = (\sec\theta + \text{cosec}\,\theta)}$.

Sol. L.H.S.= $\sin\theta(1+\tan\theta) + \cos\theta(1+\cot\theta)$

$= \sin\theta\left(1+\dfrac{\sin\theta}{\cos\theta}\right) + \cos\theta\left(1+\dfrac{\cos\theta}{\sin\theta}\right)$

$= \sin\theta\left(\dfrac{\cos\theta+\sin\theta}{\cos\theta}\right) + \cos\theta\left(\dfrac{\sin\theta+\cos\theta}{\sin\theta}\right)$

$= \dfrac{\sin^2\theta(\cos\theta+\sin\theta) + \cos^2\theta(\sin\theta+\cos\theta)}{\cos\theta.\sin\theta}$

$= \dfrac{(\sin\theta+\cos\theta)(\sin^2\theta+\cos^2\theta)}{\cos\theta\sin\theta}$

$= \dfrac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}$, $[\because \sin^2\theta+\cos^2\theta=1]$

$= \dfrac{\sin\theta}{\cos\theta.\sin\theta} + \dfrac{\cos\theta}{\cos\theta.\sin\theta}$

$= \dfrac{1}{\cos\theta} + \dfrac{1}{\sin\theta}$

$= \sec\theta + \text{cosec}\,\theta$ = R.H.S.

Proved.

18. $\mathbf{(\sin\theta - \text{cosec}\,\theta)(\cos\theta - \sec\theta)(\tan\theta + \cot\theta) = 1}$.

Sol. L.H.S.= $(\sin\theta - \text{cosec}\,\theta)(\cos\theta - \sec\theta)(\tan\theta + \cot\theta)$

$= \left(\sin\theta - \dfrac{1}{\sin\theta}\right)\left(\cos\theta - \dfrac{1}{\cos\theta}\right)\left(\tan\theta + \dfrac{1}{\tan\theta}\right)$,

$\left[\because \text{cosec}\,\theta = \dfrac{1}{\sec\theta}, \sec\theta = \dfrac{1}{\cos\theta}\right]$

$= \left(\dfrac{\sin^2\theta-1}{\sin\theta}\right)\left(\dfrac{\cos^2\theta-1}{\cos\theta}\right)\left(\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}\right)$

$= \left[-\left(\dfrac{1-\sin^2\theta}{\sin\theta}\right)\right]\left[-\left(\dfrac{1-\cos^2\theta}{\cos\theta}\right)\right]\left(\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}\right)$

$= \left(\dfrac{\sin^2\theta}{\sin\theta}\right)\left(\dfrac{\sin^2\theta}{\cos\theta}\right)\left(\dfrac{1}{\sin\theta\cos\theta}\right)$,

[$\because \sin^2\theta + \cos^2\theta = 1$, $1 - \sin2\,\theta = \cos2\,\theta$ and $1 - \cos2\,\theta = \sin2\,\theta$]

= 1 = R.H.S. **Proved.**

19. $\cot\theta + \tan\theta = \operatorname{cosec}\theta \sec\theta$.

Sol. L.H.S.= $\cot\theta + \tan\theta$

$$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$$

$$\left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{1}{\sin\theta}.\frac{1}{\cos\theta}$$

$= \operatorname{cosec}\theta. \sec\theta$ = R.H.S. **Proved.**

20. $\dfrac{\sin\theta}{(1-\cos\theta)} = \operatorname{cosec}\theta + \cot\theta$.

Sol. L.H.S. = $\dfrac{\sin\theta}{(1-\cos\theta)}$

Multiplying denominator and numerator by $(1 + \cos\theta)$, we get

$$\text{L.H.S.} = \frac{\sin\theta}{(1-\cos\theta)} \times \frac{(1+\cos\theta)}{(1+\cos\theta)}$$

$$= \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$$

$$= \frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}$$

$$[\because 1 - \cos^2\theta = \sin^2\theta]$$

$$= \frac{1+\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$= \operatorname{cosec}\theta + \cot\theta$ = R.H.S. **Proved.**

21. $\dfrac{\cos\theta}{1+\sin\theta} = \dfrac{1-\sin\theta}{\cos\theta}$

Sol. L.H.S. = $\dfrac{\cos\theta}{1+\sin\theta}$

Multiplying numerator and denominator by $(1 - \sin\theta)$, we get

$$\text{L.H.S.} = \frac{\cos\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$$

$$= \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$$

$$= \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}$$

$$[\because 1 - \sin^2\theta = \cos^2\theta]$$

$$= \frac{1-\sin\theta}{\cos\theta} = \text{R.H.S.}$$ **Proved.**

22. $\tan^2\phi + \cot^2\phi + 2 = \sec^2\phi \operatorname{cosec}^2\phi$.

Sol. L.H.S.= $\tan^2\phi + \cot^2\phi + 2$

$$= \frac{\sin^2\phi}{\cos^2\phi} + \frac{\cos^2\phi}{\sin^2\phi} + 2,$$

$$\left[\because \tan^2\phi = \frac{\sin^2\phi}{\cos^2\phi}, \cot^2\phi = \frac{\cos^2\phi}{\sin^2\phi}\right]$$

$$= \frac{\sin^4\phi + \cos^4\phi + 2\sin^2\phi\cos^2\phi}{\sin^2\phi.\cos^2\phi}$$

$$= \frac{(\sin^2\phi + \cos^2\phi)^2}{\sin^2\phi.\cos^2\phi}$$

$$= \frac{1}{\sin^2\phi.\cos^2\phi},$$

$$[\because \sin^2\phi + \cos^2\phi = 1]$$

$$= \frac{1}{\cos^2\phi}.\frac{1}{\sin^2\phi}$$

$= \sec^2\phi.\operatorname{cosec}^2\phi$ = R.H.S. **Proved.**

23. $\dfrac{1+\sec\theta}{\sec\theta} = \dfrac{\sin^2\theta}{1-\cos\theta}$

Sol. L.H.S.= $\dfrac{1+\sec\theta}{\sec\theta}$

$$= \frac{1+\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}} \quad \left[\because \sec\theta = \frac{1}{\cos\theta}\right]$$

$$= \frac{\frac{\cos\theta+1}{\cos\theta}}{\frac{1}{\cos\theta}} = \frac{(\cos\theta+1)\times\cos\theta}{\cos\theta}$$

$$= \frac{1+\cos\theta}{1}$$

$= \frac{1+\cos\theta}{1} \times \frac{\sin^2\theta}{\sin^2\theta}$,

[Multiplying numerator and denominator by $\sin^2\theta$]

$= \frac{\sin^2\theta(1+\cos\theta)}{1-\cos^2\theta}$,

[$\because \sin^2\theta = 1 - \cos^2\theta$]

$= \frac{\sin^2\theta(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{\sin^2\theta}{1-\cos\theta}$ = R.H.S. **Proved.**

24. $\frac{\mathbf{1}}{(\mathbf{1}+\mathbf{sin}\,\theta)} + \frac{\mathbf{1}}{(\mathbf{1}-\mathbf{sin}\,\theta)}$ **= 2 sec² θ.**

Sol. L.H.S.= $\frac{1}{(1+\sin\theta)} + \frac{1}{(1-\sin\theta)}$

$= \frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$

$= \frac{2}{1-\sin^2\theta}$, [$\because 1 - \sin^2\theta = \cos^2\theta$]

$= \frac{2}{\cos^2\theta} = 2\sec^2\theta$ = R.H.S.

Proved.

25. $\frac{\mathbf{sec}\,\theta - \mathbf{1}}{\mathbf{sec}\,\theta + \mathbf{1}} = \frac{\mathbf{1} - \mathbf{cos}\,\theta}{\mathbf{1} + \mathbf{cos}\,\theta}$

Sol. L.H.S.= $\frac{\sec\theta - 1}{\sec\theta + 1}$

$= \frac{\frac{1}{\cos\theta} - 1}{\frac{1}{\cos\theta} + 1}$ $\left[\because \sec\theta = \frac{1}{\cos\theta}\right]$

$= \frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cos\theta}}$

$= \frac{(1-\cos\theta)\times\cos\theta}{\cos\theta\times(1+\cos\theta)}$

$= \frac{1-\cos\theta}{1+\cos\theta}$ = R.H.S. **Proved.**

Short Answer Type Questions

Prove the Following Questions :

26. $\frac{\mathbf{cos}^2\,\theta}{\mathbf{sin}\,\theta}$ **+ sinθ = cosec θ.**

Sol. L.H.S. = $\frac{\cos^2\theta}{\sin\theta} + \sin\theta$

$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta}$

[$\because \sin^2\theta + \cos^2\theta = 1$]

$= \frac{1}{\sin\theta} = \text{cosec}\,\theta$ = R.H.S.

Proved.

27. $\frac{\mathbf{1} - \mathbf{sin}\,\theta}{\mathbf{1} + \mathbf{sin}\,\theta}$ **= (sec θ – tan θ)²**

Sol. L.H.S. = $\frac{1-\sin\theta}{1+\sin\theta}$

Multiplying numerator and denominator by (1 – sin θ), we get

L.H.S. = $\left(\frac{1-\sin\theta}{1+\sin\theta}\right) \times \left(\frac{1-\sin\theta}{1-\sin\theta}\right)$

$= \frac{(1-\sin\theta)^2}{(1-\sin^2\theta)} = \frac{(1-\sin\theta)^2}{\cos^2\theta}$

[$\because 1 - \sin^2\theta = \cos^2\theta$]

$= \left(\frac{1-\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$

$= (\sec\theta - \tan\theta)^2$ = R.H.S.

Proved.

28. $\frac{\mathbf{1} - \mathbf{cos}\,\theta}{\mathbf{1} + \mathbf{cos}\,\theta}$ **= (cot θ – cosec θ)².**

Sol. L.H.S.= $\frac{1-\cos\theta}{1+\cos\theta}$

Multiplying numerator and denominator by (1 – cos θ), we get

L.H.S. = $\frac{1-\cos\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta}$

$= \dfrac{(1-\cos\theta)^2}{1-\cos^2\theta}$

$= \dfrac{(1-\cos\theta)^2}{\sin^2\theta}$ [$\because$ $1 - \cos^2\theta = \sin^2\theta$]

$= \left(\dfrac{1-\cos\theta}{\sin\theta}\right)^2 = \left(\dfrac{\cos\theta-1}{\sin\theta}\right)^2$

$= \left(\dfrac{\cos\theta}{\sin\theta} - \dfrac{1}{\sin\theta}\right)^2$

= (cot θ − cosec θ)^2 = R.H.S.

Proved.

29. $\left(\dfrac{\mathbf{1+\cos\theta}}{\mathbf{\sin\theta}}\right)^{\mathbf{2}} = \dfrac{\mathbf{1+\cos\theta}}{\mathbf{1-\sin\theta}}$**.**

Sol. L.H.S.= $\left(\dfrac{1+\cos\theta}{\sin\theta}\right)^2$

$= \dfrac{(1+\cos\theta)^2}{\sin^2\theta}$

$= \dfrac{(1+\cos\theta)^2}{1-\cos^2\theta}$

[$\because$ $\sin^2\theta = 1 - \cos^2\theta$]

$= \dfrac{(1+\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$

$= \dfrac{1+\cos\theta}{1-\cos\theta}$

= R.H.S. **Proved.**

30. $\dfrac{\mathbf{\tan\theta}}{\mathbf{(\sec\theta-1)}} + \dfrac{\mathbf{\tan\theta}}{\mathbf{(\sec\theta+1)}} = \mathbf{2\,cosec\,\theta}$**.**

Sol. L.H.S. $= \dfrac{\tan\theta}{(\sec\theta-1)} + \dfrac{\tan\theta}{(\sec\theta+1)}$

$= \dfrac{\frac{\sin\theta}{\cos\theta}}{\left(\frac{1}{\cos\theta}-1\right)} + \dfrac{\frac{\sin\theta}{\cos\theta}}{\left(\frac{1}{\cos\theta}+1\right)}$

$\left[\tan\theta = \dfrac{\sin\theta}{\cos\theta}, \sec\theta = \dfrac{1}{\cos\theta}\right]$

$= \dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{1-\cos\theta}{\cos\theta}} + \dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cos\theta}}$

$= \dfrac{\sin\theta\times\cos\theta}{\cos\theta\times(1-\cos\theta)} + \dfrac{\sin\theta\times\cos\theta}{\cos\theta\times(1+\cos\theta)}$

$= \dfrac{\sin\theta}{1-\cos\theta} + \dfrac{\sin\theta}{1+\cos\theta}$

$= \dfrac{\sin\theta(1+\cos\theta)+\sin\theta(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$= \dfrac{\sin\theta(1+\cos\theta+1-\cos\theta)}{1-\cos^2\theta}$

$= \dfrac{2\sin\theta}{\sin^2\theta}$ [$\because$ $1 - \cos^2\theta = \sin^2\theta$]

$= \dfrac{2}{\sin^2\theta}$ = 2 cosec θ = R.H.S.

Proved.

31. $\dfrac{\mathbf{\sin\theta}}{\mathbf{(1-\cos\theta)}} + \dfrac{\mathbf{\tan\theta}}{\mathbf{(1+\cos\theta)}}$ **= sec** θ **cosec** θ **+ cot** θ**.**

Sol. L.H.S.

$= \dfrac{\sin\theta}{(1-\cos\theta)} + \dfrac{\tan\theta}{(1+\cos\theta)}$

$= \dfrac{\sin\theta(1+\cos\theta)+\tan\theta(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$= \dfrac{\sin\theta(1+\cos\theta)+\frac{\sin\theta}{\cos\theta}(1-\cos\theta)}{1-\cos^2\theta}$

$\left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right]$

$= \dfrac{\sin\theta\left[1+\cos\theta+\frac{1}{\cos\theta}(1-\cos\theta)\right]}{\sin^2\theta}$

[$\because$ $1 - \cos^2\theta = \sin^2\theta$]

$= \dfrac{1+\cos\theta+\frac{1}{\cos\theta}-1}{\sin\theta}$

$$= \frac{\cos\theta + \frac{1}{\cos\theta}}{\sin\theta}$$

$$= \frac{\frac{\cos^2\theta + 1}{\cos\theta}}{\sin\theta} = \frac{\cos^2\theta + 1}{\sin\theta\cos\theta}$$

$$= \frac{\cos^2\theta}{\sin\theta\cos\theta} + \frac{1}{\sin\theta\cos\theta}$$

$$= \frac{\cos\theta}{\sin\theta} + \frac{1}{\cos\theta}.\frac{1}{\cos\theta}$$

$= \cot\theta + \operatorname{cosec}\theta.\ \sec\theta$ = R.H.S.

Proved.

32. $\mathbf{\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta.}$

Sol. L.H.S.= $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$$= \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - \sin^2\theta - \cos^2\theta)}$$

$[\because 1 = \sin^2\theta + \cos^2\theta]$

$$= \frac{\sin\theta(\cos^2\theta - \sin^2\theta)}{\cos\theta(\cos^2\theta - \sin^2\theta)}$$

$= \dfrac{\sin\theta}{\cos\theta} = \tan\theta$ = R.H.S. **Proved.**

33. $\mathbf{\dfrac{1 + \cos\theta + \sin\theta}{1 + \cos\theta - \sin\theta} = \dfrac{1 + \sin\theta}{\cos\theta}.}$

Sol. L.H.S.= $\dfrac{(1 + \cos\theta) + (\sin\theta)}{(1 + \cos\theta) - (\sin\theta)}$

Multiplying numerator and denominator by $\{(1 + \cos\theta) + \sin\theta\}$, we get

$$\text{L.H.S.} = \frac{(1 + \cos\theta) + \sin\theta}{(1 + \cos\theta) + \sin\theta} \times$$

$$\frac{(1 + \cos\theta) + \sin\theta}{(1 + \cos\theta) + \sin\theta}$$

$$= \frac{\{(1 + \cos\theta) + \sin\theta\}}{(1 + \cos\theta)^2 - \sin^2\theta}$$

$(1 + \cos\theta)^2 + \sin^2\theta$

$$= \frac{+ 2\sin\theta(1 + \cos\theta)}{1 + \cos^2\theta + 2\cos\theta - \sin^2\theta}$$

$= 1 + \cos^2\theta + 2\cos\theta + \sin^2\theta$

$$= \frac{+2\sin\theta + 2\sin\theta\cos\theta}{1 + \cos^2\theta - \sin^2\theta + 2\cos\theta}$$

$$= \frac{1 + 1 + 2\cos\theta + 2\sin\theta + 2\sin\theta\cos\theta}{1 + \cos^2\theta - \sin^2\theta + 2\cos\theta}$$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$2 + 2\cos\theta$

$$= \frac{+2\sin\theta + 2\sin\theta\cos\theta}{\sin^2\theta + \cos^2\theta + \cos^2\theta}$$

$- \sin^2\theta + 2\cos\theta$

$[\because 1 = \sin^2\theta + \cos^2\theta]$

$$= \frac{2(1 + \cos\theta + \sin\theta + \sin\theta\cos\theta)}{2(\cos^2\theta + \cos\theta)}$$

$$= \frac{1 + \cos\theta + \sin\theta + \sin\theta\cos\theta}{\cos^2\theta + \cos\theta}$$

$$= \frac{1(1 + \cos\theta) + \sin\theta(1 + \cos\theta)}{\cos\theta(1 + \cos\theta)}$$

$$= \frac{(1 + \cos\theta)(1 + \cos\theta)}{\cos\theta(1 + \cos\theta)}$$

$= \dfrac{1 + \sin\theta}{\cos\theta}$ = R.H.S. **Proved.**

34. $\mathbf{(\sin^4\theta - \cos^4\theta) = (\sin^2\theta - \cos^2\theta) = (2\sin^2\theta - 1) = (1 - 2\cos^2\theta).}$

Sol. L.H.S.= $\sin^4\theta - \cos^4\theta$

$= (\sin^2\theta)^2 - (\cos^2\theta)^2$

$= (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)$

$[\because a^2 - b^2 = (a - b)(a + b)]$

$= (\sin^2\theta - \cos^2\theta)$ (1),

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= \sin^2\theta - \cos^2\theta.$ **Proved.**

Again on simplification, we get

$\sin^2\theta + \cos^2\theta = \sin^2\theta - (1 - \sin^2\theta),$

$[\because \cos^2\theta = 1 - \sin^2\theta]$

$= \sin^2\theta - 1 + \sin^2\theta$

$= 2\sin^2\theta - 1.$ **Proved.**

Also, on simplification again, we get

$2\sin^2\theta - 1 = 2(1 - \cos^2\theta) - 1$

$[\because \sin^2\theta = 1 - \cos^2\theta]$

$= 2 - 2\cos^2\theta - 1$

$= 1 - 2\cos^2\theta$ = R.H.S. **Proved.**

35. $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$.

Sol. L.H.S.$= \sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$

$[\because a^3 + b^3 = (a+b)^3 - 3ab(a+b)]$

$= (1)^3 - 3\sin^2\theta\cos^2\theta(1)$,

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= 1 - 3\sin^2\theta\cos^2\theta$ = R.H.S.

Proved.

36. $\sin^8\theta - \cos^8\theta = (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$.

Sol. L.H.S. $= \sin^8\theta - \cos^8\theta$

$= (\sin^4\theta)^2 - (\cos^4\theta)^2$

$= (\sin^4\theta - \cos^4\theta)(\sin^4\theta + \cos^4\theta)$

$[\because a^2 - b^2 = (a-b)(a+b)]$

$= [(\sin^2\theta)^2 - (\cos^2\theta)^2](\sin^4\theta + \cos^4\theta)$

$= (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta)$

$= (\sin^2\theta - \cos^2\theta)(1)[(\sin^2\theta)^2 + (\cos^2\theta)^2]$,

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta]$

$[\because a^2 + b^2 = (a+b)^2 - 2ab]$

$= (\sin^2\theta - \cos^2\theta)[(1)2 - 2\sin^2\theta\cos^2\theta]$

$= (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$

= R.H.S. **Proved.**

37. $$\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} + \frac{\sin\theta - \cos^2\theta}{\sin\theta + \cos\theta} = \frac{2}{\sin^2\theta - \cos^2\theta} = \frac{2}{1 - 2\cos^2\theta}$$

Sol. L.H.S. $= \dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} + \dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}$

$$= \frac{(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2}{(\sin\theta - \cos\theta)(\sin\theta + \cos\theta)}$$

$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{1 + 1}{\sin^2\theta - \cos^2\theta}$$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$$= \frac{2}{\sin^2\theta - \cos^2\theta}$$ **Proved.**

Again on simplification, we get

$$= \frac{2}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{2}{1 - \cos^2\theta - \cos^2\theta}$$

$[\because \sin^2\theta = 1 - \cos^2\theta]$

$$= \frac{2}{1 - 2\cos^2\theta} = \text{R.H.S.}$$

Proved.

38. $\sin^2 A\cos^2 B - \cos^2 A\sin^2 B = \sin^2 A - \sin^2 B$.

Sol. L.H.S. $= \sin^2 A\cos^2 B - \cos^2 A\sin^2 B$

$= \sin^2 A(1 - \sin^2 B) - (1 - \sin^2 A)\sin^2 B$

$[\because \cos^2 B = 1 - \sin^2 B,\ \cos^2 A = 1 - \sin^2 A]$

$\sin^2 A - \sin^2 A\sin^2 B - \sin^2 B + \sin^2 A\sin^2 B$

$= \sin^2 A - \sin^2 B$ = R.H.S. **Proved.**

39. $$\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B} = 0.$$

Sol. L.H.S.

$$= \frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B}$$

$$= \frac{(\sin A - \sin B)(\sin A + \sin B) + (\cos A - \cos B)(\cos A + \cos B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{(\sin^2 A - \cos^2 A) - (\sin^2 B + \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{1-1}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{0}{(\cos A + \cos B)(\sin A + \sin B)}$$

$= 0 =$ R.H.S. **Proved.**

40. $\dfrac{\tan A + \tan B}{\cot A + \cot B} = \tan A \tan B.$

Sol. L.H.S. $= \dfrac{\tan A + \tan B}{\cot A + \cot B}$

$$= \frac{\tan A + \tan B}{\frac{1}{\tan A} + \frac{1}{\tan B}}$$

$$\left[\because \cot A = \frac{1}{\tan A}, \cot B = \frac{1}{\tan B}\right]$$

$$= \frac{\tan A + \tan B}{\frac{\tan B + \tan A}{\tan A \tan B}}$$

$$= \frac{(\tan A + \tan B) \times \tan A \tan B}{(\tan A + \tan B)}$$

$= \tan A \tan B =$ R.H.S. **Proved.**

41. $\tan^2 A - \tan^2 B$

$$= \frac{\cos^2 B - \cos^2 A}{\cos^2 B . \cos^2 A}$$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A . \cos^2 B}.$$

Sol. L.H.S. $= \tan^2 A - \tan^2 B$

$$= \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}.$$

$$\left[\because \tan^2 A = \frac{\sin^2 A}{\cos^2 A}, \tan^2 B = \frac{\sin^2 B}{\cos^2 B}\right]$$

$$= \frac{\sin^2 A . \cos^2 B - \sin^2 B . \cos^2 A}{\cos^2 A . \cos^2 B}$$

$(1 - \cos^2 A)\cos^2 B$

$$= \frac{-(1 - \cos^2 B)\cos^2 A}{\cos^2 A . \cos^2 B}$$

$[\because \sin^2 A = 1 - \cos^2 A,$
$\sin^2 B = 1 - \cos^2 B]$

$\cos^2 B - \cos^2 A \cos^2 B$

$$= \frac{-\cos^2 A + \cos^2 A \cos^2 B}{\cos^2 A . \cos^2 B}$$

$$= \frac{\cos^2 B - \cos^2 A}{\cos^2 A . \cos^2 B} \cdot$$ **Proved.**

Again on simplification, we get

$$\frac{\cos^2 B - \cos^2 A}{\cos^2 A . \cos^2 B}$$

$$= \frac{(1 - \sin^2 B) - (1 - \sin^2 A)}{\cos^2 A . \cos^2 B}$$

$$= \frac{1 - \sin^2 B - 1 + \sin^2 A}{\cos^2 A . \cos^2 B}$$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B} = \text{R.H.S.}$$

Proved.

Long Answer Type Questions

42. If tan θ + sin θ = *m* and tan θ – sin θ = *n*, show that

$m^2 - n^2 = 4\sqrt{mn}$.

Sol. L.H.S. $= (m^2 - n^2)$

$= (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$

$= 4 \tan\theta \sin\theta,$

$[\because (a+b)^2 - (a-b)^2 = 4ab]$

R.H.S. $= 4\sqrt{mn}$

$$= 4\sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}$$

$$= 4\sqrt{\tan^2\theta - \sin^2\theta}$$

$$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta},$$

$$\left[\because \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}\right]$$

$$= 4\sqrt{\frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}}$$

$$= \frac{4\sqrt{\sin^2\theta(1-\cos^2\theta)}}{\cos\theta}$$

$$= \frac{4\sqrt{\sin^2\theta.\sin^2\theta}}{\cos\theta}$$

$$= \frac{4\sin^2\theta}{\cos\theta} = \frac{4\sin\theta}{\cos\theta}\sin\theta$$

$= 4 \tan \theta.\sin \theta$

Thus, L.H.S. = R.H.S.

i.e., $(m^2 - n^2) = 4\sqrt{mn}$. **Proved.**

43. If $\sec\theta + \tan\theta = m$ and $\sec\theta - \tan\theta = n$, show that $mn = 1$.

Sol. L.H.S. $= mn$

$= (\sec\theta + \tan\theta).(\sec\theta - \tan\theta)$

$= \sec^2\theta - \tan^2\theta$

$= 1 =$ R.H.S. **Proved.**

44. If $\sin\theta + \sin^2\theta = 1$, show that $\cos^2\theta + \cos^4\theta = 1$.

Sol. Given, $\sin\theta + \sin^2\theta = 1$

or $\sin\theta = 1 - \sin^2\theta$

or $\sin\theta = \cos^2\theta$

Squaring both sides, we get

$\sin^2\theta = \cos^4\theta$

or $1 - \cos^2\theta = \cos^4\theta$, $[\because \sin^2\theta = 1 - \cos^2\theta]$

or $\cos^4\theta + \cos^2\theta = 1$. **Proved.**

45. If $x = a\cos\theta - b\sin\theta$ and $y = a\sin\theta + b\cos\theta$, prove that $x^2 + y^2 = a^2 + b^2$

Sol. L.H.S. $= x^2 + y^2$

$= (a\cos\theta - b\sin\theta)^2 + (a\sin\theta + b\cos\theta)^2$

$= a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta\cos\theta + a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta.\cos\theta$

$= a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$

$= a^2(1) + b^2(1)$, $[\because \sin^2\theta + \cos^2\theta = 1]$

$= a^2 + b^2 =$ R.H.S. **Proved.**

46. If $x = r\sin\alpha\cos\beta$, $y = r\sin\alpha\sin\beta$ and $z = r\cos\alpha$, prove that $x^2 + y^2 + z^2 = r^2$.

Sol. L.H.S. $= x^2 + y^2 + z^2$

$= (r\sin\alpha\cos\beta)^2 + (r\sin\alpha\sin\beta)^2 + (r\cos\alpha)^2$

$= r^2\sin^2\alpha\cos^2\beta + r^2\sin^2\alpha\sin^2\beta + r^2\cos^2\alpha$

$= r^2\sin^2\alpha(\cos^2\beta + \sin^2\beta) + r^2\cos^2\alpha$

$= r^2\sin^2\alpha(1) + r^2\cos^2\alpha$ $[\because \sin^2\beta + \cos^2\beta = 1]$

$= r^2(\sin^2\alpha + \cos^2\alpha)$

$= r^2(1)$ $[\because \sin^2\alpha + \cos^2\alpha = 1]$

$= r^2 =$ R.H.S. **Proved.**

47. If $\tan A + \sin A = m$ and $\tan A - \sin A = n$, prove that $(m^2 - n^2)^2 = 16mn$

Sol. L.H.S. $= (m^2 - n^2)^2$

$= [(\tan A + \sin A)^2 - (\tan A - \sin A)^2]^2$

$= [4\tan A\sin A]^2$ $[\because (a+b)^2 - (a-b)^2 = 4ab]$

$= 16\tan^2 A\sin^2 A$

R.H.S. $= 16\,mn$

$= 16(\tan A + \sin A)(\tan A - \sin A)$

$= 16(\tan^2 A - \sin^2 A)$, $[\because (a+b)(a-b) = a^2 - b^2]$

$$= 16\left[\frac{\sin^2 A}{\cos^2 A} - \sin^2 A\right]$$

$$= 16\left[\frac{\sin^2 A - \sin^2 A.\cos^2 A}{\cos^2 A}\right]$$

$$= \frac{16\sin^2 A(1-\cos^2 A)}{\cos^2 A}$$

$$= \frac{16\sin^2 A \times \sin^2 A}{\cos^2 A}$$

$$= \frac{16\sin^2 A}{\cos^2 A}.\sin^2 A$$

$= 16\tan^2 A.\sin^2 A$

Thus, L.H.S. = R.H.S. **Proved.**

48. If $\dfrac{\cos\theta}{\cos\beta} = m$ and $\dfrac{\cos\alpha}{\sin\beta} = n$, show that $(m^2 + n^2)\cos^2\beta = n^2$.

Sol. L.H.S. $= (m^2 + n^2)\cos^2\beta$

$$= \left[\left(\frac{\sin\alpha}{\cos\beta}\right)^2 + \left(\frac{\sin\alpha}{\cos\beta}\right)^2\right].\cos^2\beta$$

$$= \left[\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right].\cos^2\beta$$

$$= \cos^2\alpha\left[\frac{1}{\cos^2\beta} + \frac{1}{\sin^2\beta}\right].\cos^2\beta$$

$$= \cos^2\alpha\left[\frac{\sin^2\beta + \cos^2\beta}{\cos^2\beta\sin^2\beta}\right].\cos^2\beta,$$

$$= \cos^2\alpha\left[\frac{1}{\cos^2\beta\sin^2\beta}\right].\cos^2\beta,$$

$[\because \sin^2\beta + \cos^2\beta = 1]$

$$= \frac{\sin^2\alpha}{\sin^2\beta} = \left(\frac{\sin\alpha}{\sin\beta}\right)^2 = n^2 = \text{R.H.S.}$$

49. If a cot θ + b cosec θ = x^2 and b cot θ + a cosec θ = y^2, prove that $x^4 - y^4 = b^2 - a^2$.

Sol. L.H.S. $= x^4 - y^4$

$= (x^2)^2 - (y^2)^2$

$= (a \cot\theta + b \operatorname{cosec}\theta)^2 - (b \cot\theta + a \operatorname{cosec}\theta)^2$

$= a^2 \cot^2\theta + b^2 \operatorname{cosec}^2\theta + 2ab \cot\theta.\operatorname{cosec}\theta - (b^2 \cot^2\theta + a^2 \operatorname{cosec}^2\theta + 2ab \cot\theta.\operatorname{cosec}\theta)$

$= a^2 \cot^2\theta + b^2 \operatorname{cosec}^2\theta + 2ab \cot\theta.\operatorname{cosec}\theta - b^2 \cot^2\theta - a^2 \operatorname{cosec}^2\theta - 2ab \cot\theta.\operatorname{cosec}\theta$

$= a^2 \cot^2\theta + b^2 \operatorname{cosec}^2\theta - b^2 \cot^2\theta - a^2 \operatorname{cosec}^2\theta$

$= \cot^2\theta\,(a^2 - b^2) + \operatorname{cosec}^2\theta\,(b^2 - a^2)$

$= -\cot^2\theta(b^2 - a^2) + \operatorname{cosec}^2\theta(b^2 - a^2)$

$= (b^2 - a^2)(\operatorname{cosec}^2\theta - \cot^2\theta)$

$= (b^2 - a^2)(1), [\because \operatorname{cosec}^2\theta - \cot^2\theta = 1]$

$= b^2 - a^2 =$ R.H.S. **Proved.**

50. If $x = a$ sec θ + b tan θ and $y = a$ tan θ + b sec θ, then prove that $x^2 - y^2 = a^2 - b^2$.

Sol. L.H.S. $= x^2 - y^2$

$= (a \sec\theta + b \tan\theta)^2 - (a \tan\theta + b \sec\theta)^2$

$= a^2 \sec^2\theta + b^2 \tan^2\theta + 2ab \sec\theta.\tan\theta - (a^2 \tan^2\theta + b^2 \sec^2\theta + 2ab \tan\theta.\sec\theta)$

$= a^2 \sec^2\theta + b^2 \tan^2\theta + 2ab \sec\theta.\tan\theta - a^2 \tan^2\theta - b^2 \sec^2\theta - 2ab \tan\theta.\sec\theta$

$= a^2 \sec^2\theta + b^2 \tan^2\theta - a^2 \tan^2\theta - b^2 \sec^2\theta$

$= \sec^2\theta\,(a^2 - b^2) - \tan^2\theta\,(a^2 - b^2)$

$= (a^2 - b^2)(\sec^2\theta - \tan^2\theta)$

$= (a^2 - b^2)(1), [\because \sec^2\theta - \tan\theta = 1]$

$= (a^2 - b^2) =$ R.H.S. **Proved.**

51. If $x = a$ sec θ, $y = b$ tan θ, prove that $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Sol. L.H.S. $= \frac{x^2}{a^2} - \frac{y^2}{b^2}$

$$= \frac{a^2\sec^2\theta}{a^2} - \frac{b^2\tan^2\theta}{b^2}$$

$[\because x = a \sec\theta, y = b \tan\theta]$

$= \sec^2\theta - \tan^2\theta$

$[\because \sec^2\theta - \tan^2\theta = 1]$

$= 1 =$ R.H.S. **Proved.**

52. If $x = a$ sin θ, $y = b$ tan θ, prove that $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$.

Sol. L.H.S. $= \frac{a^2}{x^2} - \frac{b^2}{y^2}$

$$= \frac{a^2}{(a\sin\theta)^2} - \frac{b^2}{(b\tan\theta)^2},$$

$[\because x = a \sin\theta \text{ and } y = b \tan\theta]$

$$= \frac{a^2}{a^2\sin^2\theta} - \frac{b^2}{b^2\tan^2\theta}$$

$$= \frac{1}{\sin^2\theta} - \frac{1}{\tan^2\theta}$$

$$= \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}.$$

$\left[\because \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}\right]$

$$= \frac{1 - \cos^2\theta}{\sin^2\theta} = \frac{\sin^2\theta}{\sin^2\theta},$$

$[\because 1 - \cos^2\theta = \sin^2\theta]$

$= 1 =$ R.H.S. **Proved.**

53. If $x = h + a \cos\theta, y = k + b \sin\theta$, prove that :

$$\left(\frac{x-h}{a}\right)^2 + \left(\frac{y-k}{b}\right)^2 = 1.$$

Sol. L.H.S.

$$= \left(\frac{x-h}{a}\right)^2 + \left(\frac{y-k}{b}\right)^2$$

$$= \left(\frac{h + a\cos\theta - h}{a}\right)^2 + \left(\frac{k + b\sin\theta - k}{b}\right)^2$$

$[\because x = h + a \cos\theta,\ y = h + b \sin\theta]$

$$= \left(\frac{a\cos\theta}{a}\right)^2 + \left(\frac{b\sin\theta}{b}\right)^2$$

$= \cos^2\theta + \sin^2\theta$

$= 1 =$ R.H.S. **Proved.**

54. Prove that $\sec A - \tan A = \dfrac{1}{\sec A + \tan A}$.

Sol. R.H.S. $= \dfrac{1}{\sec A + \tan A} \times \dfrac{\sec A - \tan A}{\sec A - \tan A}$

$$= \frac{\sec A - \tan A}{\sec^2 A - \tan^2 A}$$

$$= \frac{\sec A - \tan A}{1}$$

$= \sec A - \tan A =$ L.H.S.

55. If $x = a\cos^3\theta, y = b\sin^3\theta$, prove that

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1.$$

Sol. L.H.S. $= \left(\dfrac{x}{a}\right)^{2/3} + \left(\dfrac{y}{b}\right)^{2/3}$

$$= \left(\frac{a\cos^3\theta}{a}\right)^{2/3} + \left(\frac{b\sin^3\theta}{b}\right)^{2/3}$$

$[\because x = a\cos^3\theta, y = b\sin^2\theta]$

$= (\cos^3\theta)^{2/3} + (\sin\theta)^{2/3}$

$= \cos^2\theta + \sin^2\theta,$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= 1 =$ R.H.S. **Proved.**

Determine whether the following equations are identities :

56. $\sin^2\theta + \sin\theta = 1$.

Sol. Given, $\sin^2\theta + \sin\theta = 1$

The variable θ in this equation can take values $0° \le \theta \le 90°$.

When $\theta = 30°$, we have

L.H.S. $= \sin^2 30° + \sin 30°$

$$= \left(\frac{1}{2}\right)^2 + \frac{1}{2} \quad \left[\because \sin 30° = \frac{1}{2}\right]$$

$$= \frac{1}{4} + \frac{1}{2} = \frac{1+2}{4} = \frac{3}{4}$$

But R.H.S. = 1

$\therefore$ The two sides are unequal (*i.e.,* L.H.S. $\ne$ R.H.S.), when $\theta = 30°$

Hence, the given equation is not an identity. **Ans.**

57. $\cot^2\theta + \cos\theta = \sin^2\theta$.

Sol. The given equation is

$\cot^2\theta + \cos\theta = \sin^2\theta$

The variable θ in this equation can take values $0° < \theta \le 90°$, since $\cot 0°$ is undefined.

When $\theta = 90°$, we have

L.H.S. $= \cot^2 90° + \cos 90°$

$= (0)^2 + 0 = 0 + 0 = 0$

and R.H.S. $= \sin^2 90° = (1)^2 = 1.$

$\therefore$The two sides are unequal (*i.e.,* L.H.S. $\ne$ R.H.S.), when $\theta = 90°$

Hence the given equation is not an identity. **Ans.**

58. $\tan^4\theta + \tan^6\theta = \tan^3\theta \sec^2\theta$.

Sol. The given equation is

$\tan^4\theta + \tan^6\theta = \tan^3\theta . \sec^2\theta$

The variable θ in this equation can take values $0° \le \theta < 90°$

Also, the given equation can be written as

$\tan^4\theta(1+\tan^2\theta) = \tan^3\theta.\ \sec^2\theta$

or $\tan^4\theta.\ \sec^2\theta = \tan^3\theta.\ \sec^2\theta.$

$[\because 1+\tan^2\theta = \sec^2\theta]$

or $\tan^3\theta\sec^2\theta(\tan\theta - 10) = 0$

$\therefore$ The given equation is satisfied if and only if

$\tan^3\theta = 0$ or $\sec^2\theta = 0$

or $\tan\theta - 1 = 0$

Now, $\tan^3\theta = 0$

$\Rightarrow \tan\theta = 0 \Rightarrow \theta = 0°$

Again $\sec^2\theta = 1+\tan^2\theta \geq 1 \neq 0$

and $\tan\theta - 1 = 0$

$\Rightarrow \tan\theta = 1 \Rightarrow \theta = 45°$

Since, the given equation has only two solutions 0° and 45°, therefore it is not an identity

59. $\dfrac{\cos\theta}{\operatorname{cosec}\theta+1} + \dfrac{\cos\theta}{\operatorname{cosec}\theta-1} = 2.$

Sol. The given equation is

$$\frac{\cos\theta}{\operatorname{cosec}\theta+1} + \frac{\cos\theta}{\operatorname{cosec}\theta-1} = 2$$

The variable θ in this equation can take values $0° \leq \theta < 90°$

The given equations becomes

$\cos\theta(\operatorname{cosec}\theta - 1)$

$$\frac{+\cos\theta(\operatorname{cosec}\theta+1)}{(\operatorname{cosec}\theta+1)(\operatorname{cosec}\theta-1)} = 2$$

or $$\frac{\cos\theta(\operatorname{cosec}-1+\operatorname{cosec}\theta+1)}{\operatorname{cosec}^2\theta-1} = 2$$

or $$\frac{2\cos\theta\operatorname{cosec}\theta}{\cot^2\theta} = 2,$$

$[\because \operatorname{cosec}^2\theta - 1 = \cot^2\theta]$

or $$\frac{2\cos\theta\times\dfrac{1}{\sin\theta}}{\dfrac{\cos^2\theta}{\sin^2\theta}} = 2$$

or $$\frac{2\cos\theta\times\sin^2\theta}{\sin\theta\times\cos^2\theta} = 2$$

or $\tan\theta = 1 = \tan 45°$

$\therefore \theta = 45°.$

Hence, the solution of the given equation is $\theta = 45°$.

60. $\dfrac{\cos^2\theta}{\cot^2\theta-\cos^2\theta} = 3.$

Sol. The given equation is

$$\frac{\cos^2\theta}{\cot^2\theta-\cos^2\theta} = 3$$

The variable θ in this equation can take those values for which cos θ and cot θ are defined and $\cot^2\theta - \cos^2\theta \neq 0$, *i.e.,* $0° < \theta < 90°$

$$\therefore \quad \frac{\cos^2\theta}{\dfrac{\cos^2\theta}{\sin^2\theta}-\cos^2\theta} = 3,$$

$$\left(\because \cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta}\right)$$

or $$\frac{\cos^2\theta}{\cos^2\theta\left(\dfrac{1}{\sin^2}-1\right)} = 3$$

or $$\frac{\sin^2\theta}{1-\sin^2\theta} = 3$$

or $$\frac{\sin^2\theta}{\cos^2\theta} = 3$$

or $\tan^2\theta = 3$

Taking square root on both sides, we get

$\tan\theta = \sqrt{3}$

or $\tan\theta = \tan 60°,$

$[\because \tan 60° = \sqrt{3}]$

$\therefore \theta = 60°$

Hence, the solution of the given equation is $\theta = 60°$. **Ans.**

61. $\dfrac{\cos^2\theta - 3\cos\theta + 2}{\sin^2\theta} = 1$

Sol. The given equation is

$$\frac{\cos^2\theta - 3\cos\theta + 2}{\sin^2\theta} = 1$$

The variable θ in this equation can take values $0° < \theta \le 90°$

$\therefore \quad \cos^2\theta - 3\cos\theta + 2 = \sin^2\theta$

or $\quad \cos^2\theta - 3\cos\theta + 2 = 1 - \cos^2\theta$

$[\because \sin^2\theta = 1 - \cos^2\theta]$

or $\quad 2\cos^2\theta - 3\cos\theta + 1 = 0$

or $2\cos^2\theta - 2\cos\theta - \cos\theta + 1 = 0$

or $2\cos\theta(\cos\theta - 1) - 1(\cos\theta - 1) = 0$

or $\quad (\cos\theta - 1)(2\cos\theta - 1) = 0$

$\Rightarrow \quad \cos\theta - 1 = 0$

or $\quad 2\cos\theta - 1 = 0$

Now, $\quad \cos\theta - 1 = 0$

$\Rightarrow \quad \cos\theta = 1 = \cos 0°$

$\therefore \quad \theta = 0°$

Since, $0° < \theta \le 90°$, therefore $\theta \ne 0°$

Again, $2\cos\theta - 1 = 0$

$\Rightarrow \cos\theta = \frac{1}{2} \qquad \left[\because \cos 60° = \frac{1}{2}\right]$

$= \cos 60°,$

$\therefore \quad \theta = 60°$

Hence, the solution of the given equation is $\theta = 60°$. **Ans.**

62. $\mathbf{\sin^2\theta - 2\cos\theta + \frac{1}{4} = 0.}$

Sol. The given equation is

$$\sin^2\theta - 2\cos\theta + \frac{1}{4} = 0.$$

The variable θ in this equation can take values $0° \le \theta < 90°$

$\therefore \quad 4\sin^2\theta - 8\cos\theta + 1 = 0$

or $4(1 - \cos^2\theta) - 8\cos\theta + 1 = 0,$

$[\because \sin^2\theta = 1 - \cos^2\theta]$

or $\quad 4 - 4\cos^2\theta - 8\cos\theta + 1 = 0$

or $\quad 4\cos^2\theta + 8\cos\theta - 5 = 0$

which is a quadratic equation in $\cos\theta$

$$\therefore \quad \cos\theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

[Here $b = 8$, $a = 4$, $c = -5$]

$$= \frac{-8 \pm \sqrt{(8)^2 - 2(4)(-5)}}{2(4)}$$

$$= \frac{-8 \pm \sqrt{64 + 80}}{8}$$

$$= \frac{-8 \pm \sqrt{144}}{8}$$

$$= \frac{-8 \pm 12}{8} = \frac{4}{8}, \frac{-20}{8}$$

$$= \frac{1}{2}, \frac{-5}{2}$$

But $\cos\theta = \frac{-5}{2}$ is not possible,

since, $0 \le \cos\theta \le 1$

$\therefore \quad \cos\theta = \frac{1}{2} = \cos 60°$

$\left[\because \cos 60° = \frac{1}{2}\right]$

$\Rightarrow \quad \theta = 60°$

Hence, the solution of the given equation is $\theta = 60°$. **Ans.**

63. $\mathbf{\frac{\sin\alpha}{1 - \cos\alpha} + \frac{\sin\alpha}{1 + \cos\alpha} = 4.}$

Sol. $\frac{\sin\alpha}{1 - \cos\alpha} + \frac{\sin\alpha}{1 + \cos\alpha}$

$$\Rightarrow \frac{\sin\alpha}{1 - \cos\alpha} \times \frac{1 + \cos\alpha}{1 + \cos\alpha} + \frac{\sin\alpha}{1 + \cos\alpha} \times \frac{1 - \cos\alpha}{1 - \cos\alpha} = 4$$

$$\Rightarrow \frac{\sin\alpha(1 + \cos\alpha)}{1 - \cos^2\alpha} + \frac{\sin\alpha(1 - \cos\alpha)}{1 - \cos^2\alpha} = 4$$

$$= \frac{\sin\alpha(1 + \cos\alpha)}{\sin^2\alpha} + \frac{\sin\alpha(1 - \cos\alpha)}{\sin^2\alpha} = 4$$

$$\Rightarrow \frac{1 + \cos\alpha}{\sin\alpha} + \frac{1 - \cos\alpha}{\sin\alpha} = 4$$

$\Rightarrow 1 + \cos\alpha + 1 - \cos\alpha = 4\sin\alpha$

$\Rightarrow \quad 2 = 4\sin\alpha$

$\Rightarrow \quad \sin\alpha = \frac{1}{2} = \sin 30°$

$\Rightarrow \quad \alpha = 30°.$

Hence the solution of the given trigonometric equation is $\alpha = 30°$.

Ans.

❑

11 Some Applications of Trigonometry

EXERCISE 11.1

Multiple Choice Type Questions

1. If the altitude of the sun is 45º, then the length of shadow of a *h* metre high tower standing on a plane will be :

(a) h metre (b) $h\sqrt{3}$ metre

(c) $\frac{h}{\sqrt{3}}$ metre (d) $\sqrt{3h}$ metre.

Ans. (a) h metre.

2. At any time the length of shadow of a pole is equal to the length of the pole, then Sun's altitude will be :

(a) 30º (b) 45º

(c) 60º (d) 90º.

Ans. (b) 45º.

3. At any instant, the length of a pole is $\sqrt{3}$ times of its shadow, the Sun's altitude will be :

(a) 30º (b) 45º

(c) 60º (d) None of these.

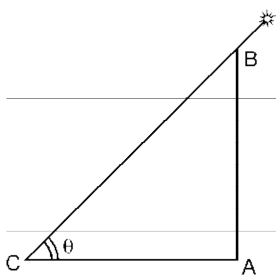
Ans. (c) 60º.

Very Short Answer Type Questions

4. The height of a tower is equal to its shadow, find the Sun's altitude at that instant.

Sol. Let AB be the tower and AC be its shadow. Let sun's altitude be θ.

Given AB = AC



$$\therefore \quad \frac{AB}{AC} = \tan\theta$$

$$\Rightarrow \quad \tan\theta = \frac{AB}{AB} = 1$$

$$= \tan 45^{\circ}$$

$$\therefore \quad \theta = 45^{\circ}$$

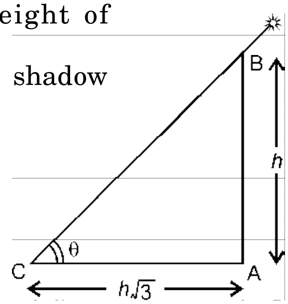
Hence, Sun's altitude is 45º. **Ans.**

5. If the shadow of a pole is $\frac{1}{\sqrt{3}}$ times of its height, find the angle of elevation of the Sun.

Sol. Let the height of pole = h

length of shadow $= \frac{h}{\sqrt{3}}$.

Let θ denotes the angle of elevation of sum



$$\text{In } \Delta ABC \tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan\theta = \frac{h\sqrt{3}}{h}$$

$$\tan\theta = \sqrt{3}$$

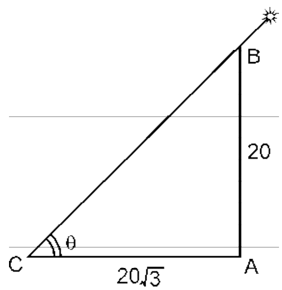
$\tan 60° = 60°$. [∵ Angle of elevation = 60°]

Ans.

6. The length of the shadow of a tower is $20\sqrt{3}$ metre. If the height of the tower be 20 metre then find the angle of elevation of the Sun at that time.

Sol. Let AB be the tower and AC is its shadow, such that AB = 20 m, AC = $20\sqrt{3}$ m Let Sun's altitude be θ, then

$$\tan\theta = \frac{AB}{AC}$$



$$= \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$= \tan 30°$$

$\therefore \quad \theta = 30°$

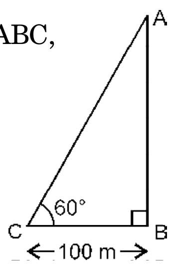
Hence angle of elevation of Sun be 30°. **Ans.**

Short Answer Type Questions

7. The angle of elevation of the top of a tower at a distance of 100 metres from its foot on a horizontal plane is found to be 60°. Find the height of the tower.

Sol. Let AB be a tower, BC = 100 metres and $\angle ACB = 60°$.

From right angled ΔABC, we have



$$\tan 60° = \frac{AB}{BC}$$

or $\quad \sqrt{3} = \frac{AB}{100}$

or $\quad AB = 100\sqrt{3}$ metres.

Hence, the height of the tower is $100\sqrt{3}$ metres. **Ans.**

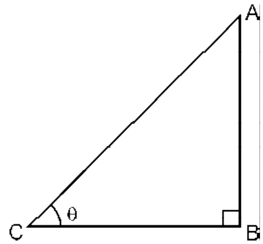
8. Determine the angle of elevation of the top of the flagstaff from a point whose distance from the flagstaff is equal to the height of the flagstaff.

Sol. Let AB be the flag-staff. Let θ be the angle of elevation.

According to the question, we have

$$AB = BC$$

Now, from right angled ΔABC,



$$\tan \theta = \frac{AB}{BC}$$

$$= \frac{BC}{BC} = 1, \quad [\because AB = BC]$$

or $\quad \theta = 45°, \quad [\because 1 = \tan 45°]$

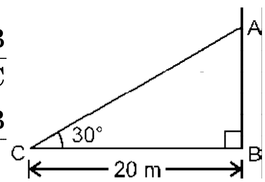
Hence, the required angle of elevation is 45°. **Ans.**

9. At a point 20 metre away from the foot of a tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower.

Sol. Let AB be the tower.

Here, BC = 20 m and $\angle ABC = 30°$.

Now, from right angled ΔABC, we have



$$\tan 30° = \frac{AB}{BC}$$

or $\quad \frac{1}{\sqrt{3}} = \frac{AB}{20}$

or $\quad AB = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3}$

$$= 11{\cdot}55 \text{ m.}$$

Hence, the height of the tower is 11·55 m. **Ans.**

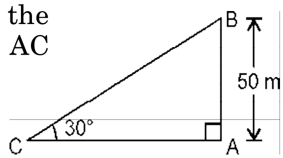
10. Find the length of the shadow of a minar 50 metre high, when the altitude of the Sun is 30°.

Sol. Let AB be the minar and AC its shadow.

Here,

$\angle ACB = 30°$

and AB = 50 m.



Now, from right angled triangle BAC, we have

$$\tan 30° = \frac{AB}{AC}$$

or $\frac{1}{\sqrt{3}} = \frac{50}{AC}$

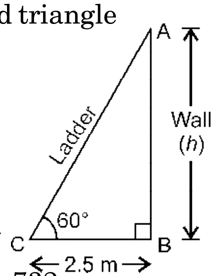
or $AC = 50\sqrt{3}$
$= 50 \times 1{\cdot}732$
$= 86{\cdot}6$ m. **Ans.**

11. A ladder is placed against a wall such that it just reaches the top of the wall. The foot of the ladder is 2·5 metre away from the wall and the ladder is inclined at an angle of 60 with the ground. Find the length of the ladder and the height of the wall.

Sol. Let the height of the wall (AB) = h metres.

∴ Distance of the foot of the ladder from the wall (BC) = 2·5 metres and ∠ACB = 60°.

From right angled triangle ABC, we have



$\tan 60° = \frac{AB}{BC}$

or $\sqrt{3} = \frac{h}{2{\cdot}5}$

or $h = 2{\cdot}5\sqrt{3}$
$= 2{\cdot}5 \times 1{\cdot}732$
$= 4{\cdot}33.$

Again, in right angled triangle ABC, we have

$AC^2 = AB^2 + BC^2$
$= (4{\cdot}33)^2 + (2{\cdot}5)^2$
$= 18{\cdot}75 + 6{\cdot}25 = 25$

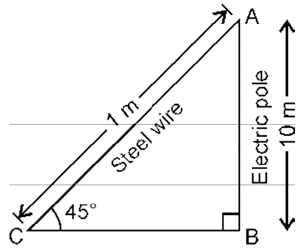
∴ $AC = \sqrt{25} \approx 5$ m.

Hence, the height of the wall is 4·33 m and the length of the ladder is 5 m. **Ans.**

12. An electric pole is 10 metre high. A steel wire tied to the top of the pole is affixed at a point on the ground to keep the pole upright. If the steel wire makes an angle of 45° with the horizontal through the foot of the pole, find the length of the steel wire.

Sol. Let the length of the steel wire (AC) = l metres. Height of the electric pole (AB) = 10 metres and ∠ACB = 45°.

In right angled triangle ABC, we have



$\sin 45° = \frac{AB}{AC}$

or $\frac{1}{\sqrt{2}} = \frac{10}{l}$

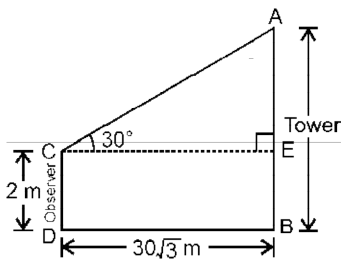
or $l = 10\sqrt{2}$
$= 10 \times 1{\cdot}414$
$= 14{\cdot}14$ m.

Hence, the length of the steel wire is 14·14 m. **Ans.**

13. An observer m tall is $30\sqrt{3}$ metre away from a tower. The angle of elevation from his eye to the top of the tower is 30°. Determine the height of the tower.

Sol. Let AB be the tower.

Height of observer (CD) = 2 m, BD $= 30\sqrt{3}$ m and ∠ACE = 30°.



Also, CD = EB

∴ AE = AB – EB
= AB – CD
= (AB – 2) m

and $EC = BD = 30\sqrt{3}$ m.

Now, in right angled triangle AEC, we have

$\tan 30° = \frac{AE}{EC}$

or $\frac{1}{\sqrt{3}} = \frac{AB-2}{30\sqrt{3}}$

or $AB - 2 = 30$

i.e., $AB = 32$ m,

Hence, the height of the tower is 32 m. **Ans.**

14. An observer is standing 58·5 metre away from a tower 60 metre high. The angle of elevation from his eye to the top of the tower is 45°. Find the height of observer.

Sol. Let the height of the tower (AB) = 60 m and the height of observer (CD) = h metres.

Also, BD = 58·5 m and ∠ACE = 45°.

From the adjoining figure,

$CD = EB$

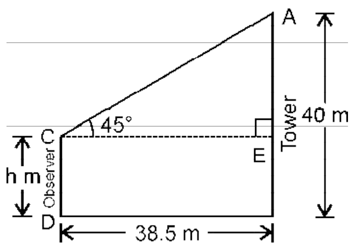
and $EC = BD = 58{\cdot}5$ m

Also, $AE = AB - EB$

$= AB - CD$

$= (60 - h)$ m.

Now, in right angled triangle AEC, we have



$$\tan 45° = \frac{AE}{EC}$$

or $1 = \frac{60-h}{58{\cdot}5}$

or $h = 60 - 58{\cdot}5 = 1{\cdot}5$ m.

Hence, the height of the observer is 1·5 m. **Ans.**

15. A straight highway leads to the foot of a 50 metre tall tower. From the top of the tower, the angles of depression of two cars on the highway are 30° and 60° respectively. What is the distance between two cars and how far is each car from the tower ?

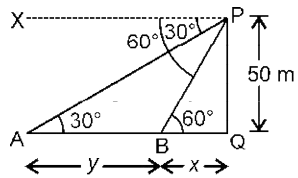
Sol. Let PQ be the tower and A, B be the positions of the two cars. Let QB = x metres and AB = y metres.

∠XPA = ∠QAP = 30°

and ∠XPB = ∠QBP = 60°

and PQ = 50 m.

Now, in right angled triangle PQB, we have



$$\cot 60° = \frac{QB}{PQ}$$

or $\frac{1}{\sqrt{3}} = \frac{x}{50}$

or $x = \frac{50}{\sqrt{3}} = \frac{50\sqrt{3}}{3}$

$= \frac{50 \times 1{\cdot}732}{3}$

$= 28{\cdot}86 \approx 28{\cdot}9$ m.

Again, in right angled triangle PQA, we have

$$\cot 30° = \frac{AQ}{PQ}$$

$\sqrt{3} = \frac{x+y}{50}$ [∵ AQ = BQ + AB = $x + y$]

or $x + y = 50\sqrt{3} = 86{\cdot}6$ m.

Substituting the value of x in above relation, we get

$y = 50\sqrt{3} - 28{\cdot}9$

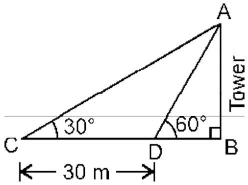
$= 50 \times 1{\cdot}732 - 28{\cdot}9$

$= 86{\cdot}6 - 28{\cdot}9 = 57{\cdot}7$

Hence, the distance between two cars is 57·7 m and the distances of the cars from the tower are 28·9 m and 86·6 m. **Ans.**

16. The angle of elevation of the top of a tower, at a instant, on the ground is 30°. After walking 30 metre towards the tower, the angle of elevation becomes 60°. What is the height of the tower ?

Sol. Let AB be the height of the tower.
Distance between the two points (CD) = 30 m.
Also, $\angle ACB = 30^\circ$ and $\angle ADB = 60^\circ$.
Now, in right angled triangle ABD, we have



$$\tan 60^\circ = \frac{AB}{BD}$$

or $$\sqrt{3} = \frac{AB}{BD}$$

or $$BD = \frac{AB}{\sqrt{3}}.$$

Also, in right angled triangle ABC, we have

$$\tan 30^\circ = \frac{AB}{BC}$$

or $$\frac{1}{\sqrt{3}} = \frac{AB}{\frac{AB}{\sqrt{3}} + 30}$$

$$\left[\because BC = BD + DC = \left(\frac{AB}{\sqrt{3}} + 30\right) m\right]$$

or $$\frac{1}{\sqrt{3}} = \frac{AB \times \sqrt{3}}{AB + 30\sqrt{3}}$$

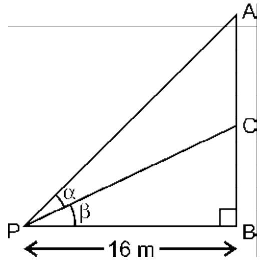
or $AB + 30\sqrt{3} = 3AB$

i.e., $$AB = 15\sqrt{3}$$
$$= 15 \times 1{\cdot}732$$
$$= 25{\cdot}98 \text{ m}.$$

Hence, the height of the tower is 25·98 m. **Ans.**

17. In the adjoining figure, the parts AC and CB are parts of a minar. AC and CB make angles α and β at the point P as shown. If $\tan \alpha = \frac{1}{2}$, $\tan \beta = \frac{1}{3}$; find the height of the minar if PB = 16 metres.

Sol. From the given right angled triangle ABP, we have



$$\tan(\alpha + \beta) = \frac{AB}{PB}$$

or $$\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha . \tan \beta} = \frac{AB}{16}$$

or $$\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}} = \frac{AB}{16}$$

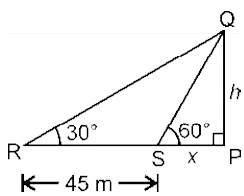
or $$\frac{\frac{5}{6}}{\frac{5}{6}} = \frac{AB}{16}$$

or $$AB = 16 \text{ m}.$$

Hence, the height of the minar is 16 m. **Ans.**

18. The shadow of a tower, standing on level ground, is found to be 45 metre longer when Sun's altitude (angle of elevation of the Sun) is 30° than when it was 60°. Find the height of the tower.

Sol. Let PQ be the tower and PR, PS be its shadows when Sun's altitudes are 30° and 60° respectively.



Let PQ = h, PS = x and PR = $(x + 45)$ m.
Then, in right angled triangle QPS, we have

$$\tan 60^\circ = \frac{PQ}{PS}$$

or $\sqrt{3} = \frac{h}{x}$

i.e., $x = \frac{h}{\sqrt{3}}$.

Also, in right angled triangle QPR, we have

$$\tan 30° = \frac{PQ}{PR}$$

or $\frac{1}{\sqrt{3}} = \frac{h}{x + 45}$

or $x = h\sqrt{3} - 45$.

Substituting $x = \frac{h}{\sqrt{3}}$ in above relation, we get

$$\frac{h}{\sqrt{3}} = \sqrt{3}h - 45$$

or $h - 3h = - 45\sqrt{3}$

or $2h = 45\sqrt{3}$

$\therefore$ $h = \frac{45 \times 1{\cdot}732}{2}$

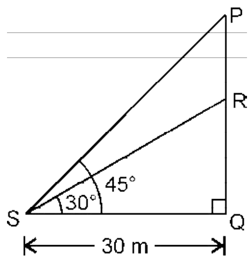
$= 38{\cdot}97$.

Hence, the height of the tower is 38·97 m. **Ans.**

19. From a point 30 m away from the foot of an unfinished temple, the angle of elevation of the top is 30°. How much high should the temple be raised so that the angle of elevation of the same point be 45° ?

Sol. Let RQ be the temple and height raised of the temple be RP.

$\angle PSQ = 45°$, $\angle RSQ = 30°$ and QS = 30 m.



Now, in right angled triangle RQS, we have

$$\tan 30° = \frac{QR}{QS}$$

or $\frac{1}{\sqrt{3}} = \frac{QR}{30}$

or $QR = \frac{30}{\sqrt{3}} = 10\sqrt{3}$.

Also, in right angled triangle PQS, we have

$$\tan 45° = \frac{PQ}{QS}$$

or $1 = \frac{PR + 10\sqrt{3}}{30}$

[$\because$ PQ = PR + QR = (PR + $10\sqrt{3}$) m]

or $PR = 30 - 10\sqrt{3}$

$= 30 - 10 \times 1{\cdot}732$

$= 12{\cdot}68$ m

Hence, required height is 12·68 m. **Ans.**

20. A man wants to find the breadth of a river. He stands on the bank and sees on the opposite bank, a minar and finds the angle of elevation of its top as 45°. When he recedes 30 m and sees the top of the minar, the angle of elevation becomes 30°. Find the breadth of the river.

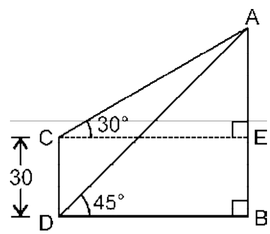
Sol. Let AB be the minar and BD be the river.

$\angle ADB = 45°$, $\angle ACE = 30°$

and DC = 30 m.

Now, in right angled triangle ABD, we have

$$\tan 45° = \frac{AB}{BD}$$



or $1 = \dfrac{AB}{BD}$

i.e., AB = BD.

Also, in right angled triangle AEC, we have

$$\tan 30° = \frac{AE}{CE}$$

or $\dfrac{1}{\sqrt{3}} = \dfrac{AB - 30}{CE}$

[$\because$ AE = AB – EB = AB – CD = AB – 30]

or $\dfrac{1}{\sqrt{3}} = \dfrac{CE - 30}{CE}$

[But, AB = BD = CE]

or $CE = \sqrt{3}\, CE - 30\sqrt{3}$

or $CE\,(1 - \sqrt{3}) = -\,30\sqrt{3}$

or $CE = \dfrac{30\sqrt{3}}{\sqrt{3} - 1}$

$= \dfrac{30\sqrt{3}\,(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \dfrac{30\sqrt{3}\,(\sqrt{3}+1)}{2}$

$= 15\sqrt{3}\,(\sqrt{3}+1)$

$= 15 \times 3 + 15\sqrt{3}$

$= 45 + 15 \times 1{\cdot}732$

$= 70{\cdot}98$ m.

Hence, the breadth of the river is 70·98 m. **Ans.**

21. The angles of elevation of the top of a rock at the top and foot of a 100 m high tower are respectively 30° and 45°. Find the height of the rock.

Sol. Let AB be the height of the tower, *i.e.*, AB = 100 m, and height of the rock, CD = h metres.

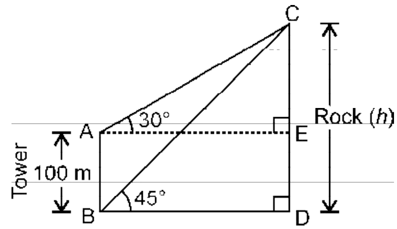
$\angle CAE = 30°$ and $\angle CBD = 45°$.

Here, AE = BD

and DE = AB = 100 m.

CE = CD – DE = (CD – 100) m.

Now, in right angled triangle CDB, we have



$$\tan 45° = \frac{CD}{BD}$$

or $1 = \dfrac{CD}{BD}$

or CD = BD.

Also, in right angled triangle CEA, we have

$$\tan 30° = \frac{CE}{AE}$$

or $\dfrac{1}{\sqrt{3}} = \dfrac{CD - 100}{AE}$

or $AE = \sqrt{3}\,(CD - 100)$

But, AE = BD = CD

$\therefore\ \sqrt{3}\,(CD - 100) = CD$

or $CD\,(\sqrt{3} - 1) = 100\sqrt{3}$

$CD = \dfrac{100\sqrt{3}}{\sqrt{3} - 1}$

$= \dfrac{100\sqrt{3}\,(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \dfrac{300 + 100\sqrt{3}}{2}$

$= 150 + 50\sqrt{3}$

$= 150 + 50 \times 1{\cdot}732$

$= 150 + 86{\cdot}6$

$= 236{\cdot}6$ m.

Hence, the height of the rock is 236·6 m. **Ans.**

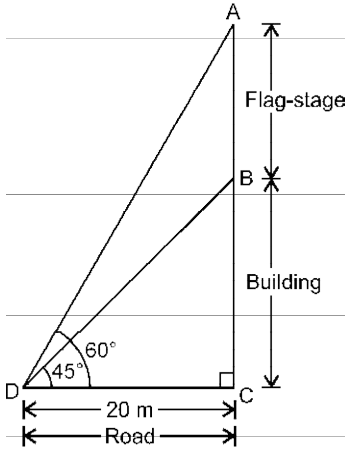
22. The angles of elevation of the top and the bottom of and flagstaff fixed on a building are 60° and 45° to a man standing on the other end of a road 20 m wide. Find the height of the flagstaff.

Sol. Let AB be the flag-staff, BC be the building and CD be the road.

$\therefore$ $\angle BDC = 45^\circ$ $\angle ADC = 60^\circ$ and $CD = 20$ m.

Now, in right angled triangle BCD, we have

$$\tan 45^\circ = \frac{BC}{CD}$$



or $$1 = \frac{BC}{20}$$

or $$BC = 20 \text{ m.}$$

Also, in right angled triangle ACD, we have

$$\tan 60^\circ = \frac{AC}{CD}$$

or $$\sqrt{3} = \frac{AB + BC}{20}$$

[$\because$ AC = AB + BC]

or $$AB + BC = 20\sqrt{3}$$

or $$AB = 20\sqrt{3} - 20$$

[$\because$ BC = 20 m]

or $$AB = 20(\sqrt{3} - 1)$$
$$= 20(1{\cdot}732 - 1)$$
$$= 14{\cdot}64 \text{ m.}$$

Hence, the length of flag-staff is 14·64 m. **Ans.**

23. The angles of elevation of an artificial earth satellite, as measured from two earth stations are on the same side of it are 30° and 60°. If the distance between the earth stations is 4000 km, find the height of the satellite.

Sol. Let the height of the satellite (AB) be h km.

Distance between the two earth stations (CD) = 4000 km.

$\angle ADB = 30^\circ$ and $\angle ACB = 60^\circ$.

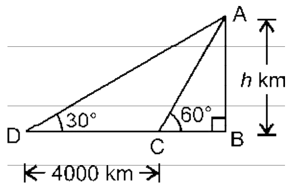
Now, in right angled triangle ABC, we have

$$\tan 60^\circ = \frac{AB}{BC}$$

or $$\sqrt{3} = \frac{h}{BC}$$

or $$BC = \frac{h}{\sqrt{3}}.$$

Also, in right angled triangle ABD, we have



$$\tan 30^\circ = \frac{AB}{BD}$$

or $$\frac{1}{\sqrt{3}} = \frac{h}{\frac{h}{\sqrt{3}} + 4000}$$

$$\left[\because BD = BC + CD = \left(\frac{h}{\sqrt{3}} + 4000\right) \text{ km.}\right]$$

or $$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}h}{h + 4000\sqrt{3}}$$

or $$2h = 4000\sqrt{3}$$

or $$h = 2000\sqrt{3}$$
$$= 2000 \times 1{\cdot}732$$
$$= 3464 \text{ km.}$$

Hence, the height of the satellite is 3464 km. **Ans.**

24. The angle of depression of the top of a pole, 7 m high from the top of a building is 60° and the angle of depression of the foot of the building from the top of

the pole is 45°. Find the height of the building and the distance between the building and the pole.

Sol. Let AB be the building and CD be the pole.

Then, CD = 7 m, ∠XAC = ∠ACE = 45° and ∠XAD = ∠ADB = 60°.

Also, EC = BD and EB = CD = 7 m.

Now, in right angled triangle ABD, we have

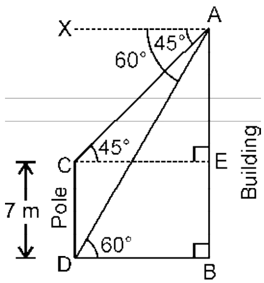
$$\tan 60° = \frac{AB}{BD}$$

or $$\sqrt{3} = \frac{AB}{BD}$$

or $$BD = \frac{AB}{\sqrt{3}}$$

or $$EC = \frac{AB}{\sqrt{3}}, \quad [\because BD = EC]$$

Also, in right angled triangle AEC, we have



$$\tan 45° = \frac{AE}{EC}$$

or $$1 = \frac{AB - 7}{EC} \quad [\because AE = AB - EB = (AB - 7) \text{ m}]$$

or $$EC = AB - 7$$

or $$\frac{AB}{\sqrt{3}} = AB - 7, \quad \left[\because EC = \frac{AB}{\sqrt{3}}\right]$$

or $$AB\left(\frac{1}{\sqrt{3}} - 1\right) = -7$$

or $$AB = \frac{7\sqrt{3}}{\sqrt{3} - 1}$$

or $$AB = \frac{7\sqrt{3}(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$= \frac{7 \times 3 + 7\sqrt{3}}{2}$$

$$= \frac{21 + 7\sqrt{3}}{2}$$

$$= \frac{21 + 12{\cdot}124}{2}$$

$$= \frac{33{\cdot}124}{2}$$

$$= 16{\cdot}562 \approx 16{\cdot}56 \text{ m}.$$

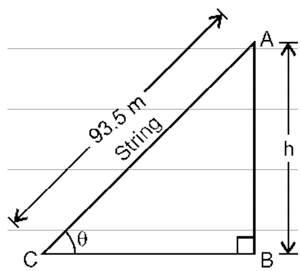
∴ $$EC = \frac{AB}{\sqrt{3}}$$

$$= \frac{16{\cdot}56}{1{\cdot}732} = 9{\cdot}56 \text{m}.$$

Hence, the height of the building is 16·56 m, and the distance between the building and the pole is 9·56 m. **Ans.**

25. The length of a string between a kite and a point on the ground is 93·5 m. If the string makes an angle θ with the level ground such that tan θ = (15/8), how high is the kite ?

Sol. Let the height of the kite from the ground, (AB) = *h* metres and AC be the string, *i.e.*, AC = 93·5 m. Also, let ∠ACB = θ.



∴ $$\tan\theta = \frac{15}{8}.$$

∴ $$\sin\theta = \frac{15}{\sqrt{(15)^2 + (8)^2}}$$

$$= \frac{15}{\sqrt{289}} = \frac{15}{17}.$$

Now, in right angled triangle ABC, we have

$$\sin\theta = \frac{AB}{AC}$$

or $$\frac{15}{17} = \frac{h}{93 \cdot 5}$$

or $$h = \frac{15 \times 93 \cdot 5}{17}$$

$$= 82 \cdot 5 \text{ m.}$$

Hence, the height of the kite from the ground is 82·50 m. **Ans.**

26. There is a small island in the middle of 100 m wide river. There is a tall tree on the island. Points C and D are directly opposite to each other on the two banks and in the line with the tree. If the angles of elevation of the top of the tree at C and D are 30° and 45° respectively, find the height of the tree.

Sol. Let AB be the tree and let AB = h metres.

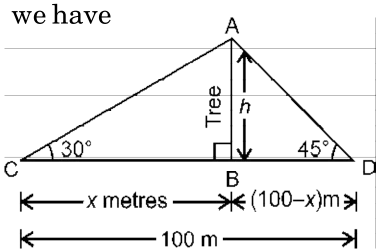
Distance between two points, *i.e.*, CD = 100 metres.

$\angle ACB = 30°$ and $\angle ADB = 45°$.

Let BC = x metres, then

BD = $(100 - x)$ metres.

Now, in right angled triangle ABC, we have



$$\tan 30° = \frac{AB}{BC}$$

or $$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

or $$x = \sqrt{3}h \qquad \text{...(i)}$$

Also, in right angled triangle ABD, we have

$$\tan 45° = \frac{AB}{BD}$$

or $$1 = \frac{h}{100 - x}$$

or $$x = 100 - h. \qquad \text{...(ii)}$$

Now, equating (i) and (ii), we get

$$\sqrt{3}h = 100 - h$$

or $$h(\sqrt{3} + 1) = 100$$

or $$h = \frac{100}{(\sqrt{3} + 1)}$$

or $$h = \frac{100(\sqrt{3} - 1)}{(\sqrt{3} + 1)(\sqrt{3} - 1)}$$

$$= 50(\sqrt{3} - 1)$$
$$= 50(1 \cdot 732 - 1)$$
$$= 50 \times 0 \cdot 732$$
$$= 36 \cdot 6 \text{ m.}$$

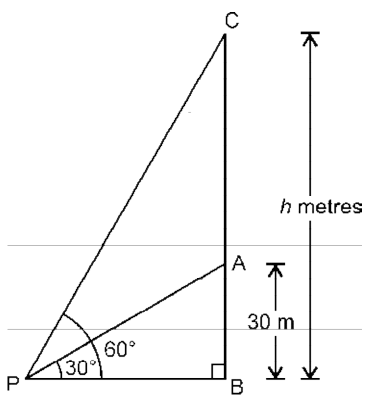
Hence, the height of the tree is 36·6 m. **Ans.**

27. At a point P on the ground, the angles of elevation of the top of a 10 m tall building, and of a helicopter covering some distance over the top of the building, are 30° and 60° respectively. Find the height of the helicopter above the ground.

Sol. Let C be the position of the helicopter and its height from the ground be h metres. AB is the building *i.e.*, AB = 10 m.

$\angle APB = 30°$ and $\angle CPB = 60°$.

Now, in right angled triangle ABP, we have



$$\tan 30° = \frac{AB}{BP}$$

or $$\frac{1}{\sqrt{3}} = \frac{10}{BP}$$

or $$BP = 10\sqrt{3} = 10 \times 1{\cdot}732 = 17{\cdot}32 \text{ m}$$

Also, in right angled triangle CBP, we have

$$\tan 60° = \frac{BC}{BP}$$

or $$\sqrt{3} = \frac{h}{17{\cdot}32}$$

or $$h = 17{\cdot}32 \times \sqrt{3} = 30 \text{ m.}$$

Hence, the height of the helicopter above the ground is 30 m. **Ans.**

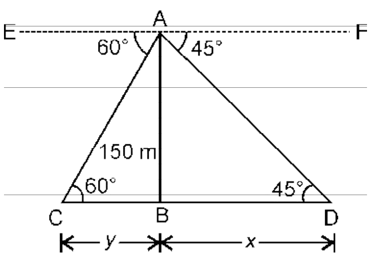
28. Two ships are sailing on opposite sides of a light house 150 m high and in the same line with the foot of the light house. Find the distance between the ships when the angles of depression of the ships are observed from the top of light house are 60° and 45°.

Sol. Let AB be the height of the light house and C, D be the postions of two ships.

Let BD = x metres and BC = y metres.

$\therefore$ $\angle EAC = \angle ACB = 60°$

and $\angle FAD = \angle ADB = 45°$.



Now, in right angled triangle ABD, we have

$$\tan 45° = \frac{AB}{BD}$$

or $$1 = \frac{150}{x}$$

or $x = 150$ m.

Also, in right angled triangle ABC, we have

$$\tan 60° = \frac{AB}{BC}$$

or $$\sqrt{3} = \frac{150}{y}$$

or $$y = \frac{150}{\sqrt{3}} = 50\sqrt{3} \text{ m.}$$

$\therefore$ Distance between two ships $= x + y$

$$= 150 + 50\sqrt{3} = 150 + 86{\cdot}6 = 236{\cdot}6 \text{ m.}$$

Hence, the distance between the two ships is 236·6 m. **Ans.**

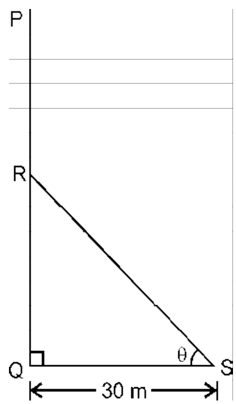
29. The top of a tree, broken by the wind, struck the ground at a distance of 30 m from the foot of the tree. Find the total height of the tree if the angle at which the top struck the ground is (i) 30°, (ii) 45°.

Sol. Let PQ be the tree. It is broken at R and the broken part touches the ground at S, *i.e.*, QS = 30 m. Also, PR = RS.

If the broken part makes angle θ from the ground.

(i) When $\theta = 30°$.

In right angled triangle RQS, we have



$$\tan 30° = \frac{RQ}{QS}$$

or $$\frac{1}{\sqrt{3}} = \frac{RQ}{30}$$

or $$RQ = \frac{30}{\sqrt{3}} = 10\sqrt{3} = 10 \times 1{\cdot}732 = 17{\cdot}32 \text{ m.}$$

Again, in right angled triangle RQS, we have

$$\cos 30° = \frac{QS}{RS}$$

or $\frac{\sqrt{3}}{2} = \frac{30}{RS}$

or $RS = \frac{30 \times 2}{\sqrt{3}} = 20\sqrt{3}$

$= 20 \times 1{\cdot}732$
$= 34{\cdot}64$ m.

$\therefore$ Height of the tree = PQ = PR + RQ
= RS + RQ, [$\because$ PR = RS]
$= 34{\cdot}64 + 17{\cdot}32$
$= 51{\cdot}96$ m. **Ans.**

(ii) When $\theta = 45°$.

In right angled triangle RQS, we have

$\tan 45° = \frac{RQ}{30}$

or RQ = 30 m.

Again, in right angled triangle RQS, we have

$\cos 45° = \frac{QS}{RS}$

or $\frac{1}{\sqrt{2}} = \frac{30}{RS}$

or $RS = 30\sqrt{2}$
$= 30 \times 1{\cdot}414$
$= 42{\cdot}42$ m.

$\therefore$ Height of the tree = PQ = PR + RQ
= RS + RQ [$\because$ PR = RS]
$= 42{\cdot}42 + 30 = 72{\cdot}42$ m.
Ans.

30. A ladder 10 m long reaches a point of a wall which is 10 m below from the top of the wall. The angle of depression of the foot of the ladder as observed from the top of the wall is 60°. Find the height of the wall.

Sol. Let BC be ladder, *i.e.*, BC = 10 m and AC = 10 m.

$\angle XAB = \angle ABD = 60°$.

In ΔBCA, we have

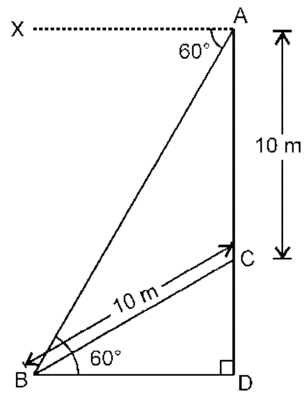
AC = BC

$\therefore$ $\angle CAB = \angle CBA$.

Now, we have $\angle ABD = 60°$ and $\angle ADB = 90°$.

$\therefore$ $\angle DAB = 180° - (60° + 90°)$
$= 30°$.

$\therefore$ $\angle DBC = \angle ABD - \angle CBA$
$= 60° - 30° = 30°$
[$\because$ $\angle CBA = \angle CAB = 30°$]



Now, in right angled triangle CDB, we have

$\sin 30° = \frac{CD}{BC}$

or $\frac{1}{2} = \frac{CD}{10}$

or CD = 5 m

$\therefore$ AD = AC + CD
= 10 + 5 = 15 m.

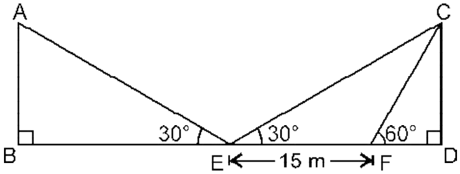
Hence, the height of the wall is 15 m. **Ans.**

31. Two lamp-posts are of equal height. A boy measured the angle of elevation of the top of each lamp-post from the mid-point of the line joining the feet of the lamp-posts as 30°. After walking 15 m towards one of them, he measured the angle of elevation of its top at the point where he stands as 60°. Find the height of each lamp-post and the distance between them.

Sol. Let AB, CD be the two lamp posts and E be the mid-point of line joining (BD) the feet of the lamp-posts.

Then, $\angle AEB = \angle CED = 30°$
$\angle CFD = 60°$ and EF = 15 m.

Now, in right angled triangle CDE, we have



$$\tan 30° = \frac{CD}{ED}$$

or $$\frac{1}{\sqrt{3}} = \frac{CD}{DE + 15}$$

[$\because$ ED = DF + 15]

or $\sqrt{3}$ CD = DF + 15. ...(i)

Again, in right angled triangle CDF, we have

$$\tan 60° = \frac{CD}{DF}$$

$$\sqrt{3} = \frac{CD}{DF}$$

or CD = $\sqrt{3}$ DF. ...(ii)

Substituting the value of CD from (ii) into (i), we get

$$\sqrt{3} \times \sqrt{3}\,DF = DF + 15$$

or 3DF – DF = 15

or $$DF = \frac{15}{2} = 7{\cdot}5 \text{ m}$$

i.e., DE = BE = 15 + 7·5 = 22·5 m.

$\therefore$ Distance between the lamp posts

= DE + EB

= 22·5 + 22·5 = 45 m.

Now, substituting the value of DF in equation (ii), we get

CD = $\sqrt{3} \times 7{\cdot}5$

= $1{\cdot}732 \times 7{\cdot}5 \approx 13$ m.

Hence, the height of each lamp-post is 13 m and the distance between them is 45 m. **Ans.**

❑

12 Area Related to Circles

EXERCISE 12.1

Multiple Choice Type Questions

1. If the perimeter of a semi-circular protractor is 36cm, that its diameter is :

(a) 10 cm (b) 12 cm

(c) 14 cm (d) 15 cm.

Sol. Perimeter of semicircle = 36 cm

$$\pi r + 2r = 36$$

$$\frac{22}{7}r + 2r = 36$$

$$22r + 14r = 36 \times 7$$

$$36r = 252$$

$$r = \frac{252}{36}$$

$$r = 7 \text{ cm.}$$

$$\text{Diameter} = 7 \times 2$$

$$= 14 \text{ cm.}$$ **Ans.**

2. The minute hand of a clock is 21cm long. The distance moved by the tip of the minute hand in 1 hour is :

(a) 21π cm (b) 42π cm

(c) 10.5 π cm (d) 7π cm.

Sol. Minute hand of a clock = 21 cm = r

Distance moved by minute hand in 1 hour

$= 2\pi r = 2 \times \pi \times 21 = 42\pi$ cm.

Short Answer Type Questions

3. If the perimeter of a semi circular protractor of Nitish is 36 cm, find its diameter.

Sol. Perimeter of semi circular protractor = 36 cm

$$\pi r + 2r = 36$$

$$\frac{22}{7}r + 2r = 36$$

$$22r + 14r = 36 \times 7$$

$$36r = 36 \times 7$$

$$r = \frac{36 \times 7}{36}$$

$$r = 7 \text{ cm.}$$

Diameter of protector = 7 × 2 = 14 cm.

4. Find the circumference of Santro's wheel whose diameter is 14 cm.

Sol. Diameter of wheel = 14 cm

Radius of wheel = 14 ÷ 2 = 7 cm

circumference of wheel = $2\pi r$

$$= 2 \times \frac{22}{7} \times 7 = 44 \text{ cm.}$$

5. Ravi has a bicycle whose wheel makes 5000 revolutions in moving 11km. Find the diameter of the wheel.

Sol. Distance covered in 5000 revolution = 11000 m

Distance covered in 1 revolution = (11000/5000) m

$$\therefore \text{Circumference of wheel} = \left(\frac{11}{5}\right) \text{m}$$

$$2\pi r = \frac{11}{5} \text{ m}$$

$$\pi d = 11/5$$

$$\frac{22}{7}d = \frac{11}{5}$$

$$d = \frac{11 \times 7}{5 \times 22}$$

$$= 0.7 \text{ m}$$

$$= 0.7 \times 100 \text{ cm} = 70 \text{ cm.}$$

6. The radius of a wheel of shreya's cycle is 84 cm. How many revolutions will it make to go 52.8 km ?

Sol. Radius of wheel = 84 cm

Distance travelled in one revolution

$$= 2\pi r$$

$$= 2 \times \frac{22}{7} \times 84$$

$$= 528 \text{ cm.}$$

Revolutions for 52.8 km =

$$\frac{52.8 \times 1000 \text{ m} \times 100 \text{cm}}{528}$$

$$= 10{,}000 \text{ revolutions.}$$

7. The diameter of wheel of surjeet's taxi is 42 cm. Find how many complete revolutions must it make to cover 396 metres ?

Sol. Diameter of wheel = 42 cm

Radius of wheel = $42 \div 2$

= 21 cm

Distance covered in one revolution

$= 2\pi r$

$= 2 \times \frac{22}{7} \times 21$

= 132 cm

Revolutions for 396 metres

$= \frac{396 \times 100 \text{cm}}{132}$

= 300 revolutions.

8. Sita is driving a cart. A wheel of the cart is making 6 revolutions per second. If the diameter of the wheel is 42cm, Find the speed of the cart.

Sol. Diameter of the wheel = 42 cm

Radius of wheel = $42 \div 2$

= 21 cm.

∴The distance travelled in one revolution

$= 2\pi r$

$= 2 \times \frac{22}{7} \times 21$

= 132 cm.

∴Distance travelled in 6 revolutions

$= 6 \times 132$

= 792 cm.

∴The distance travelled in one second = 792 cm

Speed in 1 second = 792 cm/sec.

9. A wire made of silver in the form of nose ring of radius 42 mm. It is bent into a square. Determine the side of the square.

Sol. Radius of ring = 42 mm

Perimeter of ring = $2\pi r$

$= 2 \times \frac{22}{7} \times 42$

= 264 mm.

Perimeter of square = 264 mm

side × 4 = 264

side $= \frac{264}{4}$

= 66 mm.

10. Naveen made a bangle of silver wire, of diameter 35 mm. It is rebent into a square form. Determine the length of the side of the square.

Sol. Diameter of bangle = 35 mm

Radius of bangle = $35 \div 2$

= 17.5 mm

Perimeter of bangle = $2\pi r$

$= 2 \times \frac{22}{7} \times 17.5$

= 110 mm.

Perimeter of square = side × 4

110 = side × 4

side $= \frac{110}{4}$

= 27.5 mm.

11. The circumference of the chakra of National flag exceeds its diameter by 16.8 cm. Find the radius of the chakra.

Sol. Circumference of wheel

= 16.8 + diameter

$2\pi r = 16.8 + 2r$

$2\pi r - 2r = 16.8$

$2 \times \frac{22}{7} \times r - 2r = 16.8$

$44r - 14r = 16.8 \times 7$

$30r = 117.6$

$r = \frac{117.6}{30}$

∴ Radius of chakra = 3.92 cm.

12. A bucket full of water is raised from a well by means of a rope which is wound round a pulley of diameter 77 cm. Bucket takes 1 minute 28 seconds to ascend at a uniform speed of 1.1 m/sec. Find the number of revolutions which the pulley makes in raising the bucket.

Sol. **Given :** Diameter = 77cm

Radius $= \frac{77}{2} \times \frac{1}{100}$

= 0.385 m.

Speed = 1.1 m/s

Time = 1m 28 sec = 88 sec

Distance = speed × time

$= 1.1 \times 88$

= 96.8 m.

Number of revolution

$$= \frac{\text{distance}}{\text{circumference}}$$

$$= \frac{96.8 \times 7}{2 \times 22 \times 0.385}$$

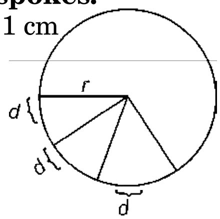
$$= 40.$$

Therefore, the wheel will make 40 revolution.

13. Manoj has a motor cycle with wheels of diameter 91cm. There are 22 spokes in the wheel. Find the length of the arc between two adjoining spokes.

Sol. $2r$ = diameter = 91 cm

$$r = \frac{91}{2}$$



All spokes are same distance to each other

wheel circumference $= 2\pi r$

$= \pi(2r)$

$= \pi \times 91$

$= 91\pi$

length between two adjoining spokes $= d = \frac{91\pi}{22}$

$$= \frac{91 \times 22}{22 \times 7}$$

= 13 cm.

14. Aman has tractor. The sum of radii of two wheels of the tractor is 98cm and the difference of their circumference is 176 cm. Find the diameter of the wheels.

Sol. Let radii are r_1 & r_2

So, $r_1 + r_2 = 98$ ...(1)

and,

$$2\pi r_1 - 2\pi r_2 = 176$$

$$2\pi(r_1 - r_2) = 176$$

$$\therefore \quad r_1 - r_2 = \frac{176}{2\pi}$$

$$r_1 - r_2 = \frac{176 \times 7}{2 \times 22}$$

$= 28$ ...(2)

on adding equation (1) & (2), we get :

$2r_1 = 126$, $r_1 = 63$.

on substituting r_1 in equation (1) we get

$63 + r_2 = 98$; $r_2 = 35$

Hence, diameter of wheels

$r_1 \times 2 = 63 \times 2$

= 126 cm.

$r_2 \times 2 = 35 \times 2$

= 70 cm.

EXERCISE 12.2

Multiple Choice Type Questions

1. The circumference of the circle is 44cm. Then the area of circle is :

(a) 276 cm² **(b) 44 cm²**

(c) 176 cm² **(d) 154 cm².**

Sol. Circumference of circle = 44cm

$$2\pi r = 44$$

$$r = \frac{44 \times 7}{2 \times 22}$$

= 7 cm

Area of circle $= \pi r^2$

$$= \frac{22}{7} \times 7 \times 7$$

$= 154$ cm².

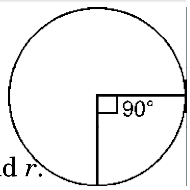
2. Area of a quadrant of circle whose circumference is 22cm is : (π = 22/7)

(a) 3.5 cm² **(b) 3.5 cm**

(c) 9.625 cm² **(d) 17.25 cm².**

Sol. Area of quadrant.

$$= \frac{1}{4} \times \text{area of circle}$$



$$= \frac{1}{4} \times (\pi r^2)$$

now, we need to find r.

It is given that

circumference $= 2\pi r$

$22 = 2\pi r$

$$r = \frac{22 \times 7}{2 \times 22}$$

= 3.5 cm

now, area of quadrant $= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5$

= 9.625 cm².

Short Answer Type Questions

3. Ram Lakhan has a field in the form of circle. The cost of ploughing the field at the rate

of ₹ 2.24 per m² is ₹ 45,056. Find the cost of fencing the field at the rate of ₹ 12.60 per metre.

Sol. Cost of ploughing = ₹ 2.24 per m²

Total cost = ₹ 45,056

$$\therefore \quad \text{Total area} = \frac{45,056}{2.24}$$

Area of circle = πr^2

$$= \frac{45,056}{2.24}$$

$$r^2 = \frac{45,056}{2.24} \times \frac{7}{22}$$

$$r^2 = 6,400$$

$$r = \sqrt{6,400}$$

$$= 80.$$

Perimeter of circle = $2\pi r$

$$= 2 \times \frac{22}{7} \times 80$$

cost of fencing field = 12.60 per metre

$$\text{Total cost} = 2 \times \frac{22}{7} \times 80 \times 12.60$$

= ₹ 6,336. **Ans.**

4. If the perimeter is half of the area of the circle numerically then find the radius of the circle.

Sol. Area of circle = πr^2

$$\text{Perimeter of circle} = \frac{\pi r^2}{2}$$

$$2\pi r = \frac{\pi r^2}{2}$$

$$r = 2 \times 2$$

= 4. **Ans.**

5. Find the area of a circular park of Janakpuri whose circumference is 77 m.

Sol. Circumference of park = 77 m

$$2\pi r = 77$$

$$r = \frac{77 \times 7}{22 \times 2}$$

= 12.25 m.

Area of part = πr^2

$$= \frac{22}{7} \times 12.25 \times 12.25$$

= 471.625 m². **Ans**

6. Find the area of a circular park whose radius is 4.5 m.

Sol. Area of circular park = πr^2

$$= \frac{22}{7} \times 4.5 \times 4.5$$

= 63.64 m². **Ans.**

7. There are four circular windows in Nisha's house, each of radius 28cm are to be fitted with glass. Calculate the cost of glass at the rate of ₹ 4.25 per sq.m.

Sol. Radius of each glass = 28cm

$$= \frac{28}{100}$$

= 0.28m

Area of each glass = πr^2

$$= \frac{22}{7} \times 0.28 \times 0.28$$

= 0.2464 m².

cost of each glass = ₹ 4.25 per m²

total cost of each glass = 4.25 × 0.2464

= ₹ 1.0472.

total cost of 4 glasses = 1.0472 × 4

= ₹ 4.19 approx.

8. Find the three places of decimals the radius of the circle whose area is the sum of the areas of the two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimeters.

Sol. For the first triangle, we have

$a = 35$, $b = 53$ and $c = 66$

$$\therefore \quad s = \frac{a+b+c}{2}$$

$$= \frac{35+53+66}{2}$$

= 77cm.

now,

Δ_1 = Area of the first triangle

$$\Rightarrow \Delta_1 = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\Rightarrow \Delta_1 = \sqrt{77(77-35)(77-53)(77-66)}$$

$$= \sqrt{77 \times 42 \times 24 \times 11}$$

$$\Rightarrow \Delta_1 = \sqrt{7 \times 11 \times 7 \times 6 \times 6 \times 4 \times 11}$$

$$= \sqrt{7^2 \times 11^2 \times 6^2 \times 2^2}$$

$\Rightarrow \Delta_1 = 7 \times 11 \times 6 \times 2$

$= 924$

For the second triangle we have

$a = 33, b = 56, c = 65$

$$s = \frac{a+b+c}{2}$$

$$= \frac{33+56+65}{2}$$

$= 77$cm.

Δ_2 = Area of the second triangle

$\Rightarrow \Delta_2 = \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \Delta_2 = \sqrt{77(77-33)(77-56)(77-65)}$

$\Rightarrow \Delta_2 = \sqrt{77 \times 44 \times 21 \times 12}$

$= \sqrt{7 \times 11 \times 4 \times 11 \times 3 \times 7 \times 3 \times 4}$

$\Rightarrow \Delta_2 = \sqrt{7^2 \times 11^2 \times 4^2 \times 3^2}$

$= 7 \times 11 \times 4 \times 3$

$= 924$ cm^2.

Let r be the radius of the circle

Then, Area of the circle = sum of the area of 2 triangles

$\Rightarrow \quad \pi r^2 = \Delta_1 + \Delta_2$

$\Rightarrow \quad \pi r^2 = 924 + 924$

$\Rightarrow \quad \frac{22}{7} \times r^2 = 1848$

$\Rightarrow \quad r^2 = \frac{1848 \times 7}{22}$

$\Rightarrow \quad r^2 = 3 \times 4 \times 7 \times 7$

$\Rightarrow \quad r = \sqrt{3 \times 2^2 \times 7^2}$

$\Rightarrow \quad r = 2 \times 7 \times \sqrt{3}$

$\Rightarrow \quad r = 14\sqrt{3}$

$\Rightarrow \quad r = 14 \times 1{\cdot}732$

$\Rightarrow \quad r = 24.25$ cm. **Ans.**

9. The radius of a circular pond is 14m. Find the radius of another circular pond whose area is one fourth of the area of this circular pond.

Sol. Radius of circular pond = 14 cm

Area of circular pond = πr^2

$$= \frac{22}{7} \times 14 \times 14$$

$= 616$.

Area of second circular pond = $616 \times \frac{1}{4}$

$= 154$.

$\pi r^2 = 154 \Rightarrow r^2 = \frac{154 \times 7}{22}$

$= r^2 = 49$

$= r = 7$ cm. **Ans.**

10. A gold smith, Vishal bent a wire made of gold in the form of square encloses an area of 196 cm². The same wire he bent in the form of a circle. Find the area of the circle.

Sol. Area of square $= 196$ cm^2

(side)$^2 = 196$

side $= \sqrt{196}$

$= 14$ cm.

Perimeter of square = side × 4

$= 14 \times 4$

$= 56$ cm.

$\therefore$ Perimeter of circle = 56 cm

$2\pi r = 56$

$$r = \frac{56 \times 7}{22 \times 2}$$

$= 8.91$ cm.

Area of circle = πr^2

$$= \frac{22}{7} \times 8.91 \times 8.91$$

$= 249.45$ cm^2.

11. The difference between circumference and diameter of a circular plot is 52.5 m. Find the area of the circular plot.

Sol. Difference between circumference and diameter = 52.5

$2\pi r - d = 52.5$

$2\pi r - 2r = 52.5$

$2r(\pi - 1) = 52.5$

$$2r\left(\frac{22}{7} - 1\right) = 52.5$$

$$2r \times \frac{15}{7} = 52.5$$

$$r = \frac{52.5 \times 7}{15 \times 2}$$

$= 12.25$.

Area of circular plot = πr^2

$$= \frac{22}{7} \times 12.25 \times 12.25$$

$= 471.625$ cm^2. **Ans.**

EXERCISE 12.3

1. A pendulum swings through an angle of 30° and describes an arc 8.8 cm in length. Find the length of the pendulum.

Sol. Length of arc = 8.8 cm

$$2\pi r \times \frac{30}{360} = 8.8$$

$$2 \times \frac{22}{7} \times r \times \frac{30}{360} = 8{\cdot}8$$

$$r = \frac{8.8 \times 360 \times 7}{30 \times 22 \times 2}$$

$= 16.8$ cm. **Ans.**

2. The short and long hand of a clock are 4 cm and 6 cm respectively. Find the sum of distances travelled by their tips in one day.

Sol. The length of the short hand and long hand are 4 cm and 6 cm respectively. Short hand is hour hand. It completes one revolution in 12 hours.

Distance travelled by short hand of the clock in one revolution will be equal to the circumference of a circle with radius 4 cm.

Distance travelled by short hand in one revolution (in 12 hours) is $2 \times \pi \times 4 = 8\pi$. Distance travelled by short hand in 24 hours

$= 2 \times 8 \times \pi$

$= 16\pi$.

Long hand is minute hand. It completes one revolution 1 hour.

Distance travelled by the long hand of the clock in one revolution will be equal to the circumference of a circle with radius 6cm. Distance travelled by long hand in one revolution (in 1 hour) is $= 2 \times \pi \times 6$

$= 12\pi$.

Distance travelled by long hand in 24 hours

$= 24 \times 12 \times \pi$

$= 288\pi$.

So, the sum of the distance travelled by their tips in one day $= 288\pi + 16\pi = 304\pi$.

Distance $= 304\pi$ or 954.56 cm.

3. The minute hand of a clock is 10cm long. Find the area swept by the minute hand between 8.00 A.M. to 8.25 A.M.

Sol. Angle described by the minute hand in 60 min = 360°.

Angle described by the minute hand in 25 min.

$$= \frac{360}{60} \times 25$$

$= 150$

Therefore,

$\theta = 150°$ and radius = 10 cm

Area swept by the minute hand in 25 minutes

$$= \left(\frac{\pi r^2 \theta}{360}\right)$$

$$= \frac{22}{7} \times 10 \times 10 \times \frac{150}{360}$$

$= 130{\cdot}95$ cm^2. **Ans.**

4. The minute-hand of a clock is 15cm long. Calculate the area swept by it in 20 minutes. Take $\pi = 3.14$.

Sol. Angle described by the minute hand in 60 min = 360°

Angle described by the minute hand in 20 minutes.

$$= \frac{360}{60} \times 20$$

$= 120°$.

Therefore,

$\theta = 120°$ and radius = 15 cm

Area swept by the minute hand in 20 minutes

$$= \left(\frac{\pi r^2 \theta}{360°}\right)$$

$$= 3.14 \times 15 \times 15 \times \frac{120}{360}$$

$= 235.5$ cm^2. **Ans.**

5. A sector is cut from a circle of radius 42 cm. The central angle of the sector is 150°. Find the length of the arc and area of the sector.

Sol. Length of arc of sector of angle θ and radius r

$$l = \frac{\theta^\circ}{180^\circ} \times \pi r$$

$$= \frac{150^\circ}{180^\circ} \times \frac{22}{7} \times 42$$

$$= 110 \text{ cm.}$$

$$\text{Area of sector} = A = \frac{\theta^\circ}{360^\circ} \times \pi r^2$$

$$A = \frac{150^\circ}{360^\circ} \times \frac{22}{7} \times 42 \times 42$$

$$= 2310 \text{cm}^2.$$

6. A circular disc. of radius 6cm is divided into three sectors with central angles 90°, 120° and 150°. What part of the whole circle is the sector with central angle 150° ? Also, calculate the ratio of the areas of the three sectors.

Sol. Given : θ = 150°, radius = 6cm

The part of the whole circle is with the central angle $150^\circ = \frac{150}{360}$

$$= \frac{5}{12}$$

$$\text{Areas} = \frac{\theta}{360^\circ} \times \pi r^2$$

$$90^\circ = \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times 6 \times 6$$

$$= \frac{198^\circ}{7}$$

$$120^\circ = \frac{120^\circ}{360^\circ} \times \frac{22}{7} \times 6 \times 6$$

$$= \frac{264}{7}$$

$$150^\circ = \frac{150^\circ}{360^\circ} \times \frac{22}{7} \times 6 \times 6$$

$$= \frac{330}{7}$$

$$\text{Ratio of areas} = \frac{198}{7} : \frac{264}{7} : \frac{330}{7}$$

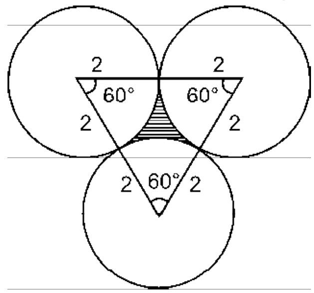
$$= 3 : 4 : 5$$

7. Three equal circles each of radius 2 cm touch one another. Find the area enclosed in between them.

Sol. Required area =

Area of triangle – 3(area of sector)

Area of equilateral triangle



$$= \frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times (2 + 2)^2$$

$$= 4\sqrt{3}.$$

$$\text{Area of sector} = \frac{\theta^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{60}{360} \times \frac{22}{7} \times 2 \times 2$$

$$= \frac{44}{21}$$

$$\text{Now, required area} = 4\sqrt{3} - 3\left(\frac{44}{21}\right)$$

$$= 4 \times 1{\cdot}732 - 6{\cdot}28$$

$$= 0.643 \text{ cm}^2.$$

8. The radius of a circle is 28cm and the area of the sector is filled with rain water is 205.4 cm². Find the central angle.

Sol. Given : Radius of circle = 28 cm

Area of sector is filled with rain water =

$$\text{Area of sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

$$205.4 = \frac{\theta}{360^\circ} \times \frac{22}{7} \times 28 \times 28$$

$$\theta = 30^\circ \text{ (approx)}$$

9. The perimeter of a sector of a circle of radius 11.2 cm and 54.4cm. Find the area of sector.

Sol. Given : Radius of circle = 11.2cm

Perimeter of sector = 54.4 cm

Perimeter of sector = $2r + \frac{\pi r\theta}{180°}$

$$54.4 = 2 \times 11.12 + \frac{\frac{22}{7}\times 11.2\times\theta}{180°}$$

$$54.4 - (2 \times 11.2) = \frac{35.2\times\theta}{180}$$

$$\frac{180(54.4-22.4)}{35.2} = \theta$$

$$= \frac{9792-4032}{35.2} = \theta$$

$$= \frac{5760}{35{\cdot}2} = \theta$$

$$\theta = 163.64.$$

$$\text{Area of sector} = \frac{\theta}{360}\times\pi r^2$$

$$= \frac{163.64}{360}\times\frac{22}{7}\times 11.2 \times 11.2$$

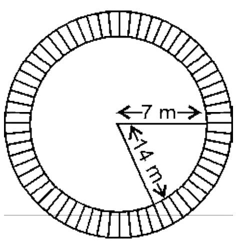
$$= \frac{451594{\cdot}03}{2520}$$

$= 179.2$ cm^2. **Ans.**

10. Flowers are to be planted in the shaded portion which is shown by sectors of two concentric circles of radii 14 m and 7m. Find the area of the shaded region.

Sol. Area of shaded portion

= area of C_1 – area of C_2



$$= \pi r_1^2 - \pi r_2^2$$

$$= \frac{22}{7}\times 14\times 14 - \frac{22}{7}\times 7\times 7$$

$$= \frac{22}{7}[14^2 - 7^2]$$

$$= \frac{22}{7}[196-49]$$

$$= \frac{22}{7}[147]$$

$$= \frac{22}{7}[21]$$

$= 462$ m^2. **Ans.**

11. The radius of a circle is 17.5cm. Find the area of the sector enclosed by two radii and an arc 44cm in length.

Sol. Given : Radius of circle (r) = 17.5cm

length of arc (l) = 44cm

Area of sector = $\frac{1}{2} \times lr$

$= \frac{1}{2} \times 44 \times$

17.5 = 385 cm^2. **Ans.**

12. The perimeter of a certain sector of a circle of radius 5.6 m is 27.2 m. Find the area of the sector.

Sol. Given : Radius of circle (r) = 5.6 m

Perimeter of sector of circle = 27.2 m

To find : Area of sector

Perimeter = $2r + \frac{\pi r\theta}{180°}$

$$27.2 = 2 \times 5.6 + \frac{\frac{22}{7}\times 5.6\times\theta}{180}$$

$$27.2 = 11.2 + \frac{17.6}{180}\theta$$

$$(27.2-11.2)\frac{180}{17.6} = \theta$$

$$\frac{16\times 180}{17.6} = \theta$$

$$\theta = 163.64.$$

$$\text{Area of a sector} = \frac{\theta}{360°}\times\pi r^2$$

$$= \frac{163.64}{360}\times\frac{22}{7}\times 5.6 \times 5.6$$

$= 44.8$ m^2. **Ans.**

13. The perimeter of a certain sector of a circle of radius 6.5cm is 31cm. Find the area of the sector.

Sol. Given : Radius of circle $(r) = 6.5$cm

Perimeter of circle = 31cm

To find : Area of sector

Perimeter of sector $= 2r + \frac{\pi r\theta}{180}$

$$31 = \frac{2\times 6.5 + \frac{22}{7}\times 6.5\times\theta}{180}$$

$$31 = \frac{13 + \frac{22}{7}\times 6.5\times\theta}{180}$$

$$31 = \frac{13 + 20.430}{180}$$

$$\frac{(31-13)180}{20{\cdot}43} = \theta$$

$$\frac{3240}{20{\cdot}43} = \theta$$

$$\theta = 158.60°$$

$$\text{Area of sector} = \frac{\theta}{360°}\times\pi r^2$$

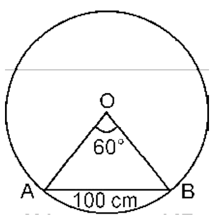
$$= \frac{158.60}{360}\times\frac{22}{7}\times 6.5\times 6.5$$

$= 58.50$ unit. **Ans.**

EXERCISE 12.4

1. A chord of a circle subtends an angle of 60° at the centre. If the length of the chord is 100cm, find the area of the major segment.

Sol. OA = OB = r



$\Rightarrow$ $\angle OAB = \angle OBA$

also, $\angle OAB + \angle OBA + \angle AOB = 180°$

$\Rightarrow$ $2\angle OAB = 180° - 60°$

$= 120°$

$\Rightarrow$ $\angle OAB = 60° = \angle OBA$

$\therefore \Delta OAB$ is equilateral

$\therefore$ OA = OB = AB

Area (minor segment) = area (sector) – area (ΔOAB)

$$= \frac{\pi r^2\theta}{360°} - \frac{\sqrt{3}a^2}{4}$$

$$= \frac{1}{4}\left(\frac{3.14\times 100\times 100\times 60}{90} - \sqrt{3}\times 100\times 100\right)$$

$$= \frac{1}{4}\left[\frac{62800 - 30000\times 1.73}{3}\right]$$

$$= \frac{1}{12}\times 62800 - 51900$$

$$= \frac{10900}{12}$$

$= 908.33\text{cm}^2$

Area (major segment) = area (circle) – area (minor segment)

$= \pi r^2 - 908.33$

$= 3.14 \times 100 \times 100 - 908.33$

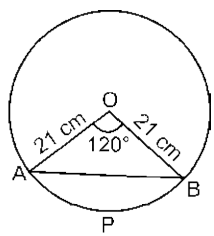
$= 30491.7 \text{ cm}^2$.

2. A chord of a cincle of radius 21cm makes an angle 120° at the centre of the circle. Find the area of the segment so formed.

Sol. In a given circle

radius (r) = 21cm

θ = 120°



Area of segment APB = Area of sector OAPB – Area of ΔOAB

$$\text{Area of sector OAPB} = \frac{\theta}{360°}\times\pi r^2$$

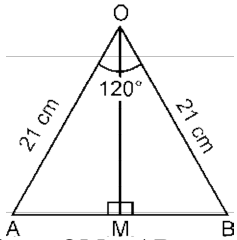
$$= \frac{120°}{360°}\times\frac{22}{7}\times 21\times 21$$

$$= \frac{1}{3}\times 22\times 3\times 21$$

$$= \frac{1386}{3}$$

$= 462 \text{ cm}^2$.

Area of $\Delta OAB = \frac{1}{2}\times \text{base}\times\text{height}$



We draw $OM \perp AB$

$\therefore \quad \angle OMB = \angle OMA = 90°$

In ΔOMA & ΔOMB

$\angle OMA = \angle OMB$ (both 90°)

$OA = OB$ (both radius)

$OM = OM$ (common)

$\therefore \quad \Delta OMA \cong \Delta OMB$ (By R.H.S. Congruency)

$\Rightarrow \quad \angle AOM = \angle BOM$ (CPCT)

$\therefore \quad \angle AOM = \angle BOM = \frac{1}{2} \angle BOA$

Also, since $\Delta OMB \cong \Delta OMA$

$\therefore \quad BM = AM$ (CPCT)

$\Rightarrow \quad BM = AM = \frac{1}{2} AB$ ...(1)

In right angle ΔOMA

$$\sin \theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\sin 60° = \frac{AM}{AO}$$

$$\frac{\sqrt{3}}{2} = \frac{AM}{21}$$

$$AM = \frac{\sqrt{3}}{2} \times 21$$

In right triangle ΔOMA

$$\cos \theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\cos 60° = \frac{OM}{AO}$$

$$\frac{1}{2} = \frac{OM}{21}$$

$$OM = \frac{1}{2} \times 21$$

from (1)

$$AM = \frac{1}{2} AB$$

$$2AM = AB$$

$$AB = 2AM$$

putting value of AM

$$AB = 2 \times \frac{\sqrt{3}}{2} \times 21$$

$$AB = \sqrt{3} \times 21$$

$$AB = 21\sqrt{3}$$

Now, Area of $\Delta AOB = \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2}$

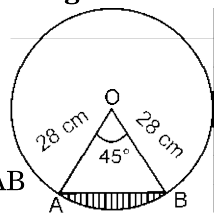
$= 190.96.$

Area of segment $= 462 - 190.96$

$= 271.04 \text{ cm}^2$. **Ans.**

3. A chord of a circle of radius 28cm subtends an angle 45° at the centre of the circle. Find the area of the minor segment.

Sol. Let AB be the chord and O is the centre of the circle



Now,

Area of sector OAB

$$= \frac{\theta}{360°} \times \pi r^2$$

$$= \frac{45°}{360°} \times \frac{22}{7} \times 28 \times 28$$

$$= \frac{1}{8} \times 22 \times 4 \times 28$$

$$= \frac{1}{2} \times 22 \times 28$$

$$= 1 \times 11 \times 28$$

$$= 308 \text{ cm}^2.$$

Again,

Area of $\Delta OAB = \frac{1}{2} \times r^2 \times \sin 45°$

$$= \frac{1}{2} \times 28 \times 28 \times \frac{1}{\sqrt{2}}$$

$$= 277.22 \text{ cm}^2$$

Now area of minor segment

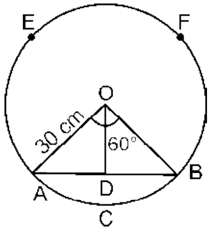
= Area of sector OAB – Area of triangle OAB

$= 308 - 277.22$

$= 30.78 \text{ cm}^2.$

4. A chord of a circle of radius 30cm makes an angle of 60° at the centre of the circle. Find the area of theminor and major segments.

Sol. Since, chord AB subtends an angle 60° to the centre.



$\therefore \angle AOB = 60°$

Draw OD ⊥ AB and its bisect

$\because \Delta OAD \cong \Delta ODB$

$\therefore \angle AOD = \angle BOD = 30°$

In right ΔAOD,

$$\sin 30° = \frac{AD}{AO}$$

$$\frac{1}{2} = \frac{AD}{30}$$

$$AD = 15 \text{ cm}$$

and, $$\cos 30 = \frac{OD}{AO}$$

$$\frac{\sqrt{3}}{2} = \frac{OD}{30}$$

$$OD = 15\sqrt{3} \text{ cm.}$$

$$AB = 2\,AD = 2 \times 15 = 30 \text{ cm.}$$

$$\therefore \text{Area of } \Delta AOB = \frac{1}{2} \times AB \times OD$$

$$= \frac{1}{2} \times 30 \times 15\sqrt{3}$$

$$= (15)^2\sqrt{3} = 389.7$$

Now, Area of sector

$$OACBO = \frac{\theta}{360}\pi r^2$$

$$= 60 \times \frac{3.14}{360} \times (30)^2$$

$$= 471 \text{ cm}$$

∴ Area of minor segment = Area of sector OACBO – Area of ΔOAB

$= 471 - 389.7 = 81.3 \text{ cm}^2$

Now, Area of major segment = Area of circle – Area of minor segment

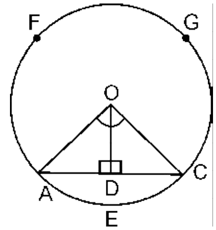
$= \pi r^2$ – Area of minor segment

$= 3.14 \times 30 \times 30 - 81.3$

$= 2744.7 \text{ cm}^2$. **Ans.**

5. A chord of a circle of radius 14cm makes a right angle at the centre. Find the areas of the minor and major segment of the circle.

Sol. Given, radius of circle = 14 cm.



A perpendicular is drawn from centre of circle to the chord of the circle, which bisect the chord.

∴ AD = DC

Also, $\angle AOD = \angle COD = 45°$

$\angle AOC = \angle AOD + \angle COD$

$= 45° + 45° = 90°.$

In right ΔAOD,

$$\sin 45° = \frac{AD}{AO}$$

$$\frac{1}{\sqrt{2}} = \frac{AD}{14}$$

$$AD = 7\sqrt{2} \text{ cm}$$

and $$\cos 45° = \frac{OD}{AO}$$

$$\frac{1}{\sqrt{2}} = \frac{OD}{14}$$

$$OD = 7\sqrt{2} \text{ cm}$$

Now, $$AC = 2AD \Rightarrow 2 \times 7\sqrt{2}$$

$$= 14\sqrt{2} \text{ cm}$$

$$\text{Now, Area of } \Delta AOC = \frac{1}{2} \times AC \times OD$$

$$= \frac{1}{2} \times 14\sqrt{2} \times 7\sqrt{2}$$

$$= 98 \text{ cm}^2$$

$$\text{Now, Area of sector OAEC} = \frac{\theta \pi r^2}{360°}$$

$$\Rightarrow \frac{90}{360} \times 3.14 \times (14)^2$$

$$\Rightarrow 153.86 \text{ cm}^2.$$

∴ Area of minor segment AEC = Area of sector OAEC – Area of ΔAOC

$= 153.86 - 98 \Rightarrow 55.86 \text{ cm}^2/56\text{cm}^2$ (Approx)

∴ Area of major segment OAFGCO = Area of circle – Area of minor segment

$= \pi r^2$ – Area of minor segment

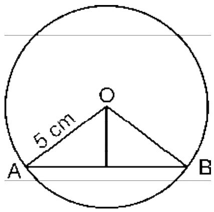
$= 3.14 \times 14 \times 14 - 55.86$

560 cm² (Approx) **Ans.**

6. The radius of a circle is 5cm. A chord of length $\sqrt{50}$ cm is drawn in the circle. Find the area of the major segment.

Sol. Here, OA = OB = 5 cm

$$AB = \sqrt{50}$$



$$AC = BC = \frac{\sqrt{50}}{2}$$

In ΔOCA

$$\sin(\angle AOC) = \frac{AC}{OA} = \frac{\sqrt{\frac{50}{2}}}{5} = \frac{1}{2}$$

$$\sin \angle AOC = 45°$$

$$\angle AOC = 45°$$

similarly,

$$\angle BOC = 45°$$

$$\angle AOB = \angle AOC + \angle BOC$$

$$= 45° + 45° = 90°$$

Area of sector of circle = $\frac{90}{360} \times \frac{22}{7} \times 5 \times 5 = \frac{275}{14}$ cm^2

Area of $\Delta AOB = \frac{1}{2} \times 5 \times 5 = \frac{25}{2}$ cm^2

area of minor segment

= area of sector – area of ΔAOB

$$= \frac{275}{14} - \frac{25}{2}$$

$$= \frac{275-175}{14}$$

$$= \frac{100}{14}$$

$$= \frac{50}{7} \text{ cm}^2.$$

Therefore,

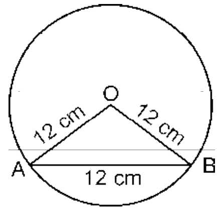
Area of major segment

= Area of circle – Area of minor segment

$$= \left(\frac{22}{7} \times 5 \times 5\right) - \frac{50}{7}$$

$= 71.428$ cm^2 **Ans.**

7. Find the lengths of the arcs cut off from a circle of radius 12 cm by a chord 12 cm long. Also find the area of the minor segment.



Sol. Here, OA = OB = 12 cm (radius)

AB= 12 cm (chord)

Hence, OAB is an equilateral triangle

$\therefore \quad \theta = 60°$

length of minor arc = $\frac{\theta}{360°} \times 2\pi r$

$$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 12$$

$= 12.56$ cm.

length of major arc = perimeter – length of minor arc

$= 2\pi r - 12.56$

$$= \left[2 \times \frac{22}{7} \times 12\right] - 12.56$$

$= 75.42 - 12.56$

$= 62.86$ m

area of minor segment

$$= \left[\frac{60}{360} \times \frac{22}{7} \times 12 \times 12\right] - \left[\frac{1}{2} \times 12 \times 12 \times \sin 60°\right]$$

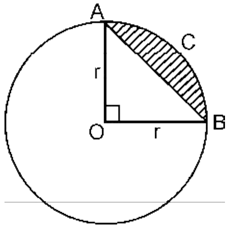
$= 75.42 - 62.352$

$= 13.068$

$= 13$ cm^2. **Ans.**

8. The perimeter of a sector of a circle with central angle 90° is 25cm. Find the area of the minor segment of the circle.

Sol. Let be the radius of a sector of a circle with centre at O.



Length of the arc ACB

$$= \frac{\theta}{360^\circ} \times 2\pi r$$

$$= \frac{90^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times r$$

$$= \frac{11}{7} r \text{ cm}$$

Perimeter of the sector AOBCA = 25cm

$$r + r + l = 25$$
$$2r + l = 25$$
$$2r + \frac{11}{7} r = 25$$
$$25r = 25 \times 7$$
$$r = 7\text{cm}$$

Area of minor segment ABCA = Area of sector AOBCA – area of ΔAOB

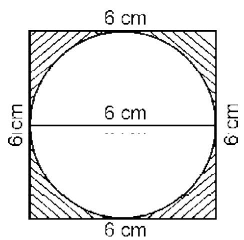
$$= \frac{\pi r^2}{4} - \frac{1}{2} \times 7 \times 7$$

$$= \frac{22}{7} \times \frac{7 \times 7}{4} - \frac{1}{2} \times 7 \times 7$$

$= 14 \text{ cm}^2$. **Ans.**

9. A circle is inscribed in a square of sides 6cm. Find the area between the square and the circle.

Sol.



Area between the square and the circle = area of square – area of circle

area of square = $(\text{side})^2 = (6)^2$

$= 6 \times 6$

$= 36\text{cm}^2$

area of circle $= \pi r^2$

∵ Circle inscribed in a circle

∴ Side of square = diameters of circle = 6 cm

∴ Radius of circle = 6 ÷ 2

= 3cm

$$\text{Area} = \frac{22}{7} \times 3 \times 3$$

$$= \frac{198}{7}$$

area between square and circle

$$= 36 - \frac{198}{7}$$

$$= \frac{54}{7}$$

$= 7\frac{5}{7}$ cm². **Ans.**

10. A rectangle whose sides 4cm and 3 cm is inscribed in a circle. Find the area enclosed between the circle and the rectangle.

Sol. Area of rectangle = length × width

$= 4 \times 3$

$= 12\text{cm}^2$

Area of circle $= \pi r^2$

Diameter $= \sqrt{4^2 + 3^2}$

$= 5$

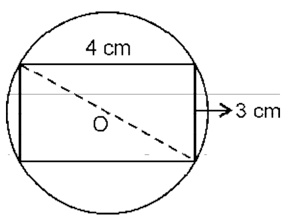
Area of circle $= \frac{22}{7} \times 2.5 \times 2.5$

= 19.64.

Area between rectangle 4 circle

= 19.64 – 12

= 7.64 cm²



EXERCISE 12.5

1. The circumference of a circular park is 314m. A 20m wide concrete track is made around it. Calculate the cost of laying turf in the park at ₹ 1.25 per sq. m and the rate of concreting track is ₹ 50 per sq. m.

Sol. Given :

Circumference of circular park = 314 m

$2\pi r = 314$

$2 \times 3.14 \times r = 314$

$$r = \frac{314}{2 \times 3.14}$$

$r = 50$ m.

$\therefore$ Outer R = 50 + 20 = 70 m

Cost of laying turf = Area of circle × cost

$= \pi r^2 \times 1.25$

$= 3.14 \times 50 \times 50 \times 1.25$

$= 9812.5$

Cost of concrete track = area of track × cost

$= [\pi(R^2 - r^2)] \times 50$

$= [3.14(70^2 - 50^2)] \times 50$

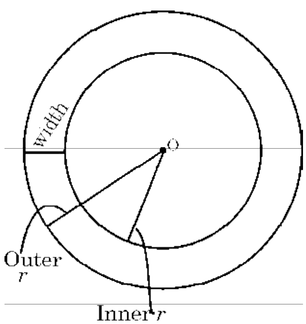
$= [3.14(70-50)(70+50)] \times 50$

$= 3.14 \times 20 \times 120 \times 50$

= ₹ 3,76,800. **Ans.**

2. A race track is in the form of a ring whose inner and outer circumference are 437 m and 503 m respectively. Find the width of the track and also its area.

Sol. Given : Inner circumference of track = 437m



Outer circumference of track = 503 m

$\because$ Circumference of circle = $2\pi r$

$$\therefore \quad r = \frac{\text{circumference}}{2\pi} \text{ of circle}$$

$$\text{inner } r = \frac{437}{2\pi}, \text{ outer } r = \frac{503}{2\pi}$$

width of track = outer r – inner r

$$= \frac{503}{2\pi} - \frac{437}{2\pi}$$

$$= \frac{503 - 437}{2\pi}$$

$$= \frac{66}{2\pi}$$

$$= \frac{66 \times 7}{2 \times 22}$$

$= 10.5$ m^2.

Area of track = Outer area – inner area

$$= \left(\pi \times \frac{503}{2\pi} \times \frac{503}{2\pi}\right) - \left(\pi \times \frac{437}{2\pi} \times \frac{437}{2\pi}\right)$$

$$= \frac{(503 \times 503) - (437 \times 437)}{4\pi}$$

$$= \frac{(503 + 437) - (503 - 437)}{4\pi}$$

$$= \frac{940 \times 66 \times 7}{4 \times 22}$$

$= 4935$ m^2. **Ans.**

3. The area of two concentric circles are 1386 sq. cm and 1886.5 sq. cm respectively. Find the width of the ring.

Sol. Given : Area of two concentric circles

= 1386 & 1886.5 sq. cm

$\because$ Area of circle = πr^2

$$\therefore \quad r = \sqrt{\frac{\text{area}}{\pi}}$$

$$r = \sqrt{\frac{1386}{\pi}} \ \& \ \sqrt{\frac{1886.5}{\pi}}$$

$$r = \sqrt{\frac{1386 \times 7}{22}} \ \& \ \sqrt{\frac{1886.5 \times 7}{22}}$$

$r = \sqrt{441}$ & $\sqrt{600.25}$

$r = 21$ & 24.5

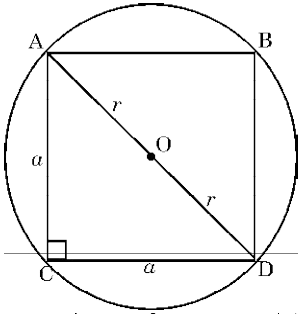
Width of ring = 24.5 – 21

= 3.5 cm.

4. A square ABCD is inscribed in a circle of radius *r*. Find the area of square.

Sol. **Given :** Radius of circle = r

To find : Area of square



Area of square = $(\text{side})^2$

let the side of square = a

$\therefore$ area = a^2

In ΔACD

$$AC^2 + CD^2 = AD^2$$
$$a^2 + a^2 = (r + r)^2$$
$$2a^2 = (2r)^2$$
$$2a^2 = 4r^2$$
$$a^2 = 2r^2$$

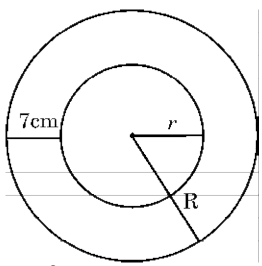
Hence, area of square = $2r^2$ sq. unit

5. A road which is 7m wide surrounds a circular park whose circumference is 352 m. Find the area of the road.

Sol. **Given :** Width of road = 7 cm

Circumference of park = 352 m

To find : Area of road



Circumference = $2\pi r$

$352 = 2\pi r$

$$r = \frac{352 \times 7}{2 \times 22}$$

= 56 m

$\therefore$ R = 56 + 7

= 63 m

Area of park = $\frac{22}{7} \times 56 \times 56$

= 9856 m^2

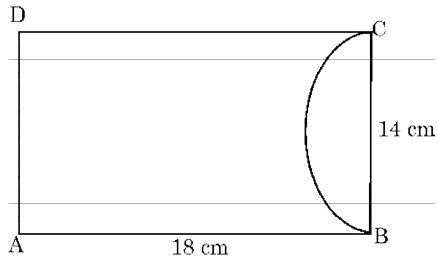
Area of road + park = $\frac{22}{7} \times 63 \times 63$

= 12474 m^2

Area of road = 12474 – 9856

= 2618 m^2.

6. A paper is the form of a rectangle ABCD in which AB = 18 cm and BC = 14 cm. A semicircular portion with BC as diameter is cut off. Find the area of the remaining paper.



Sol. **Given :** AB = 18 cm CB = 14 cm

Area of rectangle = AB × BC

= 18 × 14

= 252 cm^2

Area of semicircular portion = $\frac{\pi r^2}{2}$

$$= \frac{\frac{22}{7} \times 7 \times 7}{2}$$

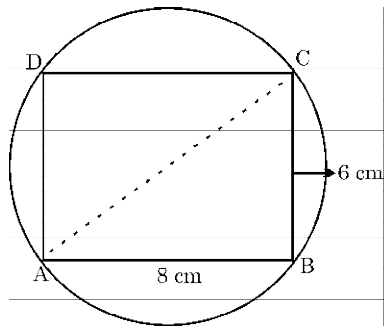
= 77 cm^2

Area of remaining paper = 252 – 77

= 175 cm^2. **Ans.**

7. A circle is drawn on a diagonal of a rectangle 8 cm × 6 cm. If the diagonal of the rectangle is the diameter of the circle, find the area of the circle.

Sol.



Sides of rectangle = AB = 8 cm, BC = 6 cm

In ΔABC

$\because \angle ABC = 90°$

$\therefore \quad AC = \sqrt{AB^2 + BC^2}$

$AC = \sqrt{8^2 + 6^2}$

$AC = \sqrt{64 + 36}$

$AC = \sqrt{100}$

$AC = 10$

$\therefore$ Diagonal of circle = 10 cm

Radius of circle = $10 \div 2$

= 5 cm

Area of circle = πr^2

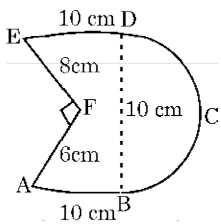
$= \frac{22}{7} \times 5 \times 5$

= 78.5 cm². **Ans.**

8. Calculate the area of shaded portion in the figure if AB = 8cm, BC = 6cm.

Sol.

9. Find the area of the region ABCDEFA shown in the given figure given that ABDE is a square of side 10 cm, BCD is a semicircle with BD as diameter, EF = 8 cm, AF = 6 cm and $\angle AFE = 90°$. Take $\pi = 3.14$.



Sol.

$\because \quad ED = AB$

$= BD = 10$ cm.

$\therefore \quad AE = 10$ cm.

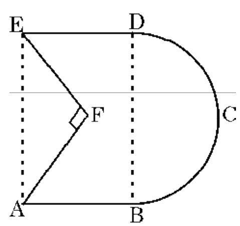
and ABDE is a square

Area of ABDE = $(side)^2$

$= (10)^2$

$= 10 \times 10$

$= 100$ cm²



Area of $\Delta AFE = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 8 \times 6$

$= 24$ cm²

Area of AFEBD = 100 – 24

= 76 cm²

Area of semicircle DCB = $\frac{\pi r^2}{2}$

$= \frac{3.14 \times 5 \times 5}{2}$

= 39.25cm²

Area of ABCDEFA = 76 + 39.25

= 115.25 cm². **Ans.**

10. From a copper which is a square of side 12.5cm, a circular disc of diameter 7cm is cut off. Find the weight of the remaining part. If 1 sq. cm of the plate weights 0.8 gm.

Sol. Given : Side of square = 12.5cm

$\therefore$ area of square = $(side)^2$

$= (12.5)^2$

$= 12.5 \times 12.5$

$= 156.25$ cm²

Diameter of circular disc. = 7 cm

radius of disc = $7 \div 2$

= 3.5 cm

area of circular disc = πr^2

$= \frac{22}{7} \times 3.5 \times 3.5$

= 38.5 cm².

Area of remaining part = 156.25 – 38.5

= 117.75 cm²

Weight of remaining part

$= 117.75 \times 0.8$

= 94.20 gm. **Ans.**

11. From a metallic plate, which is a square of side 10 cm, a circular disc of diameter 3.5 cm is cut off. Find the weight of the remaining part if 1 sq. cm of the plate weight 5 gm.

Sol. Given : Side of square = 10 cm

Area of square = $(side)^2$

$= (10)^2$

$= 10 \times 10$

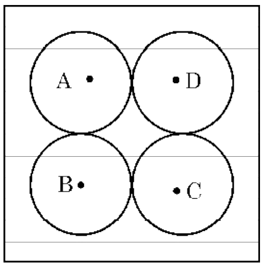
$= 100$ cm².

Diameter of circular disc = 3.5 cm
Radius of circular disc = 3.5 ÷ 2
= 1.75 cm
Area of circular disc = πr^2

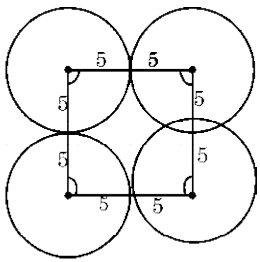
$$= \frac{22}{7} \times 1.75 \times 1.75$$

= 9.625 cm².
Area of remaining part = 100 – 9.625
= 90.375 cm².
Weight of remaining part = 90.375 × 5
= 451.875 gms. **Ans.**

12. Four equal circles, each of radius 5 cm, touch each, as shown in the figure. Find the area included between them. Take π = 3.14 cm.



Sol. Side of square = 5 + 5
= 10 cm
Area of square = (side)²



= (10)²
= 10 × 10
= 100 cm².
Radius of circle = 5 cm
θ = 90°

$$\text{Area} = \frac{\theta}{360} \times \pi r^2$$

$$\frac{90}{360°} = 3.14 \times 5 \times 5$$

= 19.625 cm².
Area of one quadrant = 19.625 cm²
Area of four quadrant = 19.625 × 4
= 78.5 cm.
Required area = 100 – 78.5
= 21.5 cm².

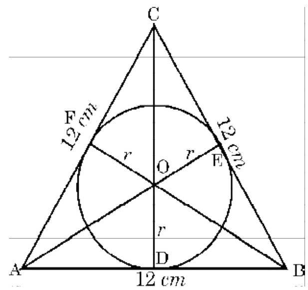
13. In an equilateral triangle of side 12cm, a circle is inscribed touching its sides. Find the area of the portion of the triangle not included in the circle. Take $\sqrt{3}$ = 1.73 and π = 3.14.

Sol. Side of triangle = 12 cm

$$\text{Area of triangle} = \frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times 12 \times 12$$

$$= 36\sqrt{3} \text{ cm}^2$$



Area of ΔAOB + Area of Δ AOC + Area of Δ BOC = Area of ΔABC

$$\frac{1}{2} \times AB \times OF + \frac{1}{2} \times AC \times OE + \frac{1}{2} \times BC \times OD = 36\sqrt{3}$$

∵ AB = AC = BC = 12 cm and
OF = OE = OD = r

$$\therefore 3 \times \frac{1}{2} \times 12 \times r = 36\sqrt{3}$$

$$r = 2\sqrt{3}$$

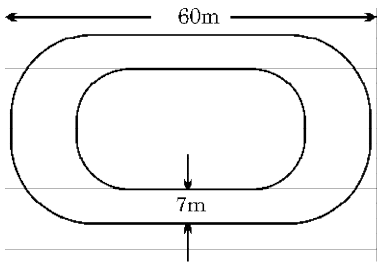
Area of circle = πr^2

$$= \frac{22}{7} \times 2\sqrt{3} \times 2\sqrt{3}$$

= 37.71 cm²
Area of remaining area
= $36\sqrt{3}$ – 37.71
= 36 × 1.73 – 37.71
= 24.57 cm². **Ans.**

14. The inside perimeter of a running track, as shown in the

figure, is 340m. The length of each straight portion is 60m, and the curved portions are semicircles. If the track is 7m wide, find the area of the track. Also, find the outer perimeter of the track.

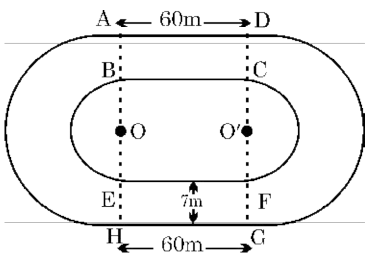


Sol. Length of inner curved portion

$= [340 - (2 \times 60)]$ m

$= 340 - 120$

$= 220$m



Therefore, the length of each inner curved surface = 110 m

Let radius be r, then

$\pi r = 110$ m

$r = 35$ m

So, inner radius (OB = O′C) = 35 m and outer radius (OA = O′D) = (55 + 7)

= 42 m

Hence, required area = Area of 2 rectangles + Area of circular range

$(2 \times 60 \times 7) + \frac{22}{7}\ [(42)^2 - (35)^2]$

$840 + \frac{22}{7}\ (42 + 35)(42 - 35)$

$840 + 1694$

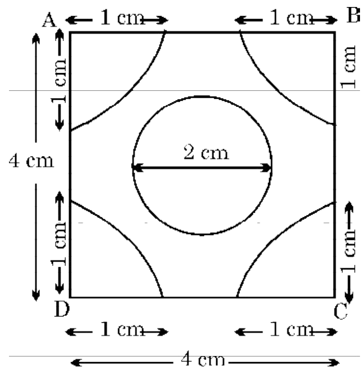
$= 2534\ m^2$

Length of outer boundary of the park

$2 \times 60 + 2 \times \frac{22}{7} \times 42$

$= 384$ m. **Ans.**

15. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in adjoining figure. Find the area of the remaining portion of the square.



Sol. Side of square = 4 cm

Radius of inner circle = $2 \div 2$

= 1 cm

Radius of quadrants of circle = 1 cm

Area of square = $(\text{side})^2$

$= (4)^2$

$= 4 \times 4$

$= 16\ cm^2$

Area of circle = πr^2

$= \frac{22}{7} \times 1 \times 1$

$= \frac{22}{7}$

Area of quadrant = $\frac{90}{360} \times \pi r^2$

$= \frac{90}{360} \times \frac{22}{7} \times 1 \times 1$

$= \frac{1}{4} \times \frac{22}{7}$

Area of 4 quadrant = $\frac{1}{4} \times \frac{22}{7} \times 4$

$= \frac{22}{7}$

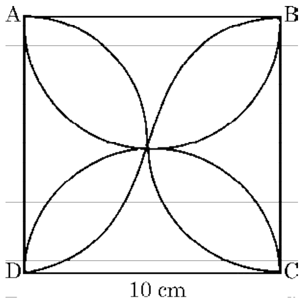
Area of remaining part =

$16 - \frac{22}{7} - \frac{22}{7}$

$$= \frac{112 - 22 - 22}{7}$$

$$= \frac{68}{7} \text{ cm}^2.$$ **Ans.**

16. Find the area of the shaded design in adjoining figure where ABCD is a square of side 10 cm and semicircles are drawn with each side of the square as diameter. (Use π = 3.14)



Sol. Given : Side of square ABCD = 10 cm

Area of square ABCD = $(\text{side})^2$
$= (10)^2$
$= 10 \times 10$
$= 100 \text{ cm}^2$

Given semicircle is drawn width side of square as diameter.

So, diameter of semicircle = Side of square = 10 cm

Radius of semicircle = $\frac{\text{Side}}{2}$
$= \frac{10}{2}$
$= 5$ cm.

Area of semicircle AD = $\frac{1}{2} \times$ area of circle

$$= \frac{1}{2} \pi r^2$$

$$= \frac{1}{2} \times \pi \times 5 \times 5$$

$$= \frac{3.14 \times 25}{2}$$

Since radius is same for semi circle AD, BC, AB, CD

Area of semicircle AD = Area of semicircle BC

Area of semicircle AB = Area of semicircle CD

$$= \frac{3.14 \times 25}{2}$$

Area of shaded region = Area of ABCD – Area of unshaded part

Area of unshaded region = [Area of square ABCD – (Area of semi circle AD + area of semi circle BC] + [Area of square ABCD – (Area of semicircle AB + area of semi circle CD)]

$\Rightarrow$ 2 (Area of square ABCD) – (Area of semi circle AD + BC + AB + CD)

$\Rightarrow 2(100) -$

$$\left(\frac{3.14 \times 25}{2} + \frac{3.14 \times 25}{2} + \frac{3.14 \times 25}{2} + \frac{3.14 \times 25}{2}\right)$$

$$\Rightarrow 200 - 4 \times \frac{3.14 \times 25}{2}$$

$\Rightarrow 200 - 2 \times 3.14 \times 25$

$\Rightarrow 200 - 157$

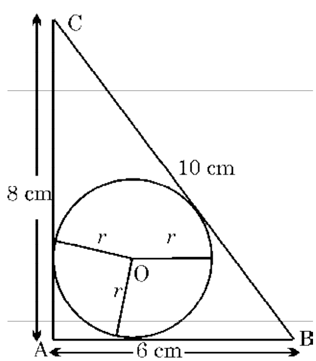
$= 43 \text{ cm}^2$

Area of Shaded region = Area of ABCD – Area of unshaded region

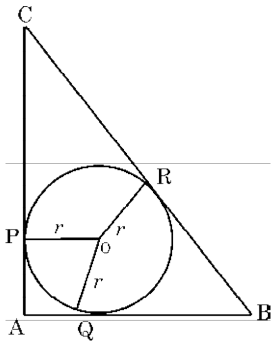
$100 - 43$

$= 57 \text{ cm}^2$. **Ans.**

17. In the given figure, $\triangle$ABC is right–angled at A with AB = 6 cm and AC = 8 cm. A circle with centre O has been inscribed inside the triangle. Find the value of *r*, the radius of the inscribed circle.



Sol. ΔABC is right triangle at A.



$$AB^2 + AC^2 = BC^2$$
$$6^2 + 8^2 = BC^2$$
$$\sqrt{36 + 64} = BC$$
$$BC = 10 \text{ cm.}$$

OP = OQ = OR = r, radius of the circle.

OP $\perp$ AC, OQ $\perp$ AB (Radius is pependicular to tangents)

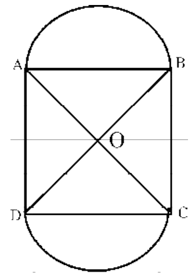
In OPAQ, $\angle A$, $\angle P$, $\angle Q$ are 90° each than the 4th angle will also be 90°.

Since OP = r, PA = AQ = r (sides of square)

Tangents from an external points are equal so,

$$BR = 6 - r \text{ and } CR = 8 - r$$
$$BC = 10 = (6 - r) + (8 - r)$$
$$10 = 14 - 2r$$
$$-4 = -2r$$
$$r = 2.$$

18. In figure, two circular flower beds have been shown on two sides of a square lawn ABCD of side 56m. If the centre of each circular flower bed is the point of intersection of the diagonals of the square lawn, find the sum of the areas of the lawns and the flower beds.



Sol. Total area= Area of OAB + Area of OCD + Area of ΔOAD + Area of ΔOBC

$$= \frac{90}{360} \times \frac{22}{7} \times \left(28\sqrt{2}\right)^2 + \frac{90}{360} \times \frac{22}{7} \times \left(28\sqrt{2}\right)^2 + \frac{1}{4} \times 56 \times 56 + \frac{1}{4} \times 56 \times 56$$

$$\Rightarrow \frac{1}{4} \times 28 \times 56 \left(\frac{22}{7} + \frac{22}{7} + 2 + 2\right)$$

$$\Rightarrow \frac{7 \times 56}{7} (22 + 22 + 14 + 14)$$

$$\Rightarrow 4032 \text{ m}^2.$$

❑

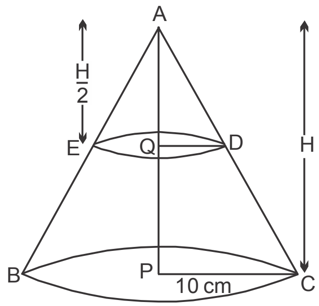
13 Surface Area and Volumes

EXERCISE 13.1

Assume $\pi = \frac{22}{7}$ unless stated otherwise.

1. A cone of radius 10 cm is divided into two parts by drawing a plane through the mid-point of its axis, parallel to its base. Compare the volumes of the two parts.

Sol. Let the height of given cone h cm on dividing it into two parts we get (i) Frustum of the cone with radius R = 10 cm and radius = 5 cm, height



$= \left(\frac{h}{2}\right)$ cm

(ii) a smaller cone of radius = 5 cm and height = $\left(\frac{h}{2}\right)$ cm

$\therefore$ Ratio of the volumes

$$= \frac{\text{volume of the smaller cone}}{\text{volume of the frustum of the cone}}$$

$$= \frac{1}{3}\pi r^2\left(\frac{h}{2}\right) : \frac{1}{3}\pi\left(\frac{h}{2}\right)[R^2 + r^2 + Rr]$$

$\therefore \; r^2 : R^2 + r^2 + Rr$

$5 \times 5 : (10 \times 10) + (5 \times 5) + (10 \times 5)$

$25 : 100 + 25 + 50$

$25 : 175$

$1 : 7$

$\therefore$ Volume of smaller cone : Volume of the frustum cone = 1 : 7.

2. Find the weight of a solid cone whose base is of diameter 14 cm and vertical height 51 cm, supposing the material of which it is made weight 10 grams per cubic cm.

Sol. Volume of cone = $\frac{1}{3}\pi r^2 h$

Diameter = 2 × radius

14 cm = 2 × radius

7cm = radius

$$\text{Volume} = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 51$$

$= 22 \times 7 \times 17$

$= 2618 \text{ cm}^3$

$\because$ 1 cm^3 weight = 10 g

$\therefore$ 2618 cm^3 = 2618 × 10

= 26180 grams

26180 grams = 26.18 kg. **Ans.**

3. How many metres of cloth 5 m wide will be required to make a conical tent, the radius of whose base is 7 m and whose height is 24 m ?

Sol. Tent is in the shape of a cone

radius (r) = 7 m,

height (h) = 24 m

$$\text{Slant height } (l) = \sqrt{(7)^2 + (24)^2}$$

$= \sqrt{49 + 576}$

$= \sqrt{625}$

$= 25 \; m^2$

C.S.A. of tent = $\pi r l$

$$= \frac{22}{7} \times 7 \times 25$$

$= 550 \text{ m}^2$.

Area of cloth = L × 5m

550 = L × 5

= 110 m.

$\therefore$ 110 m of cloth is required. **Ans.**

4. The largest sphere is to be carved out of a right circular cylinder of radius 7 cm and height 14 cm. Find the volume of the sphere.

Sol. Given : Radius of cylinder = 7cm
Height of cylinder = 14cm

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$

$= 1437.33\ cm^3$. **Ans.**

5. To construct a wall 24 m long, 0.4 m thick and 6 m high, bricks of dimensions 25cm × 16 cm × 10 cm each are used. If the mortar occupies $\frac{1}{10}^{th}$ of the volume of the wall. Find the number of bricks used.

Sol. Dimension of wall
= 24 m × 0.4 m× 6 m
Volume of wall = 24 × 0.4 × 6
= 57.6 m^3
Dimension of brick
= 25 cm × 16 cm × 10 cm
Volume of one brick
= 25 × 16 × 10
= 4000 cm^3
$1m^3$ = 100 cm × 100 cm × 100 cm
= 1000000 m^3
$1cm^3$ = 1 ÷ 1000000m^3
4000 cm^3 = 4000 ÷ 1000000 m^3
= 0.004m^3.

Volume of mortar = $\left(\frac{1}{10}\right)$ of volume of wall

$= \left(\frac{1}{10}\right) \times 57.6 = 5.76\ m^3$

Volume of all bricks
= 57.6 – 5.76
= 51.84 m^3
No. of bricks used
= 51.84 ÷ 0.004
= 12960 bricks.

6. A box open at the top has its outer dimensions 10 cm, 9 cm and 2.5 cm and its thickness is 0.5 cm, find the volume of the metal.

Sol. Outer dimention = L = 10 cm,
B = 9 cm & H = 2.5 cm
Outer volume = L × B × H
= 10×9× 2.5
= 225 cm^3.
Thickness (w) = 0.5 cm
Inner dimension =
$l = L - 2w$; $l = 10 - 2(0{\cdot}5)$; $l = 10 - 1$; $l = 9$ cm
$b = B - 2w$; $b = 9 - 2(0{\cdot}5)$; $b = 9 - 1$; $b = 8$ cm
$h = H - w$; $h = 25 - 0{\cdot}5$; $h = 2$ cm
Inner volume = $l \times b \times h$
= 9 × 8 × 2
= 144 cm^3.
Volume of the metal = outer volume – inner volume
= 225 – 144
= 81 cm^3. **Ans.**

7. Marbles of diameter 1.4 cm are dropped into a cylindrical beaker, of diameter 7 cm, contaning some water. Find the number of marbles that should be dropped into the beaker, so that the water level rises by 5.6 cm.

Sol. Diameter of spherical ball (d)
= 1.4 cm
Radius of spherical ball (r)
= 1.4 ÷ 2
= 0.7 cm

$$\text{Volume of the sphere } (v) = \frac{4}{3}\pi r^3$$

$$v = \frac{4}{3} \times \frac{22}{7} \times 0.7 \times 0.7 \times 0.7 \quad ...(1)$$

Diameter of the cylinder (D)
= 7 cm
Radius of the cylinder (R)
= 7 ÷ 2
= 3.5 cm
Let the water level rives by h
= 5.6 cm
Then,
Volume of the cylinder of height
= 5.6 cm
and radius = 3.5 cm
$V = \pi r^2 h$

$$V = \frac{22}{7} \times 3.5 \times 3.5 \times 5.6 \quad ...(2)$$

Let the number of balls dropped in beaker = n

$$n = \frac{V}{v}$$

$$n = \frac{\frac{22}{7} \times 3.5 \times 3.5 \times 5.6}{\frac{4}{3} \times \frac{22}{7} \times 0.7 \times 0.7 \times 0.7}$$

$$= \frac{215.6}{1.43}$$

$n = 150.$

Hence, number of marbles that should be dropped into the beaker = 150 marbles. **Ans.**

8. The internal and external diameters of hollow hemispherical vessel are 24 cm and 25 cm respectively. If the cost of painting 1cm^2 of the surface area is ₹ 5.25, find the total cost of painting the vessel all over.

Sol. Given : Internal diameter= 24 cm

Internal radius = 24 ÷ 2

= 12 cm.

External diameter = 25 cm

External radius = 25 ÷ 2

= 12.5 cm.

Surface area of internal bowl = $2\pi r^2$

$= 2 \times \frac{22}{7} \times 12 \times 12$

$= 905.14$ cm^2

Surface area of external bowl = $2\pi R^2$

$= 2 \times \frac{22}{7} \times 12.5 \times 12.5$

$= 982.14$cm^2

Surface area of ring = $\pi(R^2 - r^2)$

$= \frac{22}{7} \times [(12.5)^2 - (12)^2]$

$= \frac{22}{7} [(12.5 - 12)(12.5 + 12)]$

$= \frac{22}{7} \times 0.5 \times 24.5$

$= 38.5$ cm^2

Cost of internal bowl

= 905.14 × 5.25 = 4,752 ₹

Cost of external bowl

= 982.14 × 5.25 = 5,156.25 ₹

Cost of ring = 38.5 × 5.25 = 202.125₹

Cost of painting the bowl= 4752 + 5,156.25 + 202.125

= ₹ 10,110.375.

9. Lead spheres of diameter 6 cm are dropped into a cylindrical beaker containing some water and are fully sumberged. If the diameter of the beaker is 18 cm and the water rises by 40 cm, find the number of lead spheres dropped in the water.

Sol. Diameter of sphere = 6 cm

Radius of sphere = 6 ÷ 2

= 3 cm

Volume of sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 3 \times 3 \times 3$

$= 36\pi$

Diameter of cylinder = 18 cm

Radius of cylinder = 18 ÷ 2

= 9 cm

Height of cylinder = 40 cm

Volume of cylinder = $\pi r^2 h$

$= \pi \times 9 \times 9 \times 40 = 3240\pi$

Number of lead spheres dropped in water :

$$= \frac{\text{Volume of cylinder}}{\text{Volume of sphere}}$$

$$= \frac{3240\pi}{36\pi}$$

= 90 spheres.

10. The largest possible sphere is carved out from a solid cube of side 40m. Find the surface area of the sphere. (Use π = 3.14)

Sol. The largest possible carved from a cube must have diameter

= side of the cube

∴ Diameter = 40 m

∴ Radius = 40 ÷ 2

= 20 cm

Surface area of sphere

$= 4\pi r^2$

$= 4 \times 3.14 \times 20 \times 20$

$= 5024$ cm^2.

11. A vessel in the shape of a cuboid contains some water. If three identical spheres are immeersed in the water, the level of water is increased by 2 cm. If the area of the base of the cuboid is 160 cm² and its height 12 cm, determine the radius of any of the sphere.

Sol. Area of base of cuboid = 160 cm²

Volume of a sphere

$$= \frac{4}{3}\pi r^3$$

Volume of 3 spheres = $4\pi r^3$

Increased volume of water

$$= 160 \times 2$$

$$= 320 \text{ cm}^2$$

Since the volume is increased after immersing 3 spheres so, increased volume of water will be equal to the volume of 3 spheres

$$= 4\pi r^3$$

$$4 \times \frac{22}{7} \times r^3 = 320$$

$$88\, r^3 = 2240$$

$$r^3 = \frac{2240}{88}$$

$$r^3 = 25.45$$

$$r = 3\sqrt{25.45}$$

$$r = 2.94 \text{ cm}$$

EXERCISE 13.2

1. A solid is in the form of a right circular cylinder, with a hemisphere at one end and a cone at the other end. The radius of the common base is 3.5 cm and the heights of cylindrical and conical portion are 10 cm and 6 cm, respectively, find the total surface area of the solid.

Sol. Given : Radius of common base

$$= 3.5 \text{ cm}$$

Height of cylindrical part

$$= 10 \text{ cm}$$

Height of conical part (h) = 6cm

Let 'l' be the slant height of cone

$$l = \sqrt{r^2 + h^2}$$

$$l = \sqrt{(3.5)^2 + 6^2}$$

$$l = \sqrt{12.25 + 36}$$

$$l = \sqrt{48.25} \text{ cm}$$

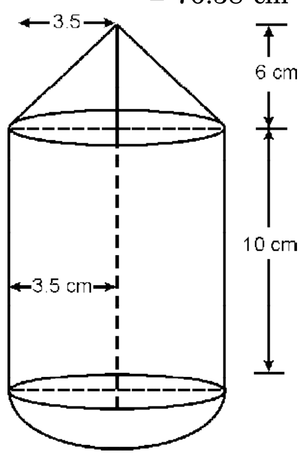
$$l = 6.95 \text{ cm.}$$

Curved surface area of cone (S_1)

$$= \pi r l$$

$$= 3.14 \times 3.5 \times 6.95$$

$$= 76.38 \text{ cm}^2$$



Curved surface area of cylinder (S_2)

$$= 2\pi r h$$

$$= 2 \times 3.14 \times 3.5 \times 10$$

$$= 219.8 \text{ cm}^2$$

Curved surface area of hemisphere

$$(S_3) = 2\pi r^2$$

$$= 2 \times 3.14 \times 3.5 \times 3.5$$

$$= 76.93 \text{ cm}^2$$

Total surface area of solid

$$= S_1 + S_2 + S_3$$

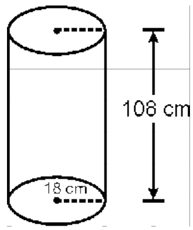
$$76.34 + 219.8 + 76.93$$

$$= 373.11 \text{ cm}^2$$

2. A solid is composed of a cylinder with hemispherical ends. If the length of the whole solid is 108 cm and the diameter of the cylinder is 36 cm, Find the cost of polishing the surface at the rate of 7 paise per cm^2. (Use π = 3.14)

Sol. Diameter of cylinder = 36 cm

$\therefore$ Radius of cylinder = 36 ÷ 2 = 18 cm



$\therefore$Radius of hemisphere = 18 cm

Height of the cylinder

= 108 – (18 + 18)

= 72 cm

TSA of the whole solid

= CSA of the cylinder + CSA of two hemisphere

$= 2\pi rh + 2(2\pi r^2)$

$= 2\pi r[h + 2r]$

= 2 × 3.1416 × 18 [72 + 2 × 18]

= 113.0976[72 + 36]

= 113.0976 × 108

= 12214.5408

$\therefore$The cost of painting = 12214.5408 × 0.07 = ₹ 855.02. **Ans.**

3. A decorative block is made of two solids—a cube and a hemisphere. The base of the block is the cube with edge of 5 cm and the hemisphere attached on the top has a diameter of 4.2 cm. If the block to be painted, find the total area to be painted.

Sol. The decorative block is a combination of a cube and a hemisphere.

For cubical portion

Each edge = 5 cm

For hemispherical portion

Diameter = 4.2 cm

Radius (r) = 4.2 ÷ 2 = 2.1 cm.

Total surface area of cube

= 6 × (edge)^2

= 6 × 5 × 5

= 150 cm^2.

Here the part of cube where the hemisphere is attached is not included in the surface area.

$\therefore$The surface area to be painted = Total surface area of cube + Area of base of hemisphere + Curved surface area of hemisphere

Total surface area

$= 150 - \pi r^2 + 2\pi r^2$

$= 150 + \pi r^2$

= 150 + [3.14 × 2.1 × 2.1]

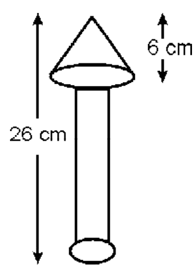
= 150 + 13.86

= 163.86 cm^2

Hence the total surface area of decorative block = 163.86 cm^2.

4. A wooden toy rocket is in the shape of a cone mounted on a cylinder. The height of the entire rocket is 26 cm while the height of the conical part is 6 cm. The base of the conical portion has a diameter of 5 cm, while the base diameter of the cylindrical portion is 3 cm. If the conical portion is to be painted orange and the cylindrical portion yellow, find the area of the rocket painted with each of these colours.

Sol. Area to be painted orange = Curved surface area of cone + base area of cone – base area of cylinder



Area to be painted yellow = curved surface area of cylinder + Area of one bottom base of the cylinder

Curved surface area of the cone

$= \pi rl$

$\because$ Diameter of conical portion

$= 5$ cm

$\therefore$ Radius of conical portion

$= 5 \div 2$

$= 2.5$ cm

Height of conical portion (h)

$= 6$cm

Slant height of conical portion (l)

$= \sqrt{h^2 + r^2}$

$l = \sqrt{6^2 + (2.5)^2}$

$l = \sqrt{36 + 6.25}$

$l = \sqrt{42.25}$

$l = 6.5$ cm.

C.S.A of cone $= 3.14 \times 2.5 \times 6.5$

$= 51.025$ cm^2

Base of the cone =

Area of circle $= \pi r^2$

= Radius of circle r

$= 2.5$ cm

Area of circle $= 3.14 \times 2.5 \times 2.5$

$= 19.625$ cm^2

Curved surface area of cylinder

$= 2\pi rh$

$\because$ Diameter of cylinder $= 3$ cm

$\therefore$ Radius of cylinder $(r) = 3 \div 2$

$= 1.5$ cm

Height of cylinder (h)

= Total height − height of cone

$= 26 - 6$

$= 20$ cm

C.S.A. of cylinder

$= 2 \times 3.14 \times 1.5 \times 20$

$= 188.4$ cm^2

Base area of the cylinder = Area of circle $= \pi r^2$

Radius of circle $(r) = 1.5$ cm

Area of circle $= 3.14 \times 1.5 \times 1.5$

$= 7.065$ cm^2

Area to be painted orange

$= 51.025 + 19.625 - 7.0625$

$= 63.58$ cm^2

Area to be painted yellow

$= 188.4 + 7.065$

$= 195.465$ cm^2. **Ans.**

5. A toy is in the form of a cone mounted on a hemisphere. The diameter of the base and the height of the cone are 6 cm and 4 cm. respectively. Determine the surface area of the toy. (use $\pi = 3.14$)

Sol. Height of cone $= 4$ cm

Diameter of cone $= 6$ cm

Radius of cone $= 6 \div 2$

$= 3$ cm

Slant height of cone

$= \sqrt{r^2 + h^2}$

$= \sqrt{3^2 + 4^2}$

$= 5$ cm

Lateral surface area of cone $= \pi rl$

$= 3.14 \times 3 \times 5$

$= 47.1$ cm^2

Diameter of hemisphere $= 6$ cm

Radius of hemisphere

$= 6 \div 2$

$= 3$ cm.

Lateral surface area of hemisphere

$= 2\pi r^2$

$= 2 \times 3.14 \times 3 \times 3$

$= 56.52$ cm^2.

The surface area of toy = lateral surface area of cone + lateral surface area hemisphere

$= 47.10 + 56.52$

$= 103.62$ cm^2.

6. A tent is in the form of a right cylinder surmounted by a cone. The diameter of cylinder is 24 m. The height of the cylindrical portion is 11m while the vertex of the cone is 16 m above the ground. Find the area of the canvas required for the tent.

Sol. Diameter of the cylinder

$= 24$ m

and radious of the cylinder

$= 24 \div 2$

$= 12$ m

It's height $= 11$ m

C.S.A of Cylindrical Portion

$= 2\pi rh$

$= 2 \times \pi \times 12 \times 11$

$= (264\ \pi)\ m^2$

Radius of the cone = 12 m

Height $= 16 - 11$

$= 5m$

Slant height of the cone

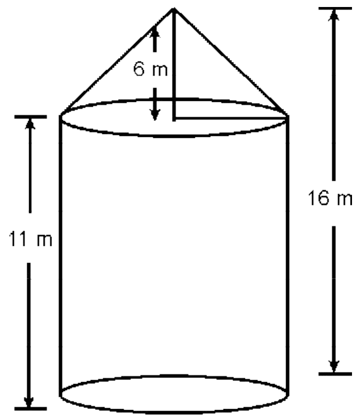
$= \sqrt{r^2 + h^2}$

$= \sqrt{12^2 + 5^2}$

$= \sqrt{144 + 25}$

$= \sqrt{169}$

$= 13m$



C.S.A of cone $= \pi rl$

$= \pi \times 12 \times 13$

$= (156\ \pi)\ m^2$

Area of Canvas = C.S.A of Cylind–rical Portion + C.S.A of Cone

$= (264\ \pi + 156\pi)\ m^2$

$= (420\ \pi)\ m^2$

$= (420 \times \frac{22}{7}\ \pi) m^2$

$= 1320\ m^2$ **Ans.**

7. From a solid cylinder whose height is 3.6 cm and diameter 2.1 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm².

Sol. The outer surface area of cylinder

$= 2\pi rh$

$= 2 \times 3.14 \times 1.05 \times 3.6$

$= 23.7384\ cm^2$

Slant height of the cone

$= \sqrt{h^2 + r^2}$

$= \sqrt{(3.6)^2 + (1.05)^2}$

$= \sqrt{12.96 + 1.10}$

$= \sqrt{14.06}$

$= 3.75$ cm.

Hence, outer surface area of cone

$= \pi rl$

$= 3.14 \times 3.75 \times 1.05$

$= 12.36375.$

Surface area of cylindrical base = πr^2

$= 3.14 \times 1.05 \times 1.05$

$= 3.46185\ cm^2$

Hence, total surface area of remaining solid= The outer surface area of the cylinder + Inner surface area of hollow portion of cylinder left + surface area of cylindrical base = 23.7384 + 12.36375 + 3.46185 = 39.564.

Hence, the total surface area of remaining solid to the nearest cm² is 40 cm². **Ans.**

8. From a solid hemisphere whose radius is 2.1 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm² and also find out the cost of painting the remaining solid green at the rate of ₹ 1.25 per square cm. (Use $\sqrt{2}$ =1.41]

Sol. TSA of remaining solid $= 2\pi r^2 + \pi rl$

slant height of cone (l)

$= \sqrt{h^2 + r^2}$

$= \sqrt{(2.1)^2 + (2.1)^2}$

$= \sqrt{4.41 + 4.41} = \sqrt{8.82}$

$= 2.97$ cm

Now, TSA of remaining solid = CSA of hemisphere + CSA of cone

$= 2\pi r^2 + \pi rl$

$= \pi r\ (2r + l)$

$= 3.14 \times 2.1\,[(2\times2.1) + 2.97]$
$= 6.594\,(4.2 + 2.97)$
$= 47.26.$

Cost of painting remaining solid
$= 47.26 \times 1.25$
$= ₹\ 59.08.$

9. A toy is in the shape of a cylinder with two equal cones stuck to each of its ends. The length of entire solid is 30 cm and diameter of cylinder and cones 10.5 cm and the length of the cylinder is 14 cm. Find its surface area.

Sol. Given : Length of entire solid
$= 30$ cm

Length of cylinder $(h) = 14$ cm

Length of two cone $(h) = 30 - 14$
$= 16$ cm.

Length of one cone $(h) = \dfrac{16}{2}$
$= 8$ cm

Diameter $= 10.5$ cm

$\therefore$ Radius $= \dfrac{10.5}{2}$
$= 5.25$ cm.

Slant height of cone $(l) = \sqrt{h^2 + r^2}$
$= \sqrt{8^2 + (5.25)^2}$
$= 9.57\ \text{cm}^2.$

Surface area of toy = Surface area of 2 cones + surface area of cylinder.
$= 2(\pi rl) + 2\pi rh$
$= 2\pi r\ (l+h)$
$= 2 \times \dfrac{22}{7} \times 5.25\,(9.57 + 14)$
$= \dfrac{231(23.57)}{7}$
$= \dfrac{5444.67}{7}$
$= 777.81\ \text{cm}^2.$

10. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The radius and height of the cylindrical part are 5 cm and 13 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Find the surface area of the toy, if the total height of the toy is 30 cm.

Sol. Given : Total height $= 30$ cm
height of cylinder $= 13$ cm
height of cone $= 30 - 18$
$= 12$ cm.

Curved surface area of hemisphere
$= 2\pi r^2$
$= 2 \times 3.14 \times 5 \times 5$
$= 157\ \text{cm}^2.$

Curved surface area of cylinder
$= 2\pi rh$
$2 \times 3.14 \times 5 \times 13$
$= 408.2\ \text{cm}^2.$

Curved surface area of cone
$= \pi rh$
$= 3.14 \times 5 \times 13$
$= 204.1\ \text{cm}^2.$

Surface area of toy
$= 157 + 408.2 + 204.1$
$= 769.3.$

Hence, surface area of toy 770 cm^2

11. Rasheed got a toy, a playing top (lattu) as his birthday present, which surprisingly had no colour on it. He wanted to colour it with his crayons. The toy is shaped like a cone surmounted by a hemisphere. The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area he has to colour.

Sol. Total surface area of the top
= curved surface area of hemisphere + curved surface area of cone.

Curved surface area of hemisphere
$= 2\pi r^2$
$= 2 \times \dfrac{22}{7} \times \dfrac{3.5}{2} \times \dfrac{3.5}{2}$
$= 19.25\ \text{cm}^2$

Curved surface area of cone $= \pi rl$

Slant height of cone $(l) = \sqrt{r^2 + h^2}$

Height of cone = Height of top – Height of hemispherial part

$= 5 - \frac{3.5}{2}$

$= 3.25$ cm

Slant height of the cone

$= \sqrt{\left(\frac{3.5}{2}\right)^2 + (3.25)^2}$

$= 3.7$ cm

C.S.A. of cone $= \frac{22}{7} \times \frac{3.5}{2} \times 3.7$

$= 20.30$ cm^2

Area of top $= 19.25 + 20.30$

$= 39.55$ cm^2. **Ans.**

12. A circus tent is in the form of a right circular cylinder and a right circular cone above it. The diameter and the height of the cylindrical part of the tent are 126 m and 5 m respectively. The total height of the tent is 21 m. Find the total surface area of the tent. Also, find the cost of the tent. If the canvas used cost ₹ 12 per square metre.

Sol. Given : Diameter of base

$= 126$ m

∴ Radius of base $= 126 \div 2$

$= 63$ m

Height of cylindrical part

$= 5$m

Total Height $= 21$ m

∴ Height of cone $= 21 - 5$

$= 16$ m

Slant Height of cone

$= \sqrt{r^2 + h^2}$

$= \sqrt{(63)^2 + (16)^2}$

$= \sqrt{3969 + 256}$

$= \sqrt{4225}$

$= 65$ m.

Curved surface area of cone

$= \pi rl$

$= \frac{22}{7} \times 63 \times 65$

$= 12870$ m^2.

Lateral surface area of cylindrical tent $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 63 \times 5$

$= 1980$ m^2

Total surface area of tent

$= 12{,}870 + 1{,}980$

$= 14{,}850$ m^2

Total cost of tent

$= 14{,}850 \times$ ₹ 12

$=$ ₹ 1,78,200. **Ans.**

13. A tent is of the shape of a right circular cylinder up to height of 3m and conical above it. The total height of the tent is 13.5 m and the radius of the base is 14m. Find the cost of cloth required to make the tent at the rate of ₹ 80 per m².

Sol. Given : Radius of base $(r) = 14$ m

Height of cylinder $(h) = 3$ m

Total height of tent $= 13.5$ m

Height of cone $= 13.5 - 3$

$= 10.5$ m

Slant hight of cone (l)

$= \sqrt{r^2 + h^2}$

$= \sqrt{(14)^2 + (10.5)^2}$

$= 17.5$ m.

Curved surface area of cone $= \pi rl$

$= \frac{22}{7} \times 14 \times 17.5$

$= 770$ m^2

Lateral surface area of cylinder

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 14 \times 3$

$= 264$ m^2

Total surface area of tent

$= 770 + 264$

$= 1{,}034$m^2.

Cost of tent

$= 1{,}034 \times$ ₹ 80

$=$ ₹ 82,720

14. A tent is of the shape of a right circular cylinder upto A height of 3 metres and then becomes a right circular cone with a maximum height of 13.5 metres above the ground. Calculate the

cost of painting the inner side of the tent at the rate of ₹ 2 per square metre, if the radius of the base is 14 metres.

Sol. Given :Radius of base (r)= 14 cm

Height of cylinder (h) = 3 m

Total height of tent = 13.5 m

Height of cone = 13.5 – 3

= 10.5 m

Slant height of cone (l)

$= \sqrt{r^2 + h^2}$

$= \sqrt{14^2 + 10.5^2}$

= 17.5 m

Curved surface area of cone = πrl

$= \frac{22}{7} \times 14 \times 17.5$

= 770 m^2

Lateral surface area of cylindrical part = $2\pi rh$

$= 2 \times \frac{22}{7} \times 14 \times 3$

= 264 m^2

Total surface area of tent

= 770 + 264

= 1034 m^2.

Cost of tent= 1034 × ₹ 2

= ₹ 2,068.

15. A medicine capsule is in the form of a cylinder with 2 hemispheres stuck to each of its ends. The length of the entire capsule is 16 mm and the diameter of the capsule is 14 mm. Find its surface area.

Sol. Given : Diameter of capsule = 14 mm

Radius of capsule = 14 ÷ 2

= 7 mm Lenght of entire capsule = 16 mm

Length of cylindrical part = Total length – radius of two hemisphere

= 16 – [2 × 7]

= 16 – 14

= 2 mm

Curved surface area of hemisphere = $2\pi r^2$

$= 2 \times \frac{22}{7} \times 7 \times 7$

= 308 mm^2.

Curved surface area of cylinder = $2\pi rh$

$= 2 \times \frac{22}{7} \times 7 \times 2$

= 88 mm^2.

Total surface area = C.S.A. of two hemisphere + C.S.A. of cylinders

= (2 × 308) + 88

= 616 + 88

= 704 mm^2

16. Two cubes each of volume 125m^3 are joined end to end. Find the surface area of the resulting cuboid.

Sol. Volume of each cube = 125 m^3

Volume of cube = $(\text{side})^3$

$125 = (\text{side})^3$

$\sqrt[3]{125}$ = side

side = 5m.

When 2 cubes are joined,

Lenght of cuboid = 5 + 5

= 10 m

Breadth of cuboid = 5 m

Height of cuboid = 5 m

Surface area of cuboid

= $2(lb + bh + hl)$

= 2 [(10 × 5) + (5 + 5) × (10 × 5)]

= 2 [50 + 25 + 50]

= 2 × 125

= 250 m^2

17. A toy is in the form of a cone of radius 7 cm mounted on a hemisphere of same radius. The total height of the toy is 15 cm. The toy is to be painted blue. If the rate of painting is ₹ 0.50 per sq.cm. Find the surface area to be painted and cost of painting.

Sol. Given : Radius of cone = 7 cm

Total height of toy = 15 cm

height of cone = 15 – 7

= 8 cm

Slant height of cone = $\sqrt{r^2 + h^2}$

$= \sqrt{7^2 + 8^2}$

$= \sqrt{49 + 64}$

$= \sqrt{113}$

= 10.63 cm.

Total surface area of toy

= $2\pi r^2 + \pi rl$

= $\pi r(2r + l)$

$= \frac{22}{7} \times 7 \; [(2 \times 7) + 10.63]$

$= 22(14 + 10.63)$

$= 22 \times 24.63$

$= 541.86 \text{ cm}^2$

Cost of painting

$= 541.86 \times 0.50$ ₹

$=$ ₹ 270.93

EXERCISE 13.3

1. A juice seller was serving his customers using glasses. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass has a hemispherical raised portion which reduced the capacity of the glass. If the height of the glass was 10 cm. Find what the apparent capacity of the glass was and what the actual capacity was. (Use $\pi = 3.14$)

Sol. Given : Diameter of glass = 5 cm

$\therefore$ Radius of glass (r) = $5 \div 2 = 2.5$ cm

Height of glass (h) = 10 cm

Actual capacity of glass = Volume of cylindrical part

$= \pi r^2 h$

$= 3.14 \times 2.5 \times 2.5 \times 10$

$= 196.25 \text{ cm}^3$.

Apparent capacity of glass = Volume of cylindrical part – Volume of hemispherical raised portion

$= 196.25 - \left[\frac{2}{3}\pi r^3\right]$

$= 196.25 - \left[\frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5\right]$

$= 196.25 - 32.59$

$= 163.66$ **Ans.**

2. A solid toy is in the form of a hemisphere surmounted by a right circular cone. Height of the cone is 2 cm and the diameter of the base is 4 cm. If a right circular cylinder circumscribes the toy, find how much more space it will cover.

Sol. Given : Height of cone = 2 cm

Diameter of base = 4 cm

Radius of base = $4 \div 2$

= 2 cm

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 2 \times 2 \times 2$

$= \frac{8}{3}\pi$

Volume of hemisphere = $\frac{2}{3}\pi r^3$

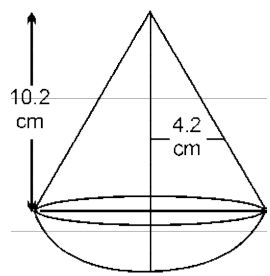
$= \frac{2}{3} \times \pi \times 2 \times 2 \times 2 = \frac{16}{3}\pi$

Volume of toy $= \frac{8\pi}{3} + \frac{16\pi}{3}$

$= \frac{24\pi}{3}$

$= 8\pi$.

3. A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden toy.



Sol. Radius of the hemisphere = 4.2 cm

Height of the toy = 10.2 cm

$\therefore$ Height of the cone = $10.2 - 4.2$

= 6 cm

Radius of the cone = 4.2 cm

Volume of the toy = volume of hemisphere + volume of cone

$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2 \; (2r + h)$

$= \frac{1}{3} \times \frac{22}{7} \times 4.2 \times 4.2\ (2 \times 4.2 + 6)$

$= \frac{388.08(14.4)}{21}$

$= \frac{5588.352}{21}$

$= 266.112\ cm^3$

$\therefore$ Volume of the toy = $266.112\ cm^3$.

4. A cylindrical container is filled with ice-cream whose, diameter is 12 cm and height is 15 cm. The whole ice-cream is distributed to 10 childrens in equal cones having hemispherical tops. If the height of conical portion is twice the diameter of its base, find the diameter of the ice-cream cone.

Sol. **Given :** Diameter of cylindrical portion (D) = 12 cm

$\therefore$ Radius of cylindrical portion (R) = 12 ÷ 2 = 6 cm

Height of cylindrical portion (H) = 15 cm

Let the radius of cone $(r) = r$

$\therefore$ Diameter of cone $= r \times 2 = 2r$

$\therefore$ Height of cone $= 2r \times 2 = 4r$

Volume of cylindrical portion = 10 (volume of conical part + volume of hemisphere)

$\Rightarrow \pi r^2 H = 10\left[\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right]$

$\Rightarrow \pi \times 6 \times 6 \times 15 = 10\left[\frac{1}{3}\pi r^2 \times 4r + \frac{2}{3}\pi r^3\right]$

$\Rightarrow 540 \times \pi = 10\left[\frac{4}{3}\pi r^3 + \frac{2}{3}\pi r^3\right]$

$\Rightarrow \frac{540}{10} \times \pi = \frac{6}{3}\pi r^3$

$\Rightarrow 54\pi = 2\pi r^3$

$\Rightarrow r^3 = \frac{54\pi}{2\pi}$

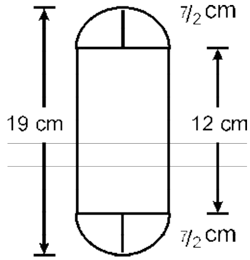
$\Rightarrow r^3 = 27$

$\Rightarrow r^3 = 3$

$\therefore$ Diameter of ice cream cone $= 3 \times 2 = 6$ cm.

5. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm and the diameter of the cylinder is 7 cm. Find the volume and total surface area of the solid.

Sol.



Given : Radius of cylinder $(r) = \frac{7}{2}$ cm

height of cylinder $(h) = 19 - 2 \times \frac{7}{2}$

$= 12$ cm

volume of solid =
volume of cylinder + volume of 2 hemispheres

$= \pi r^2 h + 2\left(\frac{2}{3}\pi r^3\right)$

$= \pi \times \left(\frac{7}{2}\right)^2 \times 12 + \frac{4\pi}{3}\left(\frac{7}{2}\right)^3$

$= 147\ \pi + 57.16\ \pi$

$= 204.16\ \pi$

$= 204.16 \times 3.14$

$= 641.67\ cm^3$.

Total surface area = curved surface area of cylinder + surface area of two hemispheres

$= 2\pi rh + 2(2\pi r^2)$

$= 2\pi r(h + 2r)$

$= 2\pi \times \frac{7}{2}(12 + 7)$

$= 2 \times \frac{22}{7} \times \frac{7}{2}(12 + 7)$

$= \frac{44}{7} \times \frac{7}{2}(12 + 7)$

$= 22 \times 19\ cm^2$

$= 418\ cm^2$. **Ans.**

6. Find the volume of a Gulab jamun in the form of a right

circular cylinder with hemispherical ends whose total length is 2.7 cm and the diameter of each hemispherical end is 0.7 cm.

Sol. Diameter of cylinder = 0·7 cm

Radius of cylinder = 0·7 ÷ 2

= 0·35 cm

Total height = 2·7 cm

Height of cylinder

= 2·7 – (2 × 0·35)

= 2 cm

Volume of cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 0{\cdot}35 \times 0{\cdot}35 \times 2$

= 0·77 cm^2

Volume of hemisphere = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (0{\cdot}35)^3$

= 0·09 cm^3

Volume of gulab jamun

= Volume of cylinder + Volume of 2 hemispheres

0·77 + (2 × 0·09)

= 0·95 cm^3. **Ans.**

7. The difference between outside and inside surface areas of cylindrical metallic pipe 14 cm long is 44 cm^2. If the pipe is made of 99 cm^3 of metal, find the outer and inner radii of the pipe.

Sol. Given : Height of metallic pipe = 14 cm. Difference between outside and inside surface areas = 44 cm^2

Difference between outside and inside curved surface area of cylinder

$= 2\pi Rh - 2\pi rh$

$44 \text{ cm}^2 = 2\pi h(R - r)$

$R - r = \frac{44 \times 7}{2 \times 22 \times 14}$

$R - r = 0{\cdot}5$ .... (1)

Given : The pipe is made of metal = 99 cm^3

Volume of cylindrical metallical pipe = $\pi R^2 h - \pi r^2 h$

$99 = \pi h(R^2 - r^2)$

$R^2 - r^2 = \frac{99 \times 7}{22 \times 14}$

$(R - r)(R + r) = 2{\cdot}25$

$0{\cdot}5(R + r) = 2{\cdot}25$

$R + r = \frac{2{\cdot}25}{0{\cdot}5}$

$R + r = 4{\cdot}5$ ......(2)

On adding equation (1) and (2) we get,

$2R = 5;\ R = 2{\cdot}5$

On substituting value of R in equation (1) we get

$2{\cdot}5 - r = 0{\cdot}5$

$r = 2{\cdot}5 - 0{\cdot}5$

$r = 2.$

Hence, outer radius = 2·5 cm and inner radius = 2 cm. **Ans.**

8. A solid iron pole having a cylindrical portion 110 cm high and of base diameter 12 cm is surmounted by a cone 9 cm high. Find the mass of the pole, given that mass of 1 cm^3 of iron is 8 g.

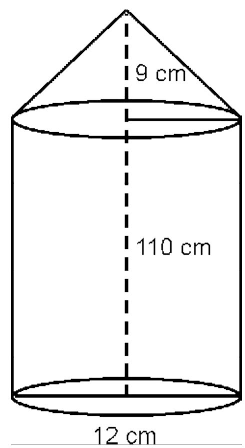
Sol. Given : Height of cylindrical portion (h) = 110 cm

Base diameter of cylindrical portion = 12 cm

∴ Radius = 12 ÷ 2

= 6 cm

Height of cone = 9 cm



Radius of base of cone

= Radius of base of cylindrical portion = 6 cm

Volume of solid = volume of cylindrical portion + volume of conical portion

$\Rightarrow \pi r^2 h + \frac{1}{3}\pi r^2 H$

$\Rightarrow \pi r^2 \left(h + \frac{H}{3}\right)$

$\Rightarrow \frac{22}{7} \times 6 \times 6 \left(110 + \frac{9}{3}\right)$

$\Rightarrow \frac{22}{7} \times 6 \times 6 \times 113$

Mass of pole = volume × mass

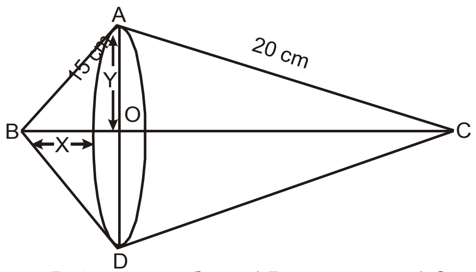
$= \frac{22}{7} \times 6 \times 6 \times 113 \times \frac{8}{1000}$ kg

$= \frac{715968}{7000}$

= 102·281 kg. **Ans.**

9. A right-angled triangle, whose sides are 15 cm and 20 cm, is made to revolve about its hypotenuse. Find the volume and the surface area of the double cones so formed. (π = 3·14)

Sol. When a right angled triangle is revolved around its hypotenuse, a double cone is formed with same radius but with different heights.



It is given that $AB = 15$ cm, $AC = 20$ cm

Let $OB = x$ and $OA = y$

In right angle triangle ABC

$BC^2 = AC^2 + AB^2$

$BC^2 = 20^2 + 15^2$

$BC^2 = 400 + 225$

$BC = \sqrt{625}$

$BC = 25$ cm

In ΔOAC

$AC^2 = OA^2 + OC^2$

$20^2 = Y^2 + (BC - OB)^2$

$400 = Y^2 + (25 - X)^2$

$400 = Y^2 + 625 - 50X + X^2$

$400 = 15^2 + 625 - 50X$

$50X = 450$

$X = 9$ cm

In ΔOAB

$AB^2 = OA^2 + OB^2$

$15^2 = X^2 + Y^2$ ....(1)

From equation (1)

$15^2 = 9^2 + y^2$

$y^2 = 225 - 81$

$y^2 = 144$

$y = 12$ cm

Also, $OC = 25 - X$

$= 25 - 13$

$= 12$ cm^2

Now, volume of cone,

$v = 1/3\ \pi r^2 h$

Hence, volume of double cone

$= 1/3\ \pi (OA)^2 \times BO + 1/3\ \pi (OA)^2 \times OC$

$= 1/3\ \pi (12)^2 \times (OB + OC)$

$= 1/3 \times 3·14 \times 144 \times 25$

$= 3768$ cm^3.

Curved surface area of cone = πrl

$= 3·14 \times 12 \times (15 + 20)$

$= 1318·8$ cm^2. **Ans.**

10. A blacksmith, Sujan made a pillar which consists of a cylindrical portion 2·8 m high and 20 cm in diameter and a cone 4·2 cm high is surmounting it. Find the weight of the pillar, if 1 cm^3 of iron weighs 7·5 grams.

Sol. Given : Height of cylindrical portion (H) = 2·8 m

= 280 cm

Diameter of cylindrical portion

= 20 cm

∴ Radius of cylindrical portion

= 20 ÷ 2

= 10 cm.

Height of cone (h) = 4·2 cm

Volume of pillar = Volume of cylinder + Volume of cone

$= \pi r^2 H + \frac{1}{3} \pi r^2 h$

$= \pi r^2 \left(H + \frac{h}{3}\right)$

$= \frac{22}{7} \times 10 \times 10 \times \left(280 + \frac{4·2}{3}\right)$

$= 88440$ cm^3.

Weights $= 88440 \times 7·5$

$= 663300$ g

$= 663·3$ kg. **Ans.**

11. A rocket which was prepared is in the form of a cylinder closed at the lower end with a cone of the same radius attached to the top. The cylinder is of the radius 2·5 m and height 21 m and the cone has the slant height 8 m. Calculate the volume of the rocket. $\left(\sqrt{231} = 15{\cdot}1987\right)$.

Sol. Curved surface area of cone = πrl

$= \frac{22}{7} \times 2{\cdot}5 \times 8$

$= 62{\cdot}86 \text{ m}^2$.

Curved surface area of cylinder

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 2{\cdot}5 \times 21$

$= 330 \text{ m}^2$.

Area of base circle of cylinder = πr^2

$= \frac{22}{7} \times 2{\cdot}5 \times 2{\cdot}5$

$= 19{\cdot}64 \text{ m}^2$.

Total surface area of the rocket = $62{\cdot}86 + 330 + 19{\cdot}64$

$= 412{\cdot}5 \text{ m}^2$

Volume of cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 2{\cdot}5 \times 2{\cdot}5 \times 21$

$= 412{\cdot}5 \text{ m}^3$

Now, $h^2 = 8^2 - (2{\cdot}5)^2$

$h^2 = 64 - 6{\cdot}25$

$h^2 = 57{\cdot}75$

$h = \sqrt{57{\cdot}75}$

$h = 7{\cdot}6$ m.

Volume of the cone = $1/3\ \pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 2{\cdot}5 \times 2{\cdot}5 \times 7{\cdot}6$

$= 49{\cdot}76 \text{ m}^3$.

Total volume of rocket = $412{\cdot}5 + 49{\cdot}76$

$= 462{\cdot}26 \text{ m}^3$. **Ans.**

12. An iron pillar has some part in the form of a right circular cylinder and remaining in the form of a right circular cone. The radius of the base of each of cone and cylinder is 8 cm. The cylindrical part is 240 cm high and the conical part is 36 cm high. Find the weight of the pillar, if one cubic cm of iron weight 7·8 grams.

Sol. The volume of the cylindrical part

$= \pi r^2 h$

$= \frac{22}{7} \times 8 \times 8 \times 240$

$= 48274{\cdot}29 \text{ cm}^3$.

The volume of the conical part = $\frac{1}{3} \pi \times$ base $\times$ area $\times$ height

$= \frac{1}{3} \times \frac{22}{7} \times 8 \times 8 \times 36$

$= 2413{\cdot}71 \text{ cm}^3$.

Total volume $= 48274{\cdot}29 + 2413{\cdot}71$

$= 50688 \text{ cm}^3$

Mass = Density × Volume

$= 7{\cdot}8 \text{ g} \times 50688$

$= 395366{\cdot}40$ grams

$= 395{\cdot}36$ kg. **Ans.**

13. A vessel is a hollow cylinder fitted with a hemispherical bottom of the same base. The depth of the cylinder is $4\frac{2}{3}$ m and the diameter of hemisphere is 3·5 m. Calculate the volume and the internal surface area of the hollow cylinder.

Sol. Given : Diameter of the hemisphere

= 3.5 m

Radius of the hemisphere = 3.5/2

= 1.75 m

Height of the hemisphere, h = 14/3 m

Total volume of the vessel = Volume of the cylinder + Volume of the hemisphere

$V = \pi r^2 h + \frac{2}{3} \pi r^3$

$V = \pi r^2 \left(h + \frac{2}{3} r\right)$

$V = \frac{22}{7} \times (1{\cdot}75)^2 \left(14/3 + \frac{2}{3} \times 1.75\right)$

$V = 67.375/7\ (14/3 + 3.5/3)$

$V = 9.625\ (17.5/3)$

$V = 168.4375/3$

$V = 56.15 \text{ m}^3$.

Internal surface area of hollow cylinder

= Curved surface area of cylinder + Curved surface area of hemisphere

$= 2\pi rh + 2\pi r^2$

$= 2\pi r\ (h + r)$

$= 2 \times \frac{22}{7} \times 1.75\ (14/3 + 1.75)$

$= 2 \times \frac{22}{7} \times 1.75\ [(14 + 15.25)/3]$

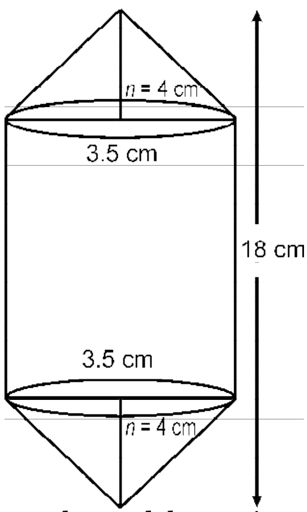
$= 44 \times 0.25\ [19.25/3]$

$= 11 \times 6.416$

$= 70.58 \text{ m}^3$ or $70\frac{7}{12} \text{ m}^3$.

Hence, The volume of the vessel $= 56 \text{ m}^3$ and, The internal surface area of the hollow cylinder = 70.58 m^3 or $70\frac{7}{12} \text{ m}^3$. **Ans.**

14. Kartik, an engineering student was asked to make a model in his workshop, which was shaped like a cylinder with two equal cones attached at its two ends; the diameter of the model is 3.5 cm and its length is 18 cm, if each cone has a height of 4 cm. Find the volume of the model that kartik made.



Sol. Since the model contains cylinder and 2 cones

Volume of air contained in model

= Volume of model

= Volume of cylinder + Volume of two cones

Volume of model

$= \pi r^2 h + 2\left(\frac{1}{3}\pi r^2 h\right)$

$= \frac{22}{7} \times 1{\cdot}75 \times 1{\cdot}75 \times 10 + 2\left(\frac{1}{3} \times \frac{22}{7} \times 1{\cdot}75 \times 1{\cdot}75 \times 4\right)$

$= \frac{22}{7} \times 1{\cdot}75 \times 1{\cdot}75\left(10 + \frac{2}{3} \times 4\right)$

$= 9{\cdot}625 \times 12{\cdot}67$

$= 121{\cdot}92 \text{ cm}^3$. **Ans.**

15. A wooden toy is in the form of a cone mounted on a hemisphere with the same radius. The diameter of the base of the conical portion is 6 cm and its height is 4 cm. Determine the volume of the toy. Also find the mass if the mass of 1 cm³ of wood is 3.5 g.

Sol. Given : Radius of cone = Radius of hemisphere

$\Rightarrow 6 \div 2 = 3$ cm.

Height of conical portion = 4 cm

Volume of cone = volume of cone + volume of hemisphere

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2\ [h + 2r]$

$= \frac{1}{3} \times \frac{22}{7} \times 3 \times 3\ (4 + 2 \times 3)$

$= \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 10$

$= 94{\cdot}29 \text{ cm}^3$.

Mass = volume × mass per cubic cm

$= 94{\cdot}29 \times 3{\cdot}5$

= 330 gm. **Ans.**

16. From a solid cylinder whose height is 8·2 cm and diameter 3·5 cm, a conical cavity of the same height and same diameter is hollowed out. Find the volume of the remaining solid to the nearest cm³.

Sol. Given : Height of cylinder = 8·2 cm

Diameter of cylinder = 3·5 cm

∴ Radius of cylinder = 3·5 ÷ 2

= 1·75 cm

Volume of remaining part

= volume of cylinder – volume of conical part

$= \pi r^2 h - \frac{1}{3}\pi r^2 h$

$= \pi r^2 h \left[1 - \frac{1}{3}\right]$

$= \frac{22}{7} \times 1{\cdot}75 \times 1{\cdot}75 \times 8{\cdot}2 \times \frac{2}{3}$

$= 52{\cdot}62$ cm^3. **Ans.**

17. Building is in the form of a cylinder surmounted by a hemispherical vaulted dome and contains $41\frac{19}{21}$ m^3 of air. If the internal diameter of the building is equal to its total height above the floor, find the height of the building.

Sol. Given : Volume of air

$= 41\frac{19}{21}$

$= \frac{880}{21}$ m^3

Internal diameter (d) = H

Internal diameter = 2r = H

Total Height of the building

H = 2r ...(1)

Height of the building =

Height of the cylinder + Radius of the Hemisphere dome

$H = h + r$

$2r = h + r$

(from equation) 1)

$h = 2r - r$

$h = r$ ...(2)

volume of air inside the building = volume of cylindrical portion + volume of hemispherical portion

$= \pi r^2 h + \frac{2\pi r^3}{3} = \frac{880}{21}$

$= \pi h^2 h + \frac{2\pi h^3}{3} = \frac{880}{21}$

$= \pi h^3 + \frac{2}{3}\pi h^3 = \frac{880}{21}$

$= \pi h^3 \left(1 + \frac{2}{3}\right) = \frac{880}{21}$

$= \pi h^3 \times \frac{5}{3} = \frac{880}{21}$

$= h^3 = \frac{880 \times 3 \times 7}{21 \times 5 \times 22}$

$= h^3 = 8$

$= h = 2.$

∴ $h = r = 2$m

Total height of the building

H = 2r; $h = 2 \times 2$

= 4 m

∴ Height = 4 m.

18. A carpenter, Abdullah prepared a model which is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm and the diameter of the cylinder is 7 cm. Find the volume and the surface area of the solid.

Sol. Given : Diameter of base = 7 cm

∴Radius of base = 7 ÷ 2

= 3·5 cm.

Total height = 19 cm

∴ Height of cylinder

= 19 – (3·5 × 2)

= 12 cm.

Total volume = volume of cylinder + volume of two hemispheres

$= \pi r^2 h + 2 \times \left(\frac{2}{3}\pi r^3\right)$

$= \pi r^2 \left(h + \frac{4r}{3}\right)$

$= \frac{22}{7} \times 3{\cdot}5 \times 3{\cdot}5 \left(12 + \frac{4}{3} \times 3{\cdot}5\right)$

$= \frac{22}{7} \times 3{\cdot}5 \times 3{\cdot}5 \times \frac{50}{3}$

$= 641{\cdot}67$ cm^3.

Surface area of solid

= Surface area of cylinder + Surface area of two hemispheres

$= 2\pi r\text{h} + 2(2\pi r^2)$

$= 2\pi r\,(h + 2r)$

$= 2 \times \frac{22}{7} \times 3{\cdot}5\,(12 + 2 \times 3{\cdot}5)$

$= 2 \times \frac{22}{7} \times 3{\cdot}5 \times 19$

$= 418\ cm^2$. **Ans.**

19. A toy is in the form of a right circular cylinder with a hemisphere on one end and a cone on other end. The height and the radius of base of the cylindrical part 15 cm and 7 cm respectively. The radius of hemisphere and the base of the conical part are same as that of the cylinder. Calculate the volume of the toy, if the height of the cone is 14 cm.

Sol. Given : Height of cylindrical part (H) = 15 cm

Radius of base (r) = 7 cm

Height of cone (h) = 14 cm

Volume of toy = Volume of hemisphere + Volume of cylinder + Volume of cone

$= \frac{2}{3}\pi r^3 + \pi r^2 H + \frac{1}{3}\pi r^2 h$

$= \pi r^2 \left[\frac{2r}{3} + H + \frac{1h}{3}\right]$

$= \frac{22}{7} \times 7 \times 7 \left[\frac{2}{3}\times 7 + 15 + \frac{1}{3}\times 14\right]$

$= \frac{22}{7} \times 7 \times 7 \left[\frac{14}{3} + 15 + \frac{14}{3}\right]$

$= \frac{22}{7} \times 7 \times 7 \times \frac{73}{3}$

$= 3747{\cdot}33\ cm^3$. **Ans.**

20. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 15 mm and the diameter of the capsule is 7 mm. Find the volume of medicine contained in it.

Sol. Given : Length of entire capsule = 15 mm

$\therefore$ Length of cylinder (h) $= 15 - (3{\cdot}5 \times 2)$

$= 8$ mm

Diameter of base = 7 mm

$\therefore$ Radius of base $= 7 \div 2$

$= 3{\cdot}5$ mm

Volume of medicine contained in capsule

= Volume of cylinder + Volume of two hemispheres

$= \pi r^2 h + 2 \times \left(\frac{2}{3}\pi r^3\right)$

$= \pi r^2 \left[h + \frac{4}{3} r\right]$

$= \frac{22}{7} \times 3{\cdot}5 \times 3{\cdot}5 \left[8 + \frac{4}{3} \times 3{\cdot}5\right]$

$= 38{\cdot}5 \times 12{\cdot}67$

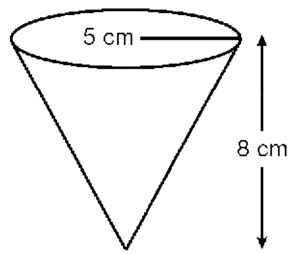
$= 487{\cdot}79\ mm^3$.

21. A vessel in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filed with water up to the rim. When lead shots, each of which is a sphere of radius 0·5 cm are dropped to the vessel, one fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Sol. Number of lead shots

$= \frac{\text{Volume of water flows out}}{\text{Volume of one lead shot}}$

Volume of water flows out $= \frac{1}{4}$ Volume of cone



$= \frac{1}{4} \times \frac{1}{3} \times \pi r^2 h$

$= \frac{1}{4} \times \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 8$

$= \frac{1}{4} \times \frac{4400}{21} = \frac{1100}{21}\ cm^3$.

Volume of one head shot = volume of sphere

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (0{\cdot}5)^3$

$= \frac{11}{21}$ cm^3

Number of lead shots

$= \frac{1100}{21} \div \frac{11}{21}$

$= \frac{1100}{21} \times \frac{21}{11}$

= 100

∴ Hence these is 100 such lead shots.

EXERCISE 13.4

1. Metallic spheres of radii 6 cm, 8 cm and 10 cm respectively are melted to form a single solid sphere. Find the radius of the resulting sphere.

Sol. Given : Radius of spheres = 6 cm, 8 cm and 10 cm.

Volume of sphere $= \frac{4}{3}\pi r^3$

Volume of all spheres = Volume of metal = Volume of sphere one + Volume of sphere two + volume of sphere 3.

$\frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$

$\frac{4}{3}\pi [r_1^3 + r_2^3 + r_3^3]$

$\frac{4}{3} \times \pi [6^3 + 8^3 + 10^3]$

$\frac{4}{3} \times \pi [216 + 512 + 1000]$

$\frac{4}{3} \times \pi \times 1728$

$= 2304\ \pi$ cm^3.

∴ Volume of new single solid sphere $= 2304\pi$

$\frac{4}{3}\pi r^3 = 2304\pi$

$r^3 = \frac{2304 \times \pi \times 3}{\pi \times 4}$

$r^3 = 1728$

$r = 12.$

Hence, Radius of resulting sphere = 12 cm.

2. The radii of the internal and external surfaces of a hollow spherical shell are 3 cm and 5 cm respectively. If it is melted and recast into a solid cylinder of height 22/3 cm, find the diameter of the cylinder.

Sol. Radius of the external surface(R) = 5 cm

Radius of the internal surface(r) = 3 cm

Volume of the sphere = Volume of cylinder

$= \frac{4}{3}\pi (R^3 - r^3) = \pi r^2 h$

$= \frac{4}{3}\pi [(5)^3 - (3)^3] = r^2 \times \frac{22}{3}$

$= \frac{4}{3}(125 - 27) = r^2 \times \frac{22}{3}$

$= \frac{4}{3}(98) = r^2 \times \frac{22}{3}$

$= \frac{392}{3} = r^2 \times \frac{22}{3}$

$r^2 = \frac{392 \times 3}{22 \times 3}$

$r^2 = 17.81$

$r = \sqrt{17{\cdot}81}$

$r = 4.22$

Diameter of the cylinder = 2 r

= 2 × 4.22

= 8.4 m. **Ans.**

3. A solid sphere of radius 'r' melted and recast into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 4 cm, its height is 24 cm and thickness 2 cm. Find the value of 'r'.

Sol. Volume of sphere $= \frac{4}{3}\pi r^3$.

Volume of hollow cylinder = πh

$[R_1^2 - R_2^2]$

Solid sphere is melted and casted into hollow cylinder. Hence volume of sphere is equal to volume of cylinder.

$$\therefore \quad \frac{4}{3}\pi r^3 = \pi h\ [R_1^2 - R_2^2]$$

$$\frac{4}{3} \times \frac{22}{7} \times r^3 = \frac{22}{7} \times 24\ [4^2 - 2^2]$$

$$\frac{4}{3} \times r^3 = 24 \times 12$$

$$r^3 = \frac{24 \times 12 \times 3}{4}$$

$$r^3 = 216$$

$$r = 6 \text{ cm.}$$ **Ans.**

4. Water is flowing at the rate of 3 km/h through a circular pipe of 20 m internal diameter into a circular of diameter 10 m and depth 2m. In how much time, will the cistern be filled ?

Sol. The length of water pipe is 3 km = 3000 m.

The diameter of pipe is 20 cm = (20/100) m.

Radius of the pipe = 10 cm = (10/100) m.

Thus, the volume of water in pipe $= \pi r^2 h$

$$= \pi \times \left(\frac{10}{100}\right)^2 \times 3000$$

$\therefore$ Volume = 30π m^3.

Thus 1 m water can be filled in

$$\text{cistern in} = \left(\frac{60}{30\pi}\right) \text{ minutes}$$

$$= \frac{2}{\pi} \text{ minutes}$$

Volume of cistern = $\pi \times 5 \times 5 \times 2$ = 50π.

Hence, it take time = $50 \times \pi \times \frac{2}{\pi}$

= 100 minutes

= 1 hours 40 minutes. **Ans.**

5. Water flows at rate of 2 km/h through a circular pipe of 14 cm internal diameter into a cistern 1·1 × 3·5 × 4 m^3. How long would it take to fill the cistern ?

Sol. The radius of the pipe

= 14 ÷ 2

= 7 cm

The area of the mouth of pipe

$= \pi r^2$

= 3·14 × 7 × 7

= 153·86 cm^2

The Volume of the tank

= 1·1 × 3·5 × 4

= 15·4 m^3

= 15400000 cm^3

Volume of water coming out of pipe

= 153·86 × x

$\therefore$ 153·86 × x = 15400000

x = 100091 cm

water flows at the rate

= 2km/h

$\Rightarrow$ 200000 cm in 1 hour

$$\therefore \text{Time taken} = \frac{100091}{200000}$$

= 0·500 hour

= 30 minutes **Ans.**

6. Water flows at the rate of 10 metres per minute through a cylindrical pipe 5 mm in diameter. How long would it take to fill a conical vessel having base diameter of 42 cm and depth of 24 cm.

Sol. Radius of the pipe = 5 ÷ 2

= 2·5 mm × 1/10 cm = 0·25 cm.

Speed of water = 10 m/min.

= 1000 cm/min.

Volume of water that flows in 1 minutes $= \pi r^2 h$

$$= \frac{22}{7} \times 0{\cdot}25 \times 0{\cdot}25 \times 1000$$

$$= \left(\frac{1375}{7}\right) \text{cm}^3$$

Radius of conical vessel = 42 ÷ 2 = 21 cm

Depth of conical vessel = height = 24 cm

Capacity of vessel = volume = 1/3 $\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 21 \times 21 \times 24$

$= 11088 \text{ cm}^3$

Therefore, Time required to fill the vessel =

$$\frac{\text{Capacity of vessel}}{\text{Volume of water flowing per minute}}$$

$$= \frac{11088}{\frac{1375}{7}}$$

$$= \frac{11088 \times 7}{1375}$$

= 56·448 minutes **Ans.**

7. A sphere of radius 4·2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Sol. Given : Radius of sphere = 4·2 cm

Radius of cylinder = 6 cm

Volume of sphere = Volume of cylinder

$$\frac{4}{3}\pi r^3 = \pi r^2 h$$

$$\frac{4}{3} \times \frac{22}{7} \times 4{\cdot}2 \times 4{\cdot}2 \times 4{\cdot}2$$

$$= \frac{22}{7} \times 6 \times 6 \times h$$

$$h = \frac{4 \times 4{\cdot}2 \times 4{\cdot}2 \times 4{\cdot}2}{3 \times 6 \times 6}$$

$h = 2{\cdot}744$ cm.

Hence, height of cylinder = 2·744 cm

8. A copper sphere of radius 3 cm is melted and recast into a right circular cone of height 3 cm. Find the radius of the base of the cone.

Sol. Given : Radius of sphere = 3 cm

Height of cone = 3 cm

Volume of sphere = Volume of cone

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\pi r^2 h$$

$$\frac{4}{3} \times \frac{22}{7} \times 3 \times 3 \times 3$$

$$= \frac{1}{3} \times \frac{22}{7} \times r^2 \times 3$$

$$r^2 = \frac{4 \times 22 \times 3 \times 3 \times 3 \times 3 \times 7}{3 \times 7 \times 22 \times 3}$$

$r^2 = 36$

$r = \sqrt{36}$

$r = 6$ cm.

Hence, radius of cone = 6 cm.

9. A solid metalic sphere of diameter 28 cm is melted and recast into a number of smaller cones, each of diameter $4\frac{2}{3}$ cm and height 3 cm. Find the number of cones so formed.

Sol. Given : Diameter of sphere = 28 cm

∴Radius of sphere (R) = 28 ÷ 2

= 14 cm

Diameter of each cone

$$= 4\frac{2}{3}$$

$$= \frac{14}{3} \text{ cm}$$

∴ Radius of each cone $(r) = \frac{14}{3} \div \frac{2}{1}$

$$= \frac{14}{3} \times \frac{1}{2}$$

$$= \frac{7}{3} \text{ cm.}$$

Let no. of cones = N

Volume of all cones = Volume of sphere

$$N\left[\frac{1}{3}\pi r^2 h\right] = \frac{4}{3}\pi R^3$$

$$N(r^2 h) = 4R^3$$

$$N = \frac{4R^3}{r^2 h}$$

$$N = \frac{4 \times 14 \times 14 \times 14}{\frac{7}{3} \times \frac{7}{3} \times 3}$$

$N = 672.$

Hence, No. of cones = 672 cones.

10. How many coins 1·75 cm in diameter and 2mm thick must be melted to form a cuboid 11cm × 10 cm × 7 cm ?

Sol. Diameter of coin = 1·75 cm

∴ Radius of coins = 1·75 ÷ 2 = 0·875 cm

Height of coin = 2 mm = 0·2 cm

Volume of one coin = $\pi r^2 h$

$$= \frac{22}{7} \times 0{\cdot}875 \times 0{\cdot}875 \times 0{\cdot}2$$

$$= 0{\cdot}48125 \text{ cm}^3$$

Volume of cuboid

= 11 cm × 10 cm × 7 cm

Let the number of coins = n

Volume of n coins

= Volume of cuboid

$$n \times 0{\cdot}48125 = 11 \times 10 \times 7$$

$$n = \frac{11 \times 10 \times 7}{0{\cdot}48125}$$

$$n = 1600.$$

∴ No. of coins = 1600 coins.

Ans.

11. A rectangular water tank of base 11m × 6m contains water upto a height of 5m. If the water in the tank is transferred into a cylindrical tank of radius 3·5m, find how high will the water level be in this tank.

Sol. Water level = 5m

∴ height = 5m

Volume of tank = $l \times b \times h$

= 5 × 6 × 11

= 330m^3

Volume of cylindrical tank will be Volume of cuboid tank = 330

$$\pi r^2 h = 330$$

$$\frac{22}{7} \times 3{\cdot}5 \times 3{\cdot}5 \times h = 330$$

$$h = \frac{330 \times 7}{3{\cdot}5 \times 3{\cdot}5 \times 22}$$

$$h = 8{\cdot}57\text{m}$$

∴ The water level in the tank will be 8·57m high.

12. A cylindrical vessel, having diameter equal to its height, is full of milk. It is poured into two identical cylindrical vessels with diameter 42 cm and height 21 cm. These vessels are filled completely. Find the diameter of the cylindrical vessel.

Sol. Given :

Diameter of each of cylindrical vessels = 42 cm

∴Radius of each vessel

= 42 ÷ 2

= 21 cm

Height of cylindrical vessel

= 21 cm

∴ Diameter of big cylindrical vessel = height of cylindrical vessel

Let diameter and height of cylindrical vessel = x

∴ Radius of big cylindrical vessel

= $x/2$

Volume of big cylindrical vessel

= Volume of 2 vessels

$$\pi R^2 h = \pi r^2 h + \pi r^2 h$$

$$\pi R^2 h = 2\pi r^2 h$$

$$\pi \times \frac{x}{2} \times \frac{x}{2} \times x$$

$$= 2 \times \pi \times 21 \times 21 \times 21$$

$$x^3 = 4 \times 2 \times 21 \times 21 \times 21$$

$$x = \sqrt[3]{2 \times 2 \times 2 \times 21 \times 21 \times 21}$$

$$x = 2 \times 21$$

$$x = 42$$

Hence, diameter and height of cylindrical vessel

= 42 cm. **Ans.**

13. A cylindrical tub of radius 5 cm and length 9·8 cm is full of water. A solid in the form of a right circular cone of height 5 cm mounted on a hemisphere of common diameter 5 cm is immersed into the tub, find the volume of water left in the tub.

Sol. Given :

Radius of cylindrical tub = 5 cm

Length of cylindrical tub height = 9·8 cm

Height of circular cone = 5 cm

Diameter of hemisphere cone = 5 cm

∴Radius of hemisphere cone 5 ÷ 2
= 2·5 cm

Volume of cylindrical tub = $\pi r^2 h$

$= \frac{22}{7} \times 5 \times 5 \times 9{\cdot}8$

= 770 cm^3

Volume of hemisphere = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 2{\cdot}5 \times 2{\cdot}5 \times 2{\cdot}5$

= 32·74 cm^3

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 2{\cdot}5 \times 2{\cdot}5 \times 5$

= 32·74 cm^3

Volume of solid = Volume of hemisphere + Volume of cone
= 32·74 + 32·74
= 65·48 cm^3

Volume of water left = Volume of cylindrical tub – Volume of solid
= 770 – 65·48
= 704·52 cm^3. **Ans.**

14. Water in a canal 1·5m wide and 6m deep is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes if 8 cm of standing water is desired.

Sol. Canal is a shape of cuboid where Breadth = 6m and Height = 1·5m

Speed of canal = 10 km/h

length of canal in 1 hour = 10 km

length of canal in 60 minutes
= 10 km

length of canal in 1 minutes

$= \left(\frac{1}{60} \times 10\right)$ km

length of canal in 30 minutes

$= \frac{30}{60} \times 10$

= 5 km

5 km = 5000m

Volume of canal
= Length × Breadth ×Height
= (5000 × 6 × 1·5) m^3

Volume of water in canal
= Volume of area irrigated × Height

5000 × 6 ×1·5 = Area irrigated × 8

Area irrigated = $\frac{5000 \times 6 \times 1{\cdot}5}{8}$

Area irrigated = 562500 m^2.

15. A 20 m deepwell with diameter 14 m is dug up and the earth from digging is evenly spread out to form a platform 20 m × 14 m. Find the height of the platform.

Sol. Diameter of the well = 14 m

∴ Radius of the well = $\frac{14}{2}$
= 7 cm.

Depth of the well = 20 m

Volume of the mud dug from the well = $\pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 20$

= 3080 m^3

Area of the rectangular platform where the mud spread = 20m ×14m

Let the height of the platform h metres.

Volume of the platform = Volume of the mud dug from the well

20 × 14 × h = 3080

$h = \frac{3080}{20 \times 14}$

$h = \frac{3080}{280}$

h = 11 m. **Ans.**

16. A well with inner radius 4 m is dug 14 m deep. Earth taken out of it has been spread evenly all around to a width of 3 m to form an embankment. Find the height of the embankment.

Sol. The well is cylindrical.

So, Volume of well = $\pi r^2 h$

Volume = $\frac{22}{7} \times 4 \times 4 \times 14$

= 704 m^3

The width of embankment is 3m

Volume of soil = 704 m^3

$\pi h\,(R^2 - r^2) = 704$

$\frac{22}{7} \times h\,(7^2 - 4^2) = 704$

$h = \frac{704 \times 7}{22 \times 33}$

h = 6·79 m. **Ans.**

17. A well with 10m inside diameter is dug 14m deep. Earth taken out of it and spread all around to a width of 5m to form an embankment. Find the height of the embankment.

Sol. Given, Radius of the well = 5 m

Height of the well = 14 m

Width of the embankment = 5 m

Therefore, radius of embankment

= 5 + 5

= 10 m

Volume of well = Volume of embankment

$\pi h\ (R^2 - r^2) = \pi r^2 h$

$\frac{22}{7} \times h\ (10^2 - 5^2)$

$= \frac{22}{7} \times 5 \times 5 \times 14$

$h = \frac{22 \times 5 \times 5 \times 14 \times 7}{22 \times 7 \times 75}$

$h = 4{\cdot}67$m. **Ans.**

18. The rain water from a roof 22 m × 20 m drains into a cylindrical vessel having diameter of base 2 m and height 3·5 m. If the vessel is just full find the rainfall in cm.

Sol. Given :

Height of cylindrical vessel = 3.5 m

Diameter of cylindrical vessel = 2m

∴ Radius of cylindrical vessel

= 2 ÷ 2

= 1 cm

length of roof = 22 m

width of roof = 20 m

Volume of rainfall on roof

= Volume of tank

length × breadth × height = $\pi r^2 h$

$2 \times 22 \times h = \frac{22}{7} \times 1 \times 1 \times 3{\cdot}5$

$h = \frac{22 \times 1 \times 1 \times 3{\cdot}5}{7 \times 2 \times 22}$

$h = 0{\cdot}25$m

$= 2{\cdot}5$ cm. **Ans.**

19. A solid iron rectangular block of dimension 4·4m, 2·6 m and 1 m is cast into a hollow cylindrical pipe of internal radius 30 cm and thickness 5 cm. Find the length of the pipe.

Sol. Volume of iron

= (440 × 260 × 100) cm^3

Internal radius of pipe (R) = 30 cm

External radius of pipe (r) = 5cm + 30cm = 35 cm

Let the length of the pipe be 'h' cm

Volume of iron in pipe = External Volume – Internal volume

$440 \times 260 \times 100 = \pi r^2 h - \pi R^2 h$

$440 \times 260 \times 100 = \pi h\ (r^2 - R^2)$

$440 \times 260 \times 100 = \frac{22}{7} \times h(35^2 - 30^2)$

$440 \times 260 \times 100 = h \times \frac{22}{7} \times (35 + 30)\ (35 - 30)$

$440 \times 260 \times 100 = h \times \frac{22}{7} \times 65 \times 5$

$h = \frac{440 \times 260 \times 100 \times 7}{65 \times 5 \times 22}$

= 11200 cm

= 112 m. **Ans.**

20. Water flows at the rate of 10m/min through a cylindrical pipe 5mm in diameter. How long would it take to fill a conical vessel whose diameter at the base of 40 cm and depth 24 cm ?

Sol. Volume of water flows = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 20 \times 20 \times 24$

$= 10057{\cdot}14$ cm^3.

Volume of water flows in 1 minute

$= \pi r^2 h$

$= \frac{22}{7} \times \frac{2{\cdot}5}{10} \times \frac{2{\cdot}5}{10} \times 1000$

$= 196{\cdot}43$ cm^3.

Therefore, time required to fill the vessel

$= \frac{10057{\cdot}14}{196{\cdot}42}$

= 51 min 12 sec. (Approx)

21. What length of a solid cylinder 2cm in diameter must be taken to recast into a pipe of length 16 cm, external diameter 20 cm and thickness 2·5 mm ?

Sol. Given :

Diameter of solid cylinder = 2 cm

Radius of solid cylinder

$= 2 \div 2$

$= 1$ cm

Volume of solid cylinder $= \pi r^2 h$

$= \pi \times 1 \times 1 \times h$

$= \pi h$

Length of hollow cylinder,

$(h) = 16$ cm

External diameter = 20 cm

$\therefore$ External radius $= 20 \div 2$

$= 10$ cm

Thickness = 2·5 mm

= 0·25 cm

Internal radius

$= 10 - 0{\cdot}25$

$= 9{\cdot}75$ cm

Volume of hollow cylinder

$= \pi (R^2 - r^2)\, h$

$= \pi [(10)^2 - (9{\cdot}75)^2] \times 16$

$= \pi \times 0{\cdot}25 \times 19{\cdot}75 \times 16$

$= 79\pi$

Volume of solid cylinder

= Volume of hollow cylinder

$\pi h = 79\pi$

$$h = \frac{79\pi}{\pi}$$

$h = 79$ cm. **Ans.**

22. Selvi's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from a sump (underground tank) which is in the shape of a cuboid. The sump has dimensions 1·57 m × 1·44 m × 95 cm. The overhead tank has its radius of 60cm and its height is 95 cm. Find the height of the water left in the sump after the overhead tank has been completely filled with water from a sump which had been full. Compare the capacity of the tank with that of the sump. (Use π = 3·14).

Sol. Volume of water left in cuboidal sump after filling the tank = Volume of cuboidal sump – Volume of cylindrical tank

Volume of cuboidal sump

Length (l) = 1·57 m

Breadth (b) = 1·44m

Height (h) = 95 cm

$$= 95 \times \frac{1}{100}$$

= 0·95m

Volume of sump

= length × Breadth × Height

$= 1{\cdot}57 \times 1{\cdot}44 \times 0{\cdot}95$

$= 2{\cdot}14776\ m^3$

Volume of overhead tank

Height of overhead tank = 95 cm

$$= 95 \times \frac{1}{100}$$

= 0·95m

Radius = 60 cm

$$= 60 \times \frac{1}{100}$$

= 0·60 m

Volume of overhead tank $= \pi r^2 h$

$$= \frac{22}{7} \times 0{\cdot}6 \times 0{\cdot}6 \times 0{\cdot}95$$

$= 1{\cdot}075 m^3$

Volume of water left in the cuboidal sump after filling the tank = Volume of cuboidal sump – Volume of cylindrical tank

$= 2{\cdot}147 - 1{\cdot}073$

$= 1{\cdot}073\ m^3$

Volume of water left in cuboidal sump = 1·073

Length × Breadth × Height

= 1·073

$1{\cdot}57 \times 1{\cdot}44 \times h = 1{\cdot}073$

$$h = \frac{1{\cdot}073}{1{\cdot}57 \times 1{\cdot}44}$$

$h = 0{\cdot}475$ m

$\therefore \quad h = 47{\cdot}5$ cm

Height of water left in sump

= 47·5 cm

Also,

$$\frac{\text{Capacity of tank}}{\text{Capacity of sump}}$$

$= \frac{\text{Volume of tank}}{\text{Volume of sump}}$

$= \frac{1{\cdot}073}{2{\cdot}147}$

$= \frac{1}{2}$. **Ans.**

23. A cone of height 24 cm and radius of base 6 cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere.

Sol. Height of cone = 24 cm

Radius of base = 6 cm

Volume of the cone

= Volume of the sphere

$$\frac{1}{3}\pi r^2 h = \frac{4}{3}\pi R^3$$

$$r^2 h = 4R^3$$

$$R^3 = \frac{r^2 h}{4}$$

$$R^3 = \frac{6 \times 6 \times 24}{4}$$

$$R^3 = 216$$

$$R = \sqrt[3]{216}$$

$R = 6$ cm. **Ans.**

24. A cylindrical vessel, 16 cm high and 9 cm of radius of the base, is filled with sand. This vessel is emptied on a marble floor and a conical heap is formed. If the height of conical heap is 12 cm. Find the radius and slant height of the heap.

Sol. Given :

Height of cylindrical vessel = 16 cm

Radius of cylindrical vessel = 9 cm

Height of conical heap = 12 cm

Volume of cylinder = Volume of cone

$$\pi r^2 h = 1/3\ \pi R^2 H$$

$$r^2 h = 1/3\ R^2 H$$

$$9 \times 9 \times 16 = \frac{1}{3} \times R^2 \times 12$$

$$R^2 = \frac{9 \times 9 \times 16 \times 3}{12}$$

$$R^2 = 324$$

$$R = \sqrt{324}$$

$R = 18$ cm.

Slant height

$$= \sqrt{R^2 + h^2}$$

$$= \sqrt{18^2 + 12^2}$$

$$= \sqrt{18 \times 18 + 12 \times 12}$$

$$= \sqrt{324 + 144}$$

$$= \sqrt{468}$$

$= 21{\cdot}63$ cm.

25. A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 18m of uniform thickness. Find the thickness of the wire.

Sol. Volume of copper rod = Volume of wire

Volume of copper rod

Copper rod is in form of cylinder with diameter of rod = 1 cm

So, radius $= \frac{1}{2}$ cm

And, length = height = 8 cm

Volume of copper rod $= \pi r^2 h$

$$= \pi \times \frac{1}{2} \times \frac{1}{2} \times 8$$

$= 2\pi \text{cm}^3$

Volume of wire

wire is form of cylinder with radius $= r$

length = height = h = 18m = 18 × 100 = 1800 cm

Volume of wire $= \pi r^2 h$

$= \pi \times r^2 \times 1800$

$= 1800\ \pi r^2\ \text{cm}^2$

Volume of copper rod

= Volume of wire

$$1800\ \pi r^2 = 2\pi$$

$$r^2 = \frac{1}{900}$$

$$r = \frac{1}{30}$$

Thickness of wire = Diameter of wire $= \frac{1}{30} \times 2$

$$= \frac{1}{15}$$

$= 0{\cdot}067$ mm.

EXERCISE 13.5

1. A friction clutch is in the form of the frustum of a cone, the radii of the ends being 8 cm. and 10 cm. and length 8 cm. Find its bearing surface and its volume.

Sol. Given that,

Radius 1 = 10 cm

Radius 2 = 8 cm

length = 8 cm

Bearing surface of frustum

$= \pi (r_1 + r_2)\, l$

$= 3{\cdot}14\ (10 + 8) \times 8$

$= 3{\cdot}14 \times 18 \times 8$

$= 452{\cdot}16 \text{ cm}^2$

Volume of frustum

$= \frac{\pi h}{3} (r_1^2 + r_2^2 + r_1 r_2)$

Height $= \sqrt{l^2 - (r_1 - r_2)^2}$

$= \sqrt{8^2 - (10 - 8)^2}$

$= \sqrt{64 - 2^2}$

$= \sqrt{64 - 4}$

$= \sqrt{60}$

$= 7{\cdot}75$ cm.

Volume $= \frac{3{\cdot}14}{3} \times 7{\cdot}75\ (10^2 + 8^2 + 10 \times 8)$

$= \frac{3{\cdot}14}{3} \times 7{\cdot}75\ (100 + 64 + 80)$

$= \frac{3{\cdot}14}{3} \times 7{\cdot}75 \times 244$

$= 1979{\cdot}25 \text{ cm}^3$. **Ans.**

2. The radii of the ends of a bucket, 30 cm high are 21 cm and 7 cm. Determine, the area of the sheet required to make this bucket.

Sol. Given : Radius of 1 bucket = 21 cm

Radius of 2 bucket = 7 cm

Height of bucket = 30 cm

Slant height $= \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{(30)^2 + (21 - 7)^2}$

$= \sqrt{900 + (14)^2}$

$= \sqrt{900 + 196}$

$= \sqrt{1096}$

$l = 33{\cdot}11$

Total surface area of sheet

$= \pi l\ (r_1 + r_2) + \pi r_2^2$

$= 3{\cdot}14 \times 33{\cdot}11\ (21 + 7) + 3{\cdot}14 \times 7 \times 7$

$= 2911{\cdot}0312 + 153{\cdot}86$

$= 3064{\cdot}8912 \text{ cm}^2$.

3. The radii of the ends of the bucket of height 24 cm are 15 cm and 5 cm respectively. Find its capacity.

Sol. Given : Height of bucket= 24 cm

Radius 1 of bucket = 15 cm

Radius 2 of bucket = 5 cm

Capacity of bucket

= Volume of bucket

$= \frac{1}{3} \pi h\ (r_1^2 + r_2^2 + r_1 r_2)$

$= \frac{1}{3} \times \frac{22}{7} \times 24\ (15^2 + 5^2 + 15 \times 5)$

$= \frac{1}{3} \times \frac{22}{7} \times 24 \times 325$

$= 8171{\cdot}43 \text{ cm}^3$. **Ans.**

4. A metallic right circular cone 20 cm high and whose vertical angle is 90° is cut into two parts at the middle point of its height by a plane parallel to the base. If the frustum so obtained be drawn into a wire of diameter 1/16 cm, find the length of the wire.

Sol. Let ABC be the metallic cone

AO = 20 cm

Cone is cut into two parts at the middle hence, height of frustum = 10 cm

So, each angle = 45°

$r_1 = 10 \qquad r_2 = 20$

$\therefore$ Volume of frustum =

$= \frac{\pi}{3} h(r_1^2 + r_2^2 + r_1 r_2)$

$= \frac{\pi}{3} \times 10\ (10^2 + 20^2 + 10 \times 20)$

$= \frac{\pi}{3} \times 10\ (100 + 400 + 200)$

$= \frac{\pi}{3} \times 10\ (700)$

$= \frac{7000\,\pi}{3}$

Let the length of the wire be l.
Given, the diameter of drawn wire from frustum is $\frac{1}{16}$ cm.

$\therefore$ Radius of wire = $\frac{1}{32}$ cm.

So, Volume of wire (cylinder)

$$= \pi\left(\frac{1}{32}\right)^2 \times l$$

$$= \frac{\pi l}{1024}$$

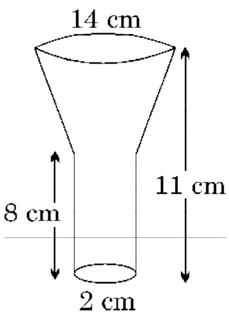
Now, according to the question,
Therefore, Volume of the metal used in wire = Volume of frustum

$$\pi \times \frac{l}{1024} = 7000\frac{\pi}{3}$$

$$l = \frac{7000 \times 1024}{3}$$

$$l = 23893{\cdot}33\text{m.}$$ **Ans.**

5. An oil funnel of tin sheet consists of a cylindrical portion 8 cm. long attached to a frustum of a cone. If the total height be 11 cm, the diameter of cylindrical portion 2 cm, and diameter of the top of the funnel 14 cm, find the area of the tin required.



Sol. Let R and r be the bigger and smaller ends of frustum, then

$$R = \frac{14}{2} = 7\text{m} \quad r = \frac{2}{2} = 1\text{cm.}$$

Let l and h be slant height and height of frustum

h = total height – height of cylindrical portion
= 11–8 = 3 cm

$$l = \sqrt{h^2 + (R-r)^2}$$

$$= \sqrt{3^2 + (7-1)^2}$$

$$= \sqrt{9 + 6^2}$$

$$= \sqrt{9 + 36}$$

$$= \sqrt{45}$$

$$= 6.71$$

Curved surface area of frustum

$$= \pi l\ (R + r)$$

$$= \frac{22}{7} \times 6.71(7+1)$$

$$= \frac{22}{7} \times 6.71 \times 8$$

$$= 168.70$$

Curved surface area of cylinder

$$= 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 1 \times 8$$

$$= 50.29$$

Total curved surface area

$$= 168.70 + 50.99$$

$$= 218.99$$

Area of the required tin

$$= 218.99 \text{ or } 219 \text{ cm}^2.$$

6. Find the volume of the frustum of cone, the areas of whose ends are 40 sq. metres and 10 sq. metres and height is 9 metres.

Sol. Area of cone $= \pi R^2$

$$40 = \pi R^2$$

$$R^2 = \frac{40 \times 7}{22}$$

$$10 = \pi r^2$$

$$r^2 = \frac{10 \times 7}{22}$$

$$R = \sqrt{\frac{40 \times 7}{22}}$$

$$r = \sqrt{\frac{10 \times 7}{22}}$$

Volume of frustum

$$= \pi\frac{h}{3}\ (R^2 + r^2 + Rr)$$

$$= \frac{22}{7} \times \frac{9}{3}\left[\left(\sqrt{\frac{40 \times 7}{22}}\right)^2 + \left(\sqrt{\frac{10 \times 7}{22}}\right)^2 + \sqrt{\frac{40 \times 7}{22}}\sqrt{\frac{10 \times 7}{22}}\right]$$

$$= \frac{22}{7} \times 3\left[\frac{40 \times 7}{22} + \frac{10 \times 7}{22} + \frac{20 \times 7}{22}\right]$$

$= \frac{22}{7} \times 3 \times \frac{490}{22}$

$= 210$ cm^3. **Ans.**

7. The diameter of the bottom of a frustum of right circular cone is 10 cm. and that of the top is 4 cm. and height is 4 cm. Find out the area of the total surface and the volume of the frustum.

Sol. Diameter of bottom of frustum = 10 cm

$\therefore$Radius of bottom of frustum (R)

$= 10 \div 2$

$= 5$ cm.

Diameter of top of frustum = 4 cm

$\therefore$ Radius of top of frustum (r)

$= 4 \div 2 = 2$ cm

Height of frustum (h) = 4 cm

Slant height of frustum (l)

$= \sqrt{h^2 + (R - r)^2}$

$= \sqrt{4^2 + (5 - 2)^2}$

$= \sqrt{16 + 3^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

$= 5$ cm.

Total surface area of frustum

$= \pi (R + r) l + \pi R^2 + \pi r^2$

$= \pi [(R + r) l + R^2 + r^2]$

$= \frac{22}{7} [(5 + 2) 5 + 5^2 + 2^2]$

$= \frac{22}{7} \times [7 \times 5 + 25 + 4]$

$= \frac{22}{7} \times [35 + 29]$

$= \frac{22}{7} \times 64$

$= 201{\cdot}14$ cm^2

Volume of frustum

$= \pi \times \frac{4}{3} (R^2 + r^2 + Rr)$

$= \frac{22}{7} \times \frac{4}{3} [5^2 + 2^2 + 5 \times 2]$

$= \frac{22}{7} \times \frac{4}{3} [25 + 4 + 10]$

$= \frac{22}{7} \times \frac{4}{3} \times 39 = 163{\cdot}43$ cm^3.

8. A bucket has top and bottom diameter of 40 cm and 20 cm respectively. Find the volume of the bucket, if its depth is 12 cm. Also, find the cost of tin sheet used for making the bucket at the rate of ₹ 1·20 per dm^2.

Sol. Top diameter of bucket = 40 cm

$\therefore$ Radius of bucket (R) = $40 \div 2$

= 20 cm

Bottom diameter of bucket = 20 cm

$\therefore$Radius of bucket = $20 \div 2$

= 10 cm

Height of bucket (H) = 12 cm

Slant height of bucket (l)

$= \sqrt{h^2 + (R - r)^2}$

$= \sqrt{12^2 + (20 - 10)^2}$

$= 15{\cdot}62$ cm

Volume of bucket

$= \pi \frac{h}{3} (R^2 + r^2 + Rr)$

$= \frac{22}{7} \times \frac{12}{3} (20^2 + 10^2 + 20 \times 10)$

$= \frac{22}{7} \times 4 (400 + 100 + 200)$

$= \frac{22}{7} \times 4 \times 700$

$= 8800$ cm^3

Curved surface area of bucket

$= \pi (R + r) l + \pi r^2$

$= \pi [(R + r) l + r^2]$

$= \frac{22}{7} [(20 + 10) 15{\cdot}62 + 10^2]$

$= \frac{22}{7} [30 \times 15{\cdot}62 + 100]$

$= 1787{\cdot}03$ cm^2

$= 17{\cdot}87$ dm^2

Cost of tin sheet

$= 17{\cdot}87 \times 1{\cdot}20$

= ₹ 21·40. **Ans.**

9. The slant height of the frustum of a cone is 12 cm and the perimeters of its plane ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Sol. Given : Slant height of frustum = 12 cm

Perimeters of plane ends = 18 cm and 6 cm

Its upper radius

$$R = 2\pi R = 18$$

$$R = \frac{9}{\pi}$$

Bottom radius

$$r = 2\pi r = 6$$

$$r = \frac{3}{\pi}$$

Curved surface area = $\pi (R + r) l$

$$= \pi \left[\frac{9}{\pi} + \frac{3}{\pi}\right] 12$$

$$= \pi \times \frac{12}{\pi} \times 12$$

$$= 12 \times 12$$

$$= 144 \text{ cm}^2.$$ **Ans.**

10. The slant height of the frustum of a cone is 5 cm and the perimeter of its plane ends are 20 cm and 12 cm. Find the curved surface area of the frustum.

Sol. Given :

Slant height of frustum (l) = 5 cm

Perimeter 1 of plane end = 20 cm

$$2\pi r_1 = 20$$

$$r_1 = \frac{10}{\pi}$$

Perimeter 2 of plane end

= 12 cm

$$2\pi r_2 = 12$$

$$r_2 = \frac{6}{\pi}$$

Curved surface area of frustum

= $\pi [R + r] l$

$$= \pi \left[\frac{10}{\pi} + \frac{6}{\pi}\right] 5$$

$$= \pi \times \frac{16}{\pi} \times 5$$

$$= 16 \times 5$$

$$= 80 \text{ cm}^2.$$

11. A shuttle cock used for playing badminton has the shape of a frustum of a cone mounted on a hemisphere. The external diameters of the frustum are 5 cm and 2 cm, the height of the entire shuttle cock is 6 cm. Find its external surface area.

Sol. Given : Diameter 1 of frustum

= 2 cm

$\therefore$ Radius 1 of frustum (r_1)

= 2 ÷ 2

= 1 cm

Diameter 2 of frustum = 5 cm

$\therefore$Radius 2 of frustum (r_2) = 5 ÷ 2

= 2·5 cm

Height of the cock (h) = 6 cm

Slant height of frustum (l)

$$= \sqrt{h^2 + (r_2 - r_1)^2}$$

$$= \sqrt{6^2 + (2{\cdot}5 - 1)^2}$$

$$= \sqrt{36 + (1{\cdot}5)^2}$$

$$= \sqrt{36 + 2{\cdot}25}$$

$$= \sqrt{38{\cdot}25}$$

= 6·18 cm.

External surface area of frustum

= Curved surface area of frustum + Curved surface area of hemisphere

$$= \pi (r_1 + r_2) l + 2\pi r_1^2$$

$$= \pi [(r_1 + r_2) l + 2r_1^2]$$

$$= \frac{22}{7} [(1 + 2{\cdot}5)\, 6{\cdot}18 + 2 \times (1)^2]$$

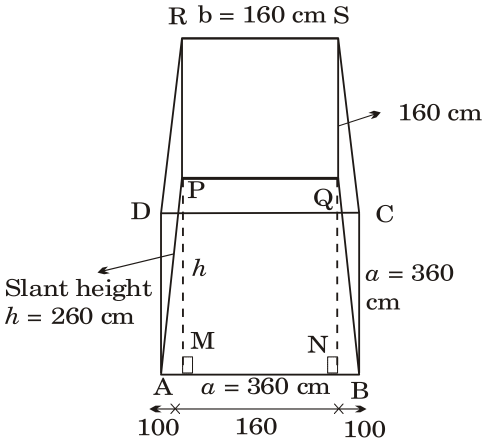
$$= \frac{22}{7} (3{\cdot}5 \times 6{\cdot}18 + 2 \times 1)$$

$$= \frac{22}{7} \times [21{\cdot}63 + 2]$$

$$= \frac{22}{7} \times 23{\cdot}63 = 74{\cdot}27 \text{ cm}^2.$$ **Ans.**

12. A pedestal is constructed in the form of the frustum of a pyramid, the sides of the square ends of frustum being 360 cm and 160 cm and its slant height 260 cm. find volume, lateral surface area including the area of the top and the cost of construction @ ₹ 50 per cubic metre and plastering it @ ₹ 10 per squar metre.

Sol. Here, ABCD is the base of Pedestal & PQRS in the top of pedestal both are square ends.



AP & BQ are slant heigh of pedestal. Draw $PM \perp AB$ & $QN \perp AB$. Then, AM = BN= 100 cm & MN = 160 cm.

Let PM in the $\perp$ height of pedestal *i.e.* PM = h

We have to required find h for right ΔAMP, By pythagoras theorem,

$$AP^2 = AM^2 + PM^2$$
$$(260)^2 = (100)^2 + h^2$$
$$h^2 = 67,600 - 10,000$$
$$h^2 = 57,600$$
$$h = \sqrt{57,600}$$
$$= 240 \text{ cm}$$

Now, Volume of Pedestal *i.e.*, frustum of pyramid = $\frac{h}{3}(a^2+b^2+ab)$

$$V = \frac{240}{3}(360^2 + 160^2 + 360 \times 160)$$
$$V = 80\,(129600 + 25600 + 57600)$$
$$V = 80(212800)$$
$$V = 17024000 \text{ cm}^3$$
$$V = 17.024 \text{ m}^3$$

Total area *i.e.*, Lateral S.A.+ Area of Top = $4 \times \left(\frac{a+b}{2}\right) \times h' + b^2$

$$= \{2 \times (360 + 160) \times 260\} + (160)^2$$
$$= \{2 \times 520 \times 260\} + (25600)$$
$$= 270400 + 25600$$
$$= 29600 \text{ cm}^2$$
$$= 29.6 \text{ m}^2.$$

Total Cost of Construction

$$= (17.024 \times ₹\,50) + (29.6 \times ₹\,10)$$
$$= 851.20 + 296.00$$
$$= ₹\,1147.20$$ **Ans.**

13. A reservoir in the form of the frustum of a right circular cone contains 44 × 10⁷ litres of water which fills it completely. The radii of the bottom and top of the reservoir are 50 metres and 100 metres respectivly. Find the depth of water. Also, find the lateral surface of the reservoir.

Sol. Radius of bottom of frustum

$$= 50 \text{ m}$$

Radius of top of frustum

$$= 100 \text{ m}$$

Volume of frustum

$$= 44 \times 10^7$$
$$\frac{1}{3} \times \pi \times h\,(R^2 + r^2 + Rr) = 44 \times 10^7$$
$$\frac{1}{3} \times \frac{22}{7} \times h\,(50^2 + 100^2 + 100 \times 50) = 44 \times 10^7$$
$$h = \frac{440000000 \times 7 \times 3}{22 \times 17500}$$
$$h = 24000$$

Slant height (l) = $\sqrt{h^2 + (R^2 - r^2)}$

$$= \sqrt{24^2 + (100^2 - 50^2)}$$
$$= \sqrt{576 + 2500}$$
$$= \sqrt{3076}$$
$$l = 55.46$$
$$l = 24\text{m.}$$

Lateral surface area of reservoir

$$= \pi\,(r_1 + r_2)\,l$$
$$= \frac{22}{7}\,(50 + 100)\,55{\cdot}46$$
$$= 26145{\cdot}9 \text{ m}^2.$$ **Ans.**

14. A tent consists of a frustum of a cone capped by a cone. If the radii of the ends of the frustum be 16 metres and 10 meters, the height of the frustum 8 metres and the slant height of the conical cap 12 metres, find the number of square metres of canvas required for the tent.

Sol. Given : Radius of top of frustum (R)

$$= 16 \text{ m}$$

Radius of bottom of frustum (r)

$$= 10 \text{ m}$$

Height of frustum (h)

$$= 8\text{m}$$

Slant height of conical cap (l)

$= 12m$

Slant height of frustum (l)

$= \sqrt{(R-r)^2 + h^2}$

$= \sqrt{(16-10)^2 + 8^2}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$= 10$ m

Surface area of frustum (S)

$= \pi L\ (R_1 + r_2)$

$= \pi \times 10\ (16 + 10)$

$= \pi \times 10 \times 26$

$= 260\pi$

Surface area of cone (s)

$= \pi rl$

$= \pi \times 10 \times 12$

$= 120\pi$

Area of canvas

$= S + s$

$= 260\pi + 120\pi$

$= 380\pi$

$= 380 \times \frac{22}{7}$

$= 1194{\cdot}29\ m^2$.

15. Hanumappa and his wife gangavva are busy making jaggery out of sugarcane. They have processed the sugarcane juice to make the molasses which is poured into moulds. It will be cooled to solidify in this shape to be sent to the market. Each mould is the shape of a frustum of a cone having the radii of its two circular faces as 30 cm and 35 cm and the height of the mould is 14 cm. If each cm^3 of molasses weight about 1·2 gm, find the weight of molasses that can be poured into each mould.

Sol. Radius of top of frustum (R)

$= 35$ cm

Radius of bottom of frustum (r)

$= 30$ cm

Height of frustum (h) = 14 cm

Volume of frustum

$= \pi \frac{h}{3} (R^2 + r^2 + Rr)$

$= \frac{22}{7} \times \frac{14}{3} (35^2 + 30^2 + 35 \times 30)$

$= \frac{22}{7} \times \frac{14}{3} (1225 + 900 + 1050)$

$= \frac{22}{7} \times \frac{14}{3} \times 3175$

$= 46566{\cdot}67\ cm^2$.

Mass of molesses that can be poured into each mould

$= 46566{\cdot}67 \times 1{\cdot}2$

$= 55880$ g

$= 55{\cdot}88$ kg. **Ans.**

16. A tent consists of a frustum of a cone capped by a cone. If the radii of the ends of the frustum be 13 metres and 7 metres, the height of the frustum 8 metres and the slant height of the conical cap 12 metres, find the number of square metres of canvas required for the tent.

Sol. Given : Height of frustum (h)

$= 8$ m

Radius of upper side (R) = 13 m

Radius of lower side (r) = 7m

Slant height of cone (L) = 12 cm

Slant height of frustum (l)

$= \sqrt{(R-r)^2 + h^2}$

$= \sqrt{(13-7)^2 + 8^2}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$= 10$ m

Curved surface area of frustum

$= \pi(R + r)\ l$

$= \pi\ (13 + 7)\ 10$

$= \pi \times 20 \times 10$

$= 200\pi\ m^2$

Curved surface area of cone

$= \pi rl$

$= \pi \times 7 \times 12$

$= 84\pi$

Total convas required for tent

$= 200\pi + 84\pi$

$= 284\pi$

$= 284 \times \frac{22}{7}$

$= 892{\cdot}57\ m^2$.

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15 Probability

EXERCISE 15.1

Multiple Choice Type Questions

1. The probability of getting a bad egg from a lot of 400 eggs is 0·035. The number of bad eggs in the lot is :

(a) 7 **(b) 14**

(c) 21 **(d) 25.**

Sol. Probability

$$= \frac{\text{No. of bad eggs in a lot}}{\text{Total no. of eggs}}$$

$$0{\cdot}035 = \frac{\text{No. of bad eggs}}{400}$$

No. of bad eggs = 400 × 0·035

= 14 eggs.

2. The probability of guessing the correct answer to a certain question is $\frac{P}{12}$. If the probability of not guessing the correct answer to same question is $\frac{3}{4}$, the value of P is :

(a) 3 **(b) 4**

(c) 2 **(d) 1.**

Sol. P (correct answer) + P (wrong answer) = 1

$$\frac{P}{12} + \frac{3}{4} = 1$$

$$\frac{P}{12} = 1 - \frac{3}{4}$$

$$\frac{P}{12} = \frac{1}{4}$$

$$P = \frac{12}{4}$$

P = 3.

3. In a throw of two die, the probability of getting a sum of 10 is :

(a) $\frac{1}{12}$ **(b) $\frac{1}{36}$**

(c) $\frac{1}{6}$ **(d) $\frac{1}{4}$.**

Sol. Total possible outcomes

= 6 × 6 = 36

Favourable outcomes

= (4, 6) (6, 4) (5, 5) = 3

Probability

$$= \frac{\text{Possible outcomes}}{\text{Favourable outcomes}}$$

$$= \frac{3}{36} = \frac{1}{12}.$$

4. The probability that a non leap year selected at random will have 53 Tuesday is :

(a) $\frac{1}{7}$ **(b) $\frac{2}{7}$**

(c) $\frac{3}{7}$ **(d) $\frac{4}{7}$.**

Sol. In a non-leap year, there are 365 days

365 days = 52 weeks + 1 days

These one day can be any day

∴ Total outcome = 7

Favourable outcome = 1

$$\text{Probability} = \frac{\text{Favourable outcome}}{\text{Total outcome}}$$

$$= \frac{1}{7}.$$

5. The probability of drawing a green coloured ball from a bag containing 6 red and 5 black balls is :

(a) 0 **(b) 1**

(c) $\frac{5}{11}$ **(d) $\frac{6}{11}$.**

Sol. Total outcomes = 6 + 5 = 11

Favourable outcomes

= 0 green balls

∴ Probability of getting green ball is 0.

6. If the probability of winning a game is 0·995, then the probability of lossing is :

(a) 1 **(b) 0·05**
(c) 0·0050 **(d) 0·0**

Sol. P (loosing game) + P (Winning game) = 1
P (loosing game) + 0·995 = 1
Probability of loosing game
= 1 – 0·995 = 0·0050.

7. If a die is thrown once, the probability of getting a no. less than 3 and greater than 2 is :
(a) 0 **(b) 1**
(c) 1/3 **(d) $\frac{2}{3}$.**

Sol. There is no number on die which is greater than 2 and less than 3.
∴ Probability = 0.

8. The probability of getting a number between 1 and 100 which is divisible by 7 is :
(a) $\frac{11}{100}$ **(b) $\frac{1}{7}$**
(c) $\frac{7}{50}$ **(d) $\frac{13}{100}$.**

Sol. Total no.of outcomes = 100
Favourable outcomes (No. divisible by 7) = 14
Probability
$$= \frac{\text{Favourable outcomes}}{\text{Total number of outcomes}}$$
$$= \frac{14}{100} = \frac{7}{50}.$$

9. If two coins are tossed simultaneously, then the probability of getting at least one head is :
(a) $\frac{3}{4}$ **(b) $\frac{1}{2}$**
(c) $\frac{1}{4}$ **(d) 1.**

Sol. Total number of outcomes = 4
Favourable outcomes = 3
(1 head, 1 tail) (1 tail, 1 head) (2 had) (2 tail)
Probability
$$= \frac{\text{Favourable outcomes}}{\text{Total no. of outcomes}}$$
$$= \frac{3}{4}.$$

10. Two dice are thrown simultaneously. Probability of getting a prime number on both dice is :
(a) $\frac{5}{18}$ **(b) $\frac{2}{9}$**
(c) $\frac{1}{3}$ **(d) $\frac{1}{4}$.**

Sol. Total no. of outcomes = 6 × 6 = 36
Prime numbers between 1 to 6 are = 2, 3 and 5
Getting a prime no. on both dice = 9
(2, 2) (2, 3) (3, 2) (2, 5) (5, 2) (3, 3) (3, 5) (5, 3) (5, 5)
Probability
$$= \frac{\text{Favourable outcomes}}{\text{Total no. of outcomes}}$$
$$= \frac{9}{36} = \frac{1}{4}.$$

Short Answer Type Questions

11. A die is thrown once. What is the probability of getting a no. 5 or 6.

Sol. Total no. of outcomes = 6
Favourable outcomes = 2
Probability
$$= \frac{\text{Favourable outcomes}}{\text{Total no. of outcomes}}$$
$$= \frac{2}{6}$$
$$= \frac{1}{3}.$$

12. A die is thrown once. Find the probability of getting :
(i) an even number
(ii) a number greater than 3
(iii) a number between 3 and 6.

Sol. (i) Total outcomes = 6
Favourable outcomes = 3 (2, 4, 6)
Probability
$$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{3}{6} = \frac{1}{2}.$$
(ii) Total outcomes = 6
Favourable outcomes = 3 (4, 5, 6)
Probability
$$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{3}{6} = \frac{1}{2}$$
(iii) Total outcomes = 6

Favourable outcomes = 2 (4, 5)
Probability

$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{2}{6} = \frac{1}{3}$

13. A die is thrown once. Find the probability of getting :
(i) an odd number
(ii) a number greater than 4
(iii) Seven.

Sol. (i) Total outcomes = 6
Favourable outcomes = 3 (1, 3, 5)
Probability

$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{3}{6} = \frac{1}{2}$

(ii) Total outcomes = 6
Favourable outcomes = 2 (5, 6)
Probability

$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{2}{6} = \frac{1}{3}$

(iii) Total outcomes = 6
Favourable outcomes = 0
∴Probability of getting seven on die = 0.

14. In a cricket match with Pakistan, Indian team has to choose a captain out of the following players :
1. Sehwag
2. Dravid
3. Sachin
4. Saurav Ganguly
5. Anil Kumble
What is the probability that :
(a) Saurav Ganguly
(b) Saurav Ganguly or Dravid
(c) Not Sachin
Will be selected as a captain of the team ?

Sol. (a) Total outcomes = 5
Favourable outcomes = 1
Probability

$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{1}{5}$

(b) Total outcomes = 5
Favourable outcomes = 2
Probability

$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{2}{5}$

(c) Total outcomes = 5
Favourable outcomes = 5 – 1 = 4
Probability

$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{4}{5}$.

15. If the probability that it will rain tomorrow is 0·85. What is the probability that it will not rain tomorrow ?

Sol. Probability (raining) + P (Not raining) = 1
0·85 + P (Not rain) = 1
Probability of not raining
= 1 – 0·85 = 0·15.

16. A card is drawn from a pack of 52 cards. What is the probability of getting :
(i) Jack
(ii) The ace of spades
(iii) a queen
(iv) a heart
(v) a red card
(vi) an ace

Sol. (i) Total no. of Jacks = 4
Total cards = 52
Probability

$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$

$= \frac{4}{52} = \frac{1}{13}$.

(ii) Total no. of spades = 1
Total cards = 52
Probability

$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{1}{52}$.

(iii) Total no. of queens = 4
Total cards = 52
Probability

$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{4}{52} = \frac{1}{13}$.

(iv) Total no. of hearts = 13
Total outcomes = 52
Probability

$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{13}{52} = \frac{1}{4}$.

(v) Total no.of red cards = 26
Total outcomes = 52
Probability

$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{26}{52} = \frac{1}{2}$.

(vi) Total no.of ace = 4

Total outcomes = 52

Probability

$$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{4}{52} = \frac{1}{13}.$$

17. When a card is drawn from a pack of 52 cards. Find the probability that it may be either a king or a queen.

Sol. Total no. of cards = 52

Total no. of queens = 4

Total no. of kings = 4

Total No. of favourable outcomes = 4 + 4 = 8

Probability

$$= \frac{\text{No. of favourable outcomes}}{\text{Total outcomes}}$$

$$= \frac{8}{52} = \frac{2}{13}.$$

18. A card is drawn from an ordinary pack and a gambler bets that it is a spade or an ace. What are the odds against his winning this bet ?

Sol. Probability $= \frac{16}{52} = \frac{4}{13}$

$$\frac{P(\overline{A})}{P(A)} = \frac{1 - P(A)}{P(A)}$$

$$= \frac{1 - \frac{4}{13}}{\frac{4}{13}}$$

$$= \frac{9}{4} = 9 : 4.$$

19. If a coin is tossed two times, what is the probability of getting 'head' at least once ?

Sol. Total outcomes = 4 (both heads) (both tails) (1 head, 1 tail) (1 tail, 1 head)

Favourable outcomes = 3 (both heads) (1 head, 1 tail) (1 tail, 1 head)

Probability

$$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$$

$$= \frac{3}{4}$$

20. A bag contains 7 red and 8 black balls. Find the probability of drawing a red ball.

Sol. Total balls = 7 + 8 = 15

Possibility of drawing red ball= 7

Probability

$$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{7}{15}.$$

21. A beg contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is :

(i) red (ii) black

Sol. (i) **Red :** Total outcomes = 3 + 5 = 8

Possible outcomes = 3

Probability

$$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{3}{8}$$

(ii) **Black :** Total outcomes = 5 + 3 = 8

Possible outcomes = 5

Probability

$$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{5}{8}.$$

22. A bag contains 4 red and 8 blue marbles. A marble is drawn at random. What is the probability of drawing.

(i) a red marble

(ii) a blue marble.

Sol. (i) Total no. of balls = 4 + 8 = 12

No. of red marbles = 4

Probability

$$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$$

$$= \frac{4}{12} = \frac{1}{3}.$$

(ii) Total no. of balls = 12

No. of blue balls = 8

Probability

$$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$$

$$= \frac{8}{12} = \frac{2}{3}.$$

23. A bag contains 4 green, 6 black and 7 white balls. A ball is drawn at random. What is the probability that it is either a green or a black ball ?

Sol. Total no. of balls = 4 + 6 + 7 = 17

No. of green balls = 4

No. of black balls = 6

Favourable outcomes = 6 + 4 = 10

Probability

$$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$$

$$= \frac{10}{17}.$$

24. There are 4 red, 3 black and 5 white balls in a bag. If a ball is drawn at random from the bag. What is the probability that it may be either red or black ?

Sol. Total no. of balls = 4 + 3 + 5 = 12

No. of red balls = 4

No. of black balls = 3

Favourable outcomes = 4 + 3 = 7

Probability

$$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{7}{12}.$$

25. A bag contains 6 black, 7 red and 2 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is :

(i) Red (ii) Black or White (iii) Not black.

Sol. (i) Total no. of balls = 6 + 7 + 2 = 15

Possibility of getting red ball = 7

Probability

$$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{7}{15}.$$

(ii) Total no. of balls = 15

Possibility of getting black ball = 6

Possibility of getting white ball = 2

Favourable outcomes = 6 + 2 = 8

Probability

$$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{8}{15}.$$

(iii) Total no. of balls = 15

Possibility of getting black ball = 6

Probability

$$= \frac{\text{unfavourable outcomes}}{\text{total outcomes}}$$

$$= \frac{15-6}{15}$$

$$= \frac{9}{15} = \frac{3}{5}.$$

26. A bag contains 5 white balls, 7 red balls, 4 black balls, and 2 blue balls. One ball is drawn at random from the bag. What is the probability that the ball drawn is :

(a) white or blue

(b) red or black

(c) not white

(d) neither white nor black

Sol. (a) Total no. of balls = 5 + 7 + 4 + 2 = 18

No. of white balls = 5

No. of blue balls = 2

Favourable outcomes = 5 + 2 = 7

Probability

$$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{7}{18}$$

(b) Total no. of balls = 18

No. of red balls = 7

No. of black balls = 4

Favourable outcomes = 7 + 4 = 11

Probability

$$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{11}{18}$$

(c) Total no. of balls = 18

No. of white balls = 5

Probability

$$= \frac{\text{unfavourable outcomes}}{\text{total outcomes}}$$

$$= \frac{18-5}{18} = \frac{13}{18}$$

(d) Total no. of balls = 18

No. of white balls = 5

No. of black balls = 4

unfavourable outcomes = 5 + 4 = 9

Probability

$$= \frac{\text{unfavourable outcomes}}{\text{total outcomes}}$$

$= \frac{18-9}{18} = \frac{9}{18} = \frac{1}{2}$

27. A box contains 40 balls of the same shape and weight. Among the balls 10 balls are white 16 are red and rest are black. What is the probability that a ball drawn from the box is not black ?

Sol. Total no. of balls = 40
No. of black balls = 40 – 10 – 16
= 14
Probability

$= \frac{\text{unfavourable outcomes}}{\text{total outcomes}}$

$= \frac{40-14}{40} = \frac{26}{40} = \frac{13}{20}$

28. Find the probability of getting (i) 9 (ii) 7 and (iii) 6 on throwing two dice.

Sol. (i) 5 cm of 9 is obtained by the following cases.
(3, 6) (4, 5) (5, 4) (6, 3) = 4
Total outcomes on throwing two dice= 6 × 6 = 36
Probability

$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$

$= \frac{4}{36} = \frac{1}{9}$

(ii) Sum of 7 is obtained in following cases :
(1, 6) (2, 5) (3, 4) (4, 3) (5, 2) (6, 1) = 6
Total outcomes = 6 × 6 = 36
Probability

$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{6}{36} = \frac{1}{6}$

(iii) Sum of 6 is obtained in following cases :
(1, 5) (2, 4) (3, 3) (4, 2) (5, 1) = 5
Total outcomes = 6 × 6 = 36
Probabillity

$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{5}{36}$

29. A black die and a white die is thrown at a same time. Write all the possible outcomes what is the probability that the sum of two numbers that turn up is 8 ?

Sol. Sum of 8 is obtained in following cases :
(6, 2) (3, 3) (4, 2) (2, 4) (2, 6) = 5
Total outcomes = 6 × 6 = 36
Probability

$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{5}{36}$

30. A black die, a red die and a green die are thrown at the same time what is the probability that the sum of three numbers that turnup is 15 ?

Sol. Total number of outcomes when three die are thrown together = 6 × 6 × 6 = 216
For sum of numbers to be 15
Possible ways are :
(6, 6, 3) (6, 3, 6) (3, 6, 6) (6, 5, 4) (6, 4, 5) (5, 4, 6) (5, 6, 4) (4, 5, 6) (4, 6, 5) (5, 5, 5)
Number of favourable outcomes = 10
Required probability

$= \frac{10}{216} = \frac{5}{108}$

31. On tossing three coins simultaneously, find the probability of getting :
(i) 3 tails (ii) 2 tails
(iii) No tails
(iv) 2 head and 1 tail
(v) at least one head.

Sol. When three coins are tossed simultaneously the possibilities are :
(H, H, H) (T, T, T) (H, H, T) (H, T, H) (T, H, H) (T, T, H) (T, H, T) (H, T, T) = 8

(i) Possibility of 3 tails = 1

Probability $= \frac{1}{8}$

(ii) Possibility of 2 tails = 3

Probability $= \frac{3}{8}$

(iii) Possible of getting no tails = 1

Probability $= \frac{1}{8}$

(iv) Possibility of 2 heads = 3
Possibility of 1 tail = 3

Probability of 2 heads = $\frac{3}{8}$

Probability of 1 tail = $\frac{3}{8}$

(v) Possibility of getting at least one head = 7

Probability = $\frac{7}{8}$

32. From a set of 17 cards numbered 1, 2, 3, ... 17 one is drawn what is the probability that its number is a multiple of 3 or 7 ?

Sol. Total outcomes = 17

Numbers multiple of 3 = 3, 6, 9, 12, 15 = 5

Number multiple of 7 = 7, 14 = 2

Favourable outcomes = 5 + 2 = 7

Probability

$= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{7}{17}$

33. 17 cards numbered 1, 2, 3 ... 16, 17 are put in a box and mixed throughly ? One person draws a card from the box. Find the probability that the number on card is :
(i) odd (ii) a prime (iii) divisible by 3
(iv) divisible by 3 or 2 both.

Sol. (i) Total odd no. in 1 to 17 is = 9

Required probability = $\frac{9}{17}$

(ii) Total prime no. in 1 to 17 is = 7

Required probability = $\frac{7}{17}$

(iii) Total no. divisible by 3 = 5

Required probability = $\frac{5}{17}$

(iv) Total no. divisible by 3 or 2 both = 2

Required probability = $\frac{2}{17}$

34. A game of chance consists of spining an arrow which is equally likely to come the rest pointing to one of the number 1, 2, 3 ... 12 what is the probability that it will point to :
(i) 10
(ii) an odd number
(iii) a number which is multiple of 3.

Sol. (i) Possible of getting 10 = 1

Required probability = $\frac{1}{12}$

(ii) Possibility of getting odd number = 6

Required probability = $\frac{6}{12} = \frac{1}{2}$

(iii) Possibility of getting no. multiple of 3 = 4

Required probability = $\frac{4}{12} = \frac{1}{3}$

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Unit 7 : Statistics and Probability

14 Statistics

EXERCISE 14.1

Multiple Choice Type Questions

1. If x_i's are the mid points of the class intervals of a grouped data f_i's are the corresponding frequencies and $\bar{x}$ is the mean, then $\sum f_i(x_i - \bar{x}) = ?$

(a) 1 **(b) 0**

(c) – 1 **(d) 2**

Sol. Mean (x)

$$= \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

$$x = \frac{(f_1x_1 + f_2x_2 + f_3x_3 \ldots f_n \times n)}{f_1 + f_2 + f_3 \ldots f_n}$$

$$x = \frac{\sum f_i x_i}{\sum f_i}, \ \Sigma f_i = n$$

$$x = \frac{\sum f_i x_i}{n}$$

$$nx = \Sigma f_i x_i \qquad \ldots(1)$$

$$\Sigma(f_i x_i - x) = (f_1x_1 - x) + (f_2x_2 - x) + \ldots (f_nx_n - x)$$

$$\Sigma(f_i x_i - x) = (f_1x_1 + f_2x_2 + \ldots + f_nx_n) - (x + x + \ldots n \text{ times})$$

$$\Sigma(f_i x_i - x) = \Sigma f_i x_i - nx$$

$$\Sigma(f_i x_i - x) = nx - nx$$

(from equation (1))

$$\Sigma(f_i x_i - x) = 0.$$

2. For finding the mean by using the formula $\bar{x} = A + h\left(\frac{\sum f_i u_i}{\sum f_i}\right)$ we have $u_i = ?$

(a) $\frac{(A - x_i)}{h}$ **(b) $\frac{(x_i - A)}{h}$**

(c) $\frac{(A + x_i)}{h}$ **(d) $h(x_1 - A)$**

Sol. $u_i = \frac{x_i - A}{h}$.

3. In the formula $\bar{x} = \left(\frac{A + \sum f_i d_i}{\sum f_i}\right)$ for finding the mean of the grouped data, the d_i's are the deviations from A of :

(a) Lower limits of the classess.

(b) Upper limits of the classes.

(c) Mid points of the classes.

(d) None of these.

Sol. d_i are the mid points of the classess.

4. Find the mean, using direct method :

Class	0–10	10–20	20–30	30–40	40–50
Frequency	3	5	9	5	3

Sol. We may prepare the table given below :

Class Interval	Frequency (f_i)	Class mark (x_i)	($f_i \times x_i$)
0—10	3	5	15
10—20	5	15	75
20—30	9	25	225
30—40	5	35	175
40—50	3	45	135
	$\Sigma f_i = 25$		$\Sigma(f_i \times x_i) = 625$

$$x_i = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

$$\text{Mean } (\bar{x}) = \frac{\sum (f_i x_i)}{\sum f_i}$$

$$= \frac{625}{25}$$

$$= 25$$

5.

Class	0–10	10–20	20–30	30–40	40–50	50–60
Frequency	7	5	6	12	8	2

Sol. We may prepare the table given below :

Class	Frequency (f_i)	Class mark (x_i)	($f_i \times x_i$)
0—10	7	5	35
10—20	5	15	75
20—30	6	25	150
30—40	12	35	420
40—50	8	45	360
50—60	2	55	110
	$\Sigma f_i = 40$		$\Sigma f_i x_i = 1150$

$$x_i = \frac{\text{Upper limit + Lower limit}}{2}$$

$$\bar{x} = \frac{\sum f_i x_i}{\sum x_i} = \frac{1150}{40} = 28{\cdot}75$$

6.

Class	10–20	20–30	30–40	40–50	50–60	60–70
Frequency	11	15	20	30	14	10

Sol. We may prepare the table given below :

Class	Frequency (f_i)	Class marks (x_i)	$f_i \times x_i$
10—20	11	15	165
20—30	15	25	375
30—40	20	35	700
40—50	30	45	1350
50—60	14	55	770
60—70	10	65	650
	$\Sigma f_i = 100$		$\Sigma f_i x_i = 4010$

$$x_i = \frac{\text{Upper limit + Lower limit}}{2}$$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{4010}{100} = 40{\cdot}1$$

7.

Marks	10–20	20–30	30–40	40–50	50–60	60–70	70–80
No. of Students	6	8	13	7	3	2	1

Sol. We may prepare table given below :

Class	Frequency (f_i)	Class Marks (x_i)	$f_i \times x_i$
10—20	6	15	90
20—30	8	25	200
30—40	13	35	455
40—50	7	45	315
50—60	3	55	165
60—70	2	65	130
70—80	1	75	75
	$\Sigma f_i = 40$		$\Sigma f_i x_i = 1430$

$$x_i = \frac{\text{Upper limit + Lower limit}}{2}$$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1430}{40} = 35{\cdot}75.$$

8.

Class	25–35	35–45	45–55	55–65	65–75
Frequency	6	10	8	12	4

Sol. We may prepare the table given below :

Class	**Frequency (f_i)**	**Class marks (x_i)**	$f_i x_i$
25—35	6	30	180
35—45	10	40	400
45—55	8	50	400
55—65	12	60	720
65—75	4	70	280
	$\Sigma f_i = 40$		$\Sigma f_i x_i = 1980$

$$x_i = \frac{\text{Upper limit + Lower limit}}{2}$$

$$\text{Mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i} = \frac{1980}{40} = 49{\cdot}5.$$

9. The mean of the following frequency distribution is 24. Find the value of P.

Marks	0–10	10–20	20–30	30–40	40–50
No. of students	15	20	35	P	10

Sol.

Marks	**No. of Students (f_i)**	**Class Marks (x_i)**	$f_i \times x_i$
0—10	15	5	75
10—20	20	15	300
20—30	35	25	875
30—40	P	35	35P
40—50	10	45	450
	$\Sigma f_i = 80 + \text{P}$		$\Sigma f_i x_i = 1700 + 35\text{P}$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

$$24 = \frac{1700 + 35\text{P}}{80 + \text{P}}$$

$$24\,(80 + \text{P}) = 1700 + 35\text{P}$$

$$1920 + 24\text{P} = 1700 + 35\text{P}$$

$$35\text{P} - 24\text{P} = 1920 - 1700$$

$$11\text{P} = 220$$

$$\text{P} = \frac{220}{11}$$

$$\text{P} = 20.$$

10. Find the missing frequencies f_1 and f_2 in the table given below, it is being given that the mean of the given frequency distribution is 50.

Class	0–20	20–40	40–60	60–80	80–100	Total
Frequency	17	f_1	32	f_2	19	120

Solution :

Class	Frequency (f_i)	Class Mark (x_i)	$d_i = x_i - a$	$f_i \times d_i$
0 – 20	17	10	– 40	– 680
20 – 40	f_1	30	– 20	$-20f_1$
40 – 60	32	$50 = a$	0	0
60 – 80	f_2	70	20	$20f_2$
80 – 100	19	90	40	760
Total	$68 + f_1 + f_2$			$80 - 20f_1 + 20f_2$

It is given that mean = 50

From the table, we have

$a = 50$, $N = 120 = 68 + f_1 + f_2$, $\Sigma f_i \times d_i = 80 - 20f_2 + 20 f_2$

$$f_1 + f_2 = 120 - 68$$

$$f_1 + f_2 = 52$$

Now, Mean $= a + \frac{1}{N} \times \Sigma f_i d_i$

$$50 = 50 + \frac{1}{120} \times 80 - 20f_1 + 20f_2$$

$$(50 - 50)120 = 80 - 20f_1 + 20f_2$$

$$20f_1 - 20 f_2 = 80$$

$$f_1 - f_2 = 4$$

Now,

$$f_1 + f_2 = 52$$

$$f_1 - f_2 = 4$$

$\therefore$ $f_1 = 28, f_2 = 24$

11. The mean of the following frequency distribution is 57·6 and the sum of the observations is 50.

Class	0–20	20–40	40–60	60–80	80–100	100–120
Frequency	7	f_1	12	f_2	8	5

Solution :

Class	Frequency (f_i)	Class Marks (x_i)	$d_i = x_i - a$	$d_i \times f_i$
0 – 20	7	10	– 40	– 280
20 – 40	f_1	30	– 20	$-20 f_1$
40 – 60	12	$50 = a$	0	0
60 – 80	f_2	70	20	$20f_2$
80 – 100	8	90	40	320
100 – 120	5	110	60	300
Total	$32 + f_1 + f_2$			$340 - 20f_1 + 20f_2$

It is given that, mean = 57·6

From the table, we have

$a = 50$, $N = 50 = 32 + f_1 + f_2$, $\Sigma d_i f_i = 340 - 20f_1 + 20f_2$

$$f_1 + f_2 = 50 - 32$$
$$f_1 + f_2 = 18$$

Now, mean $= a + \frac{1}{N} \Sigma f_i \times d_i$

$$57{\cdot}6 = 50 + \frac{1}{50}\, 340 - 20f_1 + 20f_2$$
$$(57{\cdot}6 - 50)50 = 340 - 20f_1 + 20f_2$$
$$380 - 340 = -20f_1 + 20f_2$$
$$40 = -20(f_1 - f_2)$$
$$f_1 - f_2 = -2$$

Now, $f_1 + f_2 = 18$ $\quad\therefore f_1 = 8, f_2 = 10$

$$f_1 - f_2 = -2$$

12. Find the mean using assumed – mean method :

Marks	0–10	10–20	20–30	30–40	40–50	50–60
No. of Students	12	18	27	20	17	6

Sol. Let A = 25 be the assumed mean. Then, we have :

Class Interval	No. of Students f_i	Mid value x_i	Deviation $d_i = (x_i - 25)$	$f_i \times d_i$
0–10	12	5	– 20	– 240
10–20	18	15	– 10	– 180
20–30	27	25 = A	0	0
30–40	20	35	10	200
40–50	17	45	20	340
50–60	6	55	30	180
	$\Sigma f_i = 100$			$\Sigma(f_i \times d_i) = 300$

$$\bar{x} = A + \frac{\Sigma f_i d_i}{n}$$
$$= 25 + \frac{300}{100}$$
$$= 25 + 3$$
$$= 28.$$

13.

Class	100–120	120–140	140–160	160–180	180–200
Frequency	10	20	30	15	5

Sol. Let A = 150 be the assumed mean. Then, we have :

Class interval	Frequency f_i	Mid value x_i	Deviation $d_i = (x_i - 150)$	$fi \times d_i$
100—120	10	110	– 40	– 400
120—140	20	130	– 20	– 400
140—160	30	150 = A	0	0
160—180	15	170	20	300
180—200	5	190	40	200
	$\Sigma f_i = 80$			$\Sigma(f_i d_i) = -300$

$$\bar{x} = A + \frac{\Sigma f_i d_i}{n}$$
$$= 150 + \left(\frac{-300}{80}\right)$$
$$= 150 - 3.75$$
$$= 146.25.$$

14.

Class	0–20	20–40	40–60	60–80	80–100	100–120
Frequency	20	35	52	44	38	31

Sol. Let A = 70 be the assumed mean. Then, we have

Class Interval	**Frequency (f_i)**	**Mid Value (x_i)**	**Deviation [$d_i = x_i - 70$]**	$f_i \times d_i$
0–20	20	10	– 60	– 1200
20–40	35	30	– 40	– 1400
40–60	52	50	– 20	– 1040
60–80	44	70 = A	0	0
80–100	38	90	20	760
100–120	31	110	40	1240
	$\Sigma f_i = 220$			$\Sigma(f_i \times d_i) = -1640$

$$\bar{x} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$
$$= 70 + \left(\frac{-1640}{220}\right)$$
$$= 70 - 7.45$$
$$= 62.55.$$

15. Find the arithmetic mean of each of the following frequency distribution using step deviation method :

Marks	0–10	10–20	20–30	30–40	40–50	50–60
No. of students	12	18	27	20	17	6

Sol. We Prepare the table given below : let A = 25

Class	**Frequency (f_i)**	**Mid Value (x_i)**	$u_i = \frac{x_i - 25}{10}$	$f_i \times u_i$
0—10	12	5	– 2	– 24
10—20	18	15	– 1	– 18
20—30	27	25 = A	0	0
30—40	20	35	1	20
40—50	17	45	2	34
50—60	6	55	3	18
	$\Sigma f_i = 100$			$\Sigma f_i u_i = 30$

Thus, $A = 25$, $h = 10$, $\Sigma f_i = 100$ and $\Sigma f_i \times u_i = 30$

$$\bar{x} = A + \left[h \times \frac{\Sigma f_i \times u_i}{\Sigma f_i}\right]$$

$$= 25 + \left[10 \times \frac{30}{100}\right]$$

$$= 25 + 3$$
$$= 28.$$

16.

Class	Number of Students
4—8	2
8—12	12
12—16	15
16—20	25
20—24	18
24—28	12
28—32	13
32—36	3

Sol. We may prepare following table :

Class	Frequency (f_i)	Mid value (x_i)	$u_i = \frac{x_i - 22}{4}$	$u_i \times f_i$
4—8	2	6	– 4	– 8
8—12	12	10	– 3	– 36
12—16	15	14	– 2	– 30
16—20	25	18	– 1	– 25
20—24	18	22 = A	0	0
24—28	12	26	1	12
28—32	13	30	2	26
32—36	3	34	3	9
	$\Sigma f_i = 100$			$\Sigma u_i f_i = -52$

Thus $h = 4$, $A = 22$, $\Sigma f_i = 100$ and $\Sigma u_i \times f_i = -52$

$$\bar{x} = A + \left[h \times \frac{\Sigma u_i \times f_i}{\Sigma f_i}\right]$$

$$= 22 + \left[4 \times \frac{-52}{100}\right]$$

$$= 22 - 2.08$$
$$= 19.92.$$

17.

Class	0–30	30–60	60–90	90–120	120–150	150–180
Frequency	12	21	34	52	20	11

Sol. We may prepare following table :

Class	Frequency (f_i)	Mid Value (x_i)	$u_i = \frac{x_i - 105}{30}$	$u_i \times f_i$
0—30	12	15	– 3	– 36
30—60	21	45	– 2	– 42
60—90	34	75	– 1	– 34
90—120	52	105 = A	0	0
120—150	20	135	1	20
150—180	11	165	2	22
	$\Sigma fi = 150$			$\Sigma u_i f_i = -70$

Thus, $A = 105$, $h = 30$, $\Sigma f_i = 150$, $\Sigma u_i f_i = -70$

$$\bar{x} = A + \left[h \times \frac{\Sigma u_i \times f_i}{\Sigma f_i}\right]$$

$$= 105 + \left[30 \times \frac{-70}{150}\right]$$

$$= 105 - 14$$

$$= 91$$

18.

Marks	0–14	14–28	28–42	42–56	56–70
No. of Students	7	21	35	11	16

Sol. We may prepare following table :

Class	Frequency (f_i)	mid value (x_i)	$u_i = \frac{x_i - 35}{14}$	$u_i \times x_i$
0—14	7	7	– 2	– 14
14—28	21	21	– 1	– 21
28—42	35	35 = A	0	0
42—56	11	49	1	11
56—70	16	63	2	32
	$\Sigma f_i = 90$			$\Sigma u_i \times f_i = 8$

Thus, $A = 35$, $h = 14$, $\Sigma f_i = 90$, $\Sigma u_i \times f_i = 8$

$$\bar{x} = A + \left[h \times \frac{\Sigma u_i \times f_i}{\Sigma f_i}\right]$$

$$= 35 + \left[14 \times \frac{8}{90}\right]$$

$$= 35 + 1.24$$

$$= 36.24.$$

19.

Class	10–15	15–20	20–25	25–30	30–35	35–40
Frequency	5	6	8	12	6	3

Sol. We may prepare following table :

Class	**Frequency** (f_i)	**Mid Value** (x_i)	$u_i = \frac{x_i - 22.5}{15}$	$u_i \times f_i$
10—15	5	12.5	– 0.66	– 3.33
15—20	6	17.5	– 0.33	– 1.98
20—25	8	22.5 = *A*	0	0
25—30	12	27.5	0.33	3.96
30—35	6	32.5	0.66	3.96
35—40	3	37.5	1	3
	$\Sigma fi = 40$			$\Sigma ui \times fi = 5.61$

Thus, A = 22.5, h = 15, Σfi = 40, $\Sigma u_i \times f_i$ = 5.61

$$\bar{x} = A + \left[h \times \frac{\Sigma u_i \times f_i}{\Sigma f_i}\right]$$

$$= 22.5 + \left[15 \times \frac{5.61}{40}\right]$$

$$= 22.5 + 2.10$$

$$= 24.60$$

20.

Age (in years)	18-24	24-30	30-36	36-42	42-48	48-54
No. of workers	6	8	12	8	4	2

Sol. We may prepare following table :

Age (in years)	**No. of workers** (f_i)	**Mid Value** (x_i)	$N = \frac{x_i - 39}{6}$	$f_i \times u_i$
18—24	6	21	– 3	–18
24—30	8	27	– 2	– 16
30—36	12	33	– 1	– 12
36—42	8	39 = *A*	0	0
42—48	4	45	1	4
48—54	2	51	2	4
	$\Sigma f_i = 40$			$\Sigma x f_i \times u_i = -38$

Thus, A = 39, h = 6, Σf_i = 40, $\Sigma f_i \times u_i$ = – 38

$$\bar{x} = A + \left[h \times \frac{\Sigma u_i \times f_i}{\Sigma f_i}\right]$$

$$= 39 + \left[6 \times \frac{-38}{40}\right]$$

$$= 39 - 5{\cdot}7$$

$$= 33{\cdot}3.$$

21.

Class	84—90	90—96	96—102	102—108	108—114	114—120
Frequency	15	22	20	18	20	25

Sol. We may prepare following table :

Class	Frequency (f_i)	Mid Value (x_i)	$u_i = \frac{x_i - 99}{6}$	$u_i \times fi$
84—90	15	87	– 2	– 30
90—96	22	93	– 1	– 22
96—102	20	99 = A	0	0
102—108	18	105	1	18
108—114	20	111	2	40
114—120	25	117	3	75
	$\Sigma f_i = 120$			$\Sigma u_i \times f_i = 81$

Thus, A = 99, h = 6, $\Sigma f_i = 120$, $\Sigma u_i \times f_i = 81$

$$r = A + \left[h \times \frac{\Sigma f_i \times u_i}{\Sigma f_i}\right]$$

$$= 99 + \left[6 \times \frac{81}{120}\right]$$

$$= 99 + 4{\cdot}05$$

$$= 103{\cdot}05.$$

22.

Class	500-520	520-540	540-560	560-580	580-600	600-620
Frequency	14	9	5	4	3	5

Sol. We may prepare following table :

Class	Frequency(f_i)	Mid Value(x_i)	$u_i = \frac{x_i - 550}{20}$	$u_i \times f_i$
500—520	14	510	– 2	– 28
520—540	9	530	– 1	– 9
540—560	5	550 = A	0	0
560—580	4	570	1	4
580—600	3	590	2	6
600—620	5	610	3	15
	$\Sigma fi = 40$			$\Sigma u_i \times f_i = -12$

Thus, h = 20, A = 550, $\Sigma f_i = 40$, $\Sigma u_i \times f_i = -12$

$$\bar{x} = A + \left[h \times \frac{\Sigma u_i \times f_i}{\Sigma f_i}\right]$$

$$= 550 + \left[\frac{20 \times -12}{40}\right]$$

$$= 550 - 6$$

$$= 544.$$

23. Find the mean age from the following frequency distribution.

Age (in years)	25-29	30-34	35-39	40-44	45-49	50-54	55-59
No. of persons	4	14	22	16	6	5	3

Hints : Change the given series to the exclusive series.

Sol.

Age (in years)	**Frequency (f_i)**	**Class marks (x_i)**	$u_i = \frac{x_i - 42}{5}$	$u_i \times f_i$
24·5–29·5	4	27	– 3	– 12
29·5–34·5	14	32	– 2	– 28
34·5–39·5	22	37	– 1	– 22
39·5–44·5	16	42 = A	0	0
44·5–49·5	6	47	1	6
49·5–54·5	5	52	2	10
54·5–59·5	3	57	3	9
	N = 70			$\Sigma f_i u_i = -37$

Thus, $H = 5$, $A = 42$, $N = 70$, $\Sigma f_i \times u_i = -37$

$$\bar{x} = A + \left[h \times \frac{\Sigma f_i u_i}{N}\right]$$

$$= 42 + \left[5 \times \frac{-37}{70}\right] = 42 - 2.64 = 39{\cdot}36.$$

24. The following table shows the age distribution of patients of malaria in a village during a particular month.

Age (in years)	5-14	15-24	25-34	35-44	45-54	55-64
No. of Cases	6	11	21	23	14	5

Sol. We may prepare following table :

Age	**No. of cases (f_i)**	**Mid value (x_i)**	$u_i = \frac{x_i - 39{\cdot}5}{10}$	$u_i \times f_i$
4·5—14·5	6	9·5	– 3	– 18
14·5—24·5	11	19·5	– 2	– 22
24·5—34·5	21	29·5	– 1	– 21
34·5—44·5	23	39·5 = A	0	0
44·5—54·5	14	49·5	1	14
54·5—64·5	5	59·5	2	10
	$\Sigma f_i = 80$			$\Sigma f_i \times u_i = -37$

Thus $A = 39{\cdot}5$, $h = 10$, $\Sigma f_i = 80$, $\Sigma f_i \times x_i = -39$

$$\bar{x} = A + \left[h \times \frac{\Sigma f_i \times u_i}{\Sigma f_i}\right]$$

$$= 39{\cdot}5 \left[10 \times \frac{-37}{80}\right]$$

$$= 39{\cdot}5 - 4{\cdot}625$$

$$= 34{\cdot}875.$$

EXERCISE 14·2

Multiple Choice Type Questions

1. Median = ?

(a) $l + \left[\dfrac{h \times \left(\dfrac{N}{2} - cf\right)}{f}\right]$ (b) $l + \left[\dfrac{h \times \left(cf - \dfrac{N}{2}\right)}{f}\right]$

(c) $l - \left[\dfrac{h \times \left(\dfrac{N}{2} - cf\right)}{f}\right]$ (d) None of these.

Sol. Median $= l + \left[\dfrac{h \times \left(\dfrac{N}{2} - cf\right)}{f}\right]$

2. Look at the frequency distribution table given :

Class Interval	35-45	45-55	55-65	65-75
Frequency	8	12	20	10

The median of the above distribution is :

(a) 56·5 (b) 57·5 (c) 58·5 (d) 59

Sol.

Class	**Frequency (*f*)**	**Cumulative Frequency**
35—45	8	8
45—55	12	20
55—65	20	40
65—75	10	50
	N = 50	

Now, $N = 50 \Rightarrow N/2 = 50/2 \Rightarrow 25$

The cumulative frequency just greater than 25 is 40 and corresponding classes is 55-65

$\therefore \; l = 55, h = 10, f = 20, cf = 20$ and $N/2 = 25$

$$M_e = l + \left[\frac{h \times \left(\frac{N}{2} - cf\right)}{f}\right]$$

$$= 55 + \left[10 \times \frac{(25 - 20)}{20}\right]$$

$$= 55 + 2.5$$

$$= 57{\cdot}5$$

3. The following table shows the daily wages of workers in a factory:

Daily Wages (in ₹)	0-100	100-200	200-300	300-400	400-500
No. of Workers	40	32	48	22	8

Find the median daily wages income of the worker.

Sol.

Daily wages	**No. of workers (*h*)**	**Cumulative Frequency**
0—100	40	40
100—200	32	72
200—300	48	120
300—400	22	142
400—500	8	150
	$N = 150$	

Now, $N = 150$, $N/2 = 150/2 \Rightarrow 75$

The cumulative frequency just greater than 75 is 120 and corresponding classes is 200-300

$l = 200$, $h = 100$, $f = 48$, $cf = 72$ and $N/2 = 75$

$$M_e = l + \left[\frac{h \times \left(\frac{N}{2} - cf\right)}{f}\right]$$

$$M_e = 200 + \left[100 \times \frac{(75-72)}{48}\right]$$

$$M_e = 200 + 6{\cdot}25$$

$$M_e = 206.25.$$

4. Calculate the median from the following frequency distribution :

Class	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45
Frequency	5	6	15	10	5	4	2	2

Sol.

Class	**Frequency (f_i)**	**Cumulative Frequency**
5—10	5	5
10—15	6	11
15—20	15	26
20—25	10	36
25—30	5	41
30—35	4	45
35—40	2	47
40—45	2	49
	N = 49	

Now, $N = 49 \Rightarrow N/2 = 49/2 \Rightarrow 24{\cdot}5$

The cummulative frequency just greater than 24·5 is 26 and corresponding classes is 15-20.

$l = 15$, $h = 5$, $f = 15$, $c.f. = 11$ and $N/2 = 24{\cdot}5$

$$M_e = l + \left[\frac{h \times \left(\frac{N}{2} - cf\right)}{f}\right]$$

$$= 15 + \left[\frac{5 \times (24{\cdot}5 - 11)}{15}\right]$$

$$= 15 + 4{\cdot}5$$

$$= 19{\cdot}5.$$

5. Given below is the number of units of electricity consumed in a week in a certain locality :

Consumption (in units)	65-85	85-105	105-125	125-145	145-165	165-185	185-205
Number of consumers	4	5	13	20	14	7	4

Calculate the median.

Sol.

Consumption (in units)	**No. of Consumptions**	**Cumulative frequency**
65—85	4	4
85—105	5	9
105—125	13	22
125—145	20	42
145—165	14	56
165—185	7	63
185—205	4	67
	$N = 67$	

Now, $N = 67$ and $N/2 = 67/2 \Rightarrow 33{\cdot}5$

The cumulative frequency just greater than 33·5 is 42 and corresponding class is 125-145.

$l = 125$, $h = 20$, $f = 20$, $cf = 22$ and $N/2 = 33{\cdot}5$

$$M_e = l + \left[\frac{h \times \left(\frac{N}{2} - cf\right)}{f}\right]$$

$$= 125 + \left[\frac{20 \times (33{\cdot}5 - 22)}{20}\right]$$

$$= 125 + 11{\cdot}5$$

$$= 136{\cdot}5.$$

6. Calculate the median from the following data :

Height (in cm.)	135-140	140-145	145-150	150-155	155-160	160-165	165-170	170-175
No. of boys	6	10	18	22	20	15	6	3

Sol.

Height (in cm.)	No. of Boys	Cummulative Frequency
135—140	6	6
140—145	10	16
145—150	18	34
150—155	22	56
155—160	20	76
160—165	15	91
165—170	6	97
170—175	3	100
	N = 100	

Now, N = 100, and N/2 = 100/2 $\Rightarrow$ 50.

The cumulative frequency just greater than 50 is 56 and corresponding class is 150-155.

$l = 150$, $h = 5$, $f = 22$, $cf = 34$ and N/2 = 50

$$M_e = l + \left[\frac{h \times \left(\frac{N}{2} - cf\right)}{f}\right]$$

$$= 150 + \left[\frac{5 \times (50 - 34)}{22}\right]$$

$$= 150 + 3{\cdot}64$$

$$= 153{\cdot}64.$$

7. Calculate the missing frequency from the following distribution, it being given that the median of the distribution is 24.

Class	0—10	10—20	20—30	30—40	40—50
Frequency	5	25	?	18	7

Sol.

Class Interval	Frequency	Cumulative Frequency
0-10	5	5
10-20	25	30
20-30	x	$30 + x$
30-40	18	$48 + x$
40-50	7	$55 + x$
	$N = 55 + x$	

Given median = 24 lies between in the Class interval 20-30

Then median class is 20-30

So, we have $l = 20$, $h = 10$, $f = x$, $cf = 30$

$$N/2 = \frac{55 + x}{2}$$

$$M_e = l + \left[\frac{h \times \left(\frac{N}{2} - cf\right)}{f}\right]$$

$$24 = 20 + \left[\frac{10 \times \left(\frac{55+x}{2} - 30\right)}{x}\right]$$

$$x = 25.$$

8. The median value for the following frequency distribution is 35 and the sum of the all frequencies is 170. Using the formula for median, find the missing frequencies.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	10	20	?	40	?	25	15

Sol.

Class	**Frequency**	**Cumulative Frequency**
0—10	10	10
10—20	20	30
20—30	x	$30 + x$
30—40	40	$70 + x$
40—50	y	$70 + x + y$
50—60	25	$95 + x + y$
60—70	15	$110 + x + y$
	N = 170	

Given, median = 35 lies between the class interval 30-40

Then, median class is 30-40

$l = 30$, $h = 10$, $f = 40$, $c.f. = 30 + x$

N = 170 = N/2 = 170/2 = 85

$$\text{Me} = l + \left[\frac{h \times \left(\frac{N}{2} - cf\right)}{f}\right]$$

$$35 = 30 + \left[\frac{10 \times \{(85 - 30 + x)\}}{40}\right]$$

$$35 - 30 = \frac{10 \times (85 - 30 - x)}{40}$$

$$5 = \frac{55 - x}{4}$$

$$20 = 55 - x$$

$$x = 55 - 20$$

$$x = 35$$

Since $N = 170 = 110 + x + y$

$$170 = 110 + 35 + y$$

$$y = 170 - 110 - 35$$

$$y = 25.$$

9. If the median of the following frequency distribution is 32·5, find the values of f_1 and f_2 :

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
Frequency	f_i	5	9	12	f_2	3	2	40

Sol.

Class interval	Frequencies	Cumulative Frequency
0—10	$x = f_1$	x
10—20	5	$5 + x$
20—30	9	$14 + x$
30—40	12	$26 + x$
40—50	$y = f_2$	$26 + x + y$
50—60	3	$29 + x + y$
60—70	2	$31 + x + y$
	N = 40	

Given median = 32·5 lies between class interval 30-40

$l = 30$, $h = 10$, $f = 12$, $c.f. = 14 + x$, $N/2 = 20$

$$M_e = l + \left[\frac{h \times \left(\frac{N}{2} - cf\right)}{f}\right]$$

$$32{\cdot}5 = 30 + \left[\frac{10 \times 20 - \{(14 + x)\}}{12}\right]$$

$$32{\cdot}5 - 30 = \left[\frac{5(20 - 14 - x)}{6}\right]$$

$$\frac{6 \times 2{\cdot}5}{5} = 6 - x$$

$$3 = 6 - x$$

$$x = 6 - 3$$

$$x = 3$$

$$N = 40 = 31 + x + y$$

$$40 = 31 + 3 + y$$

$$y = 40 - 31 - 3$$

$$y = 6$$

$\therefore$ $f_1 = 3$ and $f_2 = 6$

10. Calculate the median for the following data :

Age (in years)	19-25	26-32	33-39	40-46	47-53	54-60
Frequency	35	96	68	102	35	4

Hint : Convert it to exclusive form.

Sol.

Age (in years)	Frequency	Cumulative Frequency
18·5—25·5	35	35
25·5—32·5	96	131
32·5—39·5	68	199
39·5—46·5	102	301
46·5—53·5	35	336
53·5—60·5	4	340
	$N = 340$	

Now, N = 340 and N/2 = 340/2 = 170

The cumulative frequency just greater than 170 is 199 and corresponding class is 32·5 – 39·5.

l = 32·5, h = 7, f = 68, cf = 131 & N/2 = 170

$$M_e = l + \left\{\frac{h \times (N/2 - cf)}{f}\right\}$$

$$= 32{\cdot}5 + \left\{\frac{7 \times (170 - 131)}{68}\right\}$$

$$= 32{\cdot}5 + \left\{\frac{7 \times 39}{68}\right\}$$

$$= 32{\cdot}5 + 4{\cdot}01$$

$$= 36{\cdot}51.$$

11. Find the median wages for the following frequency distribution :

Wages per day (in ₹)	61–70	71–80	81–90	91–100	101–110	111–120
No. of woman workers	5	15	20	30	20	8

Hint : Convert it to exclusive form

Sol.

Wages per day (in ₹)	**No. of Women Workers**	**Cumulative Frequency**
60·5—70·5	5	5
70·5—80·5	15	20
80·5—90·5	20	40
90·5—100·5	30	70
100·5—110·5	20	90
110·5—120·5	8	98
	N = 98	

N = 98 $\therefore$ N/2 = 98/2 = 49

The cumulative frequency just greater than 49 is 70. The corresponding class is 90·5 – 100·5

l = 90·5, h = 10, f = 30, cf = 40 and N/2 = 49

$$M_e = l + \left\{\frac{h \times (N/2 - cf)}{f}\right\}$$

$$= 90{\cdot}5 + \left\{\frac{10 \times (49 - 40)}{30}\right\}$$

$$= 90{\cdot}5 + \frac{9}{3}$$

$$= 90{\cdot}5 + 3 = 93{\cdot}5.$$

12. The following table gives the marks obtained by 50 students in a class test :

Marks	11–15	16–20	21–25	26–30	31–35	36–40	41–45	46–50
No. of Students	2	3	6	7	14	12	4	2

Hint : Convert it to exclusive form

Sol.

Marks	No. of Students	Cumulative Frequency
10·5–15·5	2	2
15·5–20·5	3	5
20·5–25·5	6	11
25·5–30·5	7	18
30·5–35·5	14	32
35·5–40·5	12	44
40·5–45·5	4	48
45·5–50·5	2	50
	$N = 50$	

$N = 50$ $\therefore$ $N/2 = 50/2 = 25$

The cumulative frequency just greater than 25 is 32. The corresponding class is 30·5–35·5

$l = 30{\cdot}5$, $h = 5$, $f = 14$, $cf = 18$ and N/2 = 25

$$M_e = l + \left\{\frac{h \times (N/2 - f)}{f}\right\}$$

$$= 30{\cdot}5 + \left\{\frac{5 \times (25 - 18)}{14}\right\}$$

$$= 30{\cdot}5 + \frac{5 \times 7}{14}$$

$$= 30{\cdot}5 + \frac{35}{14}$$

$$= 30{\cdot}5 + 2{\cdot}5 = 33.$$

13. Find the median from the following data :

Class	1–5	6–10	11–15	16–20	21–25	26–30	31–35	36–40	41–45
Frequency	7	10	16	32	24	16	11	5	2

Hint : Convert it to exclusive form

Sol.

Class	Frequency	Cumulative Frequency
0·5—5·5	7	7
5·5—10·5	10	17
10·5—15·5	16	33
15·5—20·5	32	65
20·5—25·5	24	89
25·5—30·5	16	105
30·5—35·5	11	116
35·5—40·5	5	121
40·5—45·5	2	123
	$N = 123$	

$N = 123$ $\therefore$ $N/2 = 123/2 = 61{\cdot}5$

The cummulative frequency just greater than 61·5 is 65. The corresponding class is 15·5–20·5

$l = 15{\cdot}5$, $h = 5$, $f = 32$, $cf = 33$ and $N/2 = 61{\cdot}5$

$$M_e = l + \left\{\frac{h \times (N/2 - cf)}{f}\right\}$$

$$= 15{\cdot}5 + \left\{\frac{5 \times (61{\cdot}5 - 33)}{32}\right\}$$

$$= 15{\cdot}5 + \frac{5 \times 28{\cdot}5}{32}$$

$$= 15{\cdot}5 + 4{\cdot}45 = 19{\cdot}95.$$

14. Find the median from the following data :

Marks	No. of Students
Below 10	12
Below 20	32
Below 30	57
Below 40	80
Below 50	92
Below 60	116
Below 70	164
Below 80	200

Sol.

Marks	No. of Students	Cumulative Frequency
0—10	12	12
10—20	(32—12) = 20	32
20—30	(57—32) = 25	57
30—40	(80—57) = 23	80
40—50	(92—80) = 12	92
50—60	(116—92) = 24	116
60—70	(164—116) = 48	164
70—80	(200—164) = 36	200
	N = 200	

N = 200 $\therefore$ N/2 = 200/2 = 100

The cumulative frequency just greather than 100 is 116. The corresponding class is 50 – 60

$l = 50$, $h = 10$, $f = 24$, $cf = 92$ and N/2 = 100

$$M_e = l + \left\{\frac{h \times (N/2 - cf)}{f}\right\}$$

$$= 50 + \left\{\frac{10 \times (100 - 92)}{24}\right\}$$

$$= 50 + \left\{\frac{10 \times 8}{24}\right\}$$

$$= 50 + 3{\cdot}33$$

$$= 53{\cdot}33.$$

EXERCISE 14.3

1. Consider the following table :

Class interval	10–14	14–18	18–22	22–26	26–30
Frequency	5	11	16	25	19

The mode of the above data is :
(a) 23·5 (b) 24 (c) 24·4 (d) 25

Sol. The class 22–26 has maximum frequency.
So, it is the model class
$\therefore\ x_k = 22, f_k = 25, f_{k-1} = 16, f_{k+1} = 19$ and $h = 4$

$$M_0 = x_k + \left\{\frac{h \times (f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$$

$$= 22 + \left\{4 \times \frac{(25-16)}{2 \times 25 - 16 - 19}\right\}$$

$$= 22 + 4 \times \frac{9}{50 - 16 - 19}$$

$$= 22 + \frac{36}{15}$$

$= 24{\cdot}4.$

2. The mean and mode of a frequency distribution are 28 and 16 respectively. The median is :
(a) 22 (b) 23·5 (c) 24 (d) 24·5

Sol.
Mode = (3 × Median) – (2 × Mean)
16 = (3 × Median) – (2 × 28)
16 = 3 × Median – 56

$$\text{Median} = \frac{16+56}{3} = \frac{72}{3} = 24.$$

3. The median and mode of a frequency distribution are 26 and 29 respectively. Then the mean is :
(a) 27·5 (b) 24·5 (c) 28·4 (d) 25·8

Sol. Mode = (3 × Median) – (2 × Mean)
29 = (3 × 26) – (2 × Mean)
29 = 78 – (2 × Mean)

$$\text{Mean} = \frac{78-29}{2} = \frac{49}{2} = 24{\cdot}5.$$

4. For a symmetrical frequency distribution, we have :
(a) Mean < Mode < Median (b) Mean > Mode > Median
(c) Mean = Mode = Median (d) Mode = $\frac{1}{2}$ (Mean + Median)

Sol. Symmetrical distribution occurs when the values of variables occur at regular frequencies and the mean,median and mode occur at same point
$\therefore$ Mean = Mode = Median

5. Find the mode of the marks obtained by 80 students in a class test in English as given below :

Marks	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
No. of Students	3	5	16	12	13	20	6	5

Sol. The class 50–60 has maximum frequency.
So, it is the model class
$x_k = 50, f_k = 20, f_{k-1} = 13, f_{k+1} = 6, h = 10$

$$M_0 = x_k + \left\{h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$$

$$= 50 + \left\{10 \times \frac{(20 - 13)}{(2 \times 20 - 13 - 6)}\right\}$$

$$= 50 + \left\{10 \times \frac{7}{21}\right\}$$

$$= 50 + 3{\cdot}33$$
$$= 53{\cdot}33.$$

6. Find the mode of ages of 181 workers of a factory from the following frequency distribution :

Age (in years)	20–30	30–40	40–50	50–60	60–70
No. of workers	25	47	62	37	10

Sol. The class 40–50 has maximum frequency.
So, it is a model class.
$\therefore\ x_k = 40,\ f_k = 62,\ f_{k-1} = 47,\ f_{k+1} = 37,\ h = 10$

$$M_0 = x_k + \left\{h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$$

$$= 40 + \left\{10 \times \frac{(62 - 47)}{(2 \times 62 - 47 - 37)}\right\}$$

$$= 40 + 10 \times \frac{15}{40}$$

$$= 40 + 3{\cdot}75$$
$$= 43{\cdot}75.$$

7. Find the mode of the following distribution :

Class Interval	10–14	14–18	18–22	22–26	26–30	30–34	34–38	38–42
Frequency	8	6	11	20	25	22	10	4

Sol. The class 26–30 has maximum frequency.
So, it is a model class.
$\therefore\ x_k = 26,\ f_k = 25,\ f_{k-1} = 20,\ f_{k+1} = 22,\ h = 4$

$$M_0 = x_k + \left\{h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$$

$$= 26 + \left\{4 \times \frac{(25 - 20)}{(2 \times 25 - 20 - 22)}\right\}$$

$$= 26 + \left\{4 \times \frac{5}{8}\right\}$$

$$= 26 + 2{\cdot}5 = 28{\cdot}5.$$

8. Calculate the mode from the following data :

Monthly Salary (in ₹)	*No. of Employees*
0–5000	90
5000–10000	150
10000–15000	100
15000–20000	80
20000–25000	70
25000–30000	10

Sol. The class 5000–10000 has maximum frequency.
So, it is a model class.
$\therefore\ x_k = 5000,\ f_k = 150,\ f_{k-1} = 90,\ f_{k+1} = 100,\ h = 5000$

$$M_0 = x_k + \left\{h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$$

$$M_0 = 5000 + \left\{5000 \times \frac{(150 - 90)}{(2 \times 150 - 90 - 100)}\right\}$$

$= 5000 + 2727{\cdot}27.$

$= 7727{\cdot}27.$

9. Compute the mode from the following data :

Age (in years)	0–5	5–10	10–15	15–20	20–25	25–30	30–35
No. of Patients	6	11	18	24	17	13	5

Sol. The class 15–20 has maximum frequency.
So, it is a model class.
$x_k = 15,\ f_k = 24,\ f_{k-1} = 18,\ f_{k+1} = 17,\ h = 5$

$$M_0 = x_k + \left\{h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$$

$$= 15 + \left\{5 \times \frac{(24 - 18)}{(2 \times 24 - 18 - 17)}\right\}$$

$= 15 + 2{\cdot}30 = 17{\cdot}30.$

10. Compute the mode from the following series :

Size	45–55	55–65	65–75	75–85	85–95	95–105	105–115
Frequency	7	12	17	30	32	6	10

Sol. The class 85–95 has maximum frequency.
So, it is a model class.
$x_k = 85,\ f_k = 32,\ f_{k-1} = 30,\ f_{k+1} = 6,\ h = 10$

$$M_0 = x_k + \left\{h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$$

$$= 85 + \left\{10 \times \frac{(32 - 30)}{(2 \times 32 - 30 - 6)}\right\}$$

$$= 85 + \left\{10 \times \frac{2}{28}\right\}$$

$= 85 + 0{\cdot}71 = 85{\cdot}71.$

EXERCISE 14.4

1. Draw a cumulative frequency curve (less than type) for the following data and find the median from it :

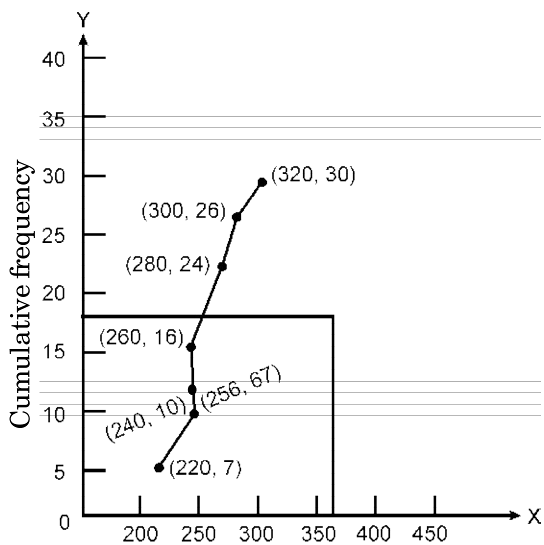
Class interval	200–220	220–240	240–260	260–280	280–300	300–320
Frequency	7	3	6	8	2	4

Sol.

Class Interval	**Frequency**	**Cumulative frequency less than type**	
200–220	7	Less than 220	7
220–240	3	Less than 240	(7 + 3) = 10
240–260	6	Less than 260	(10 + 6) = 16
260–280	8	Less than 280	(16 + 8) = 24
280–300	2	Less than 300	(24 + 2) = 26
300–320	4	Less than 320	(26 + 4) = 30
Total	N = 30		

∵ N = 30 ∴ N/2 = 30/2 = 15

On graph we plot the points (220, 7) (240, 10) (260, 16) (280, 24), (300, 26), (320, 30)



Median from the graph= 256·67

2. Following is the distribution of marks of 70 students in a periodical test :

Marks	**Number of Students**
Marks less than 10	3
Marks less than 20	11
Marks less than 30	28
Marks less than 40	48
Marks less than 50	70

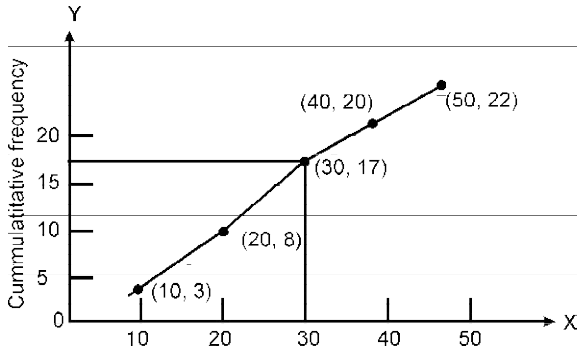
Draw a cumulative frequency curve for the above data and find the median.

Sol. We may prepare following table :

Marks	No. of Students (*cf*)	Frequency (*f*)
Less than 10	3	3
Less than 20	11	(11 – 3) = 8
Less than 30	28	(28 – 11) = 17
Less than 40	48	(48 – 28) = 20
Less than 50	70	(70 – 48) = 22

$$N = 70 \therefore \frac{N}{2} = \frac{70}{2} = 35$$

On graph we plot the points (10, 3) (20, 8) (30, 17) (40, 20) (50, 22)
Median as per graph = 33·5



Median as per graph = 33.5

3. **From the following frequency distribution, prepare the 'More than a give :**
ScoreNumber of candidates

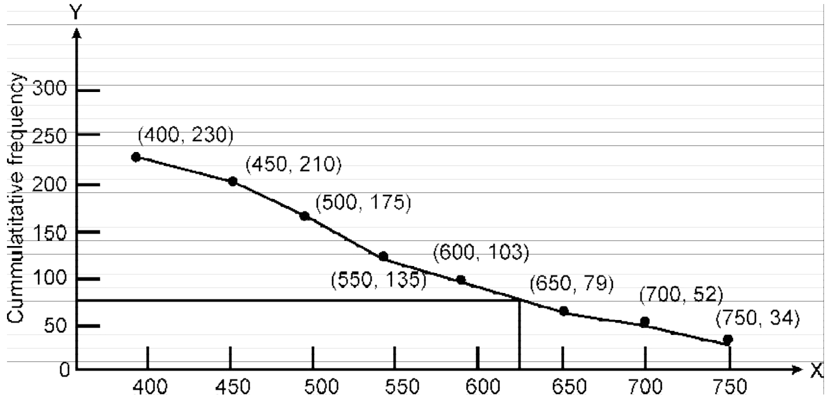
Score	Number of Candidtes
400–450	20
450–500	35
500–550	40
550–600	32
600–650	24
650–700	27
700–750	18
750–800	34
Total	230

Sol. We may prepare following table :

Score	Number of Candidate (*Cf*)	Cummulative Frequency	
400–450	20	More than 750	34
450–500	35	More than 700	(34 +18) 52
500–550	40	More than 650	(52 + 27) 79
550–600	32	More than 600	(79 + 24) 103
600–650	24	More than 550	(103 + 32) 135
650–700	27	More than 500	(135 + 40) 175
700–750	18	More than 450	(175 + 35) 210
750–800	34	More than 400	(210 + 20) 230
Total	230		

∴ N = 230 ∴ N/2 = 230/2 = 115

On the graph paper we plot the points (400, 230), (450, 210), (500, 175), (550, 135), (600, 103), (650, 79), (700, 52), (750, 34)



Median from the graph= 625

4. The marks obtained by 100 students of a class in an examination are given below :

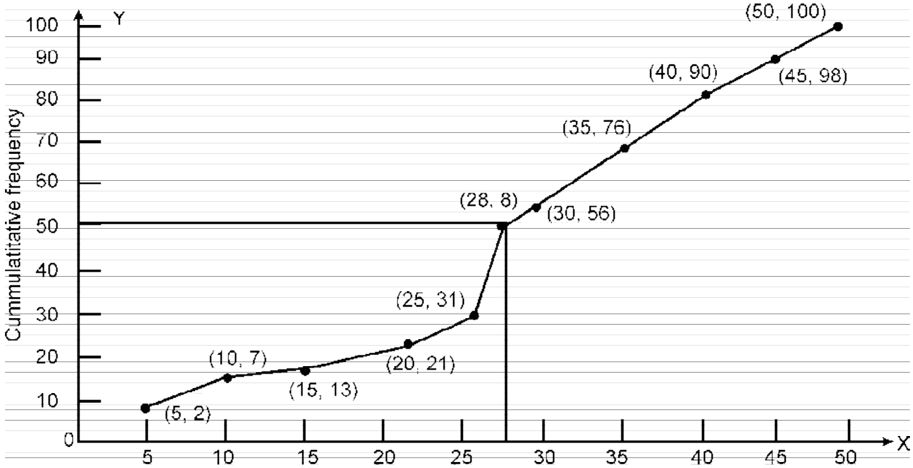
Marks	Number of contidates
0–5	2
5–10	5
10–15	6
15–20	8
20–25	10
25–30	25
30–35	20
35–40	18
40–45	4
45–50	2

Draw a cummulative frequency curves by (v) sing (i) less than series (ii) more than series. Find median.

Sol. (i)

Marks	Frequency	Cummulative Frequency	
0–5	2	Less than 5	2
5–10	5	Less than 10	7
10–15	6	Less than 15	13
15–20	8	Less than 20	21
20–25	10	Less than 25	31
25–30	25	Less than 30	56
30–35	20	Less than 35	76
35–40	18	Less than 40	94
40–45	4	Less than 45	98
45–50	2	Less than 50	100

We plot the points on graph is (5, 2), (10, 7), (15, 13), (20, 21), (25, 31), (30, 56), (35, 76), (40, 94), (45, 98), (50, 100).

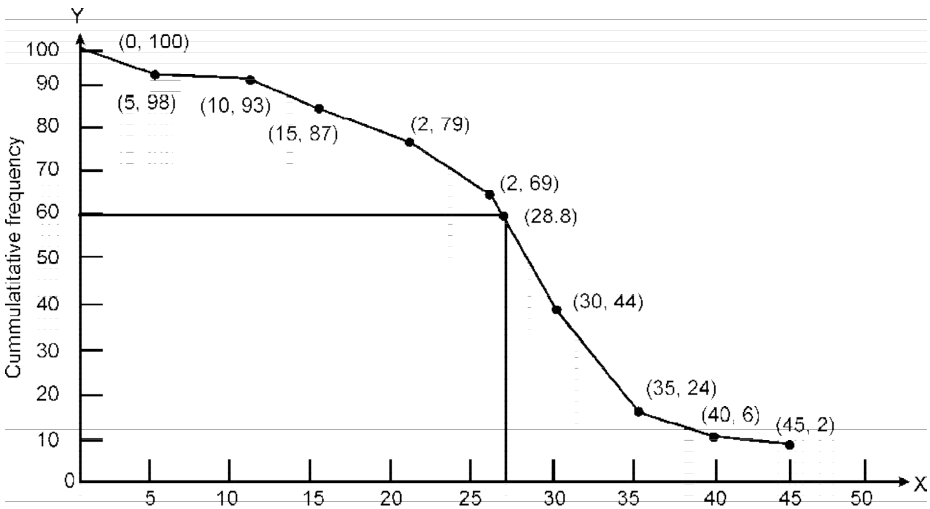


Median from the graph= 28·8

(ii)

Marks More than	Cummulative Frequency
More than 45	2
More than 40	(2 + 4) 6
More than 35	(6 + 18) 24
More than 30	(24 + 20) 44
More than 25	(44 + 25) 69
More than 20	(69 + 10) 79
More than 15	(79 + 8) 87
More than 10	(87 + 6) 93
More than 5 28.8	(93 + 5) 98
More than 0	(98 + 2) 100

On graph we plot (45, 2) (40, 6) (35, 24) (30, 44) (25, 69) (20, 79) (15, 87) (10, 93) (5, 98) (0, 100)



Median from the graph= 28·8

❏